Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.6-f^-a+b-x+c-x^2-trig-d+e-x+f-x^2-^n

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May 23, 2020 Compiled on May 23, 2020 at 8:21am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [142]. This is test number [140].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.59 (140)	% 1.41 (2)
Mathematica	% 100. (142)	% 0. (0)
Maple	% 80.28 (114)	% 19.72 (28)
Maxima	% 40.85 (58)	% 59.15 (84)
Fricas	% 80.99 (115)	% 19.01 (27)
Sympy	% 22.54 (32)	% 77.46 (110)
Giac	% 44.37 (63)	% 55.63 (79)

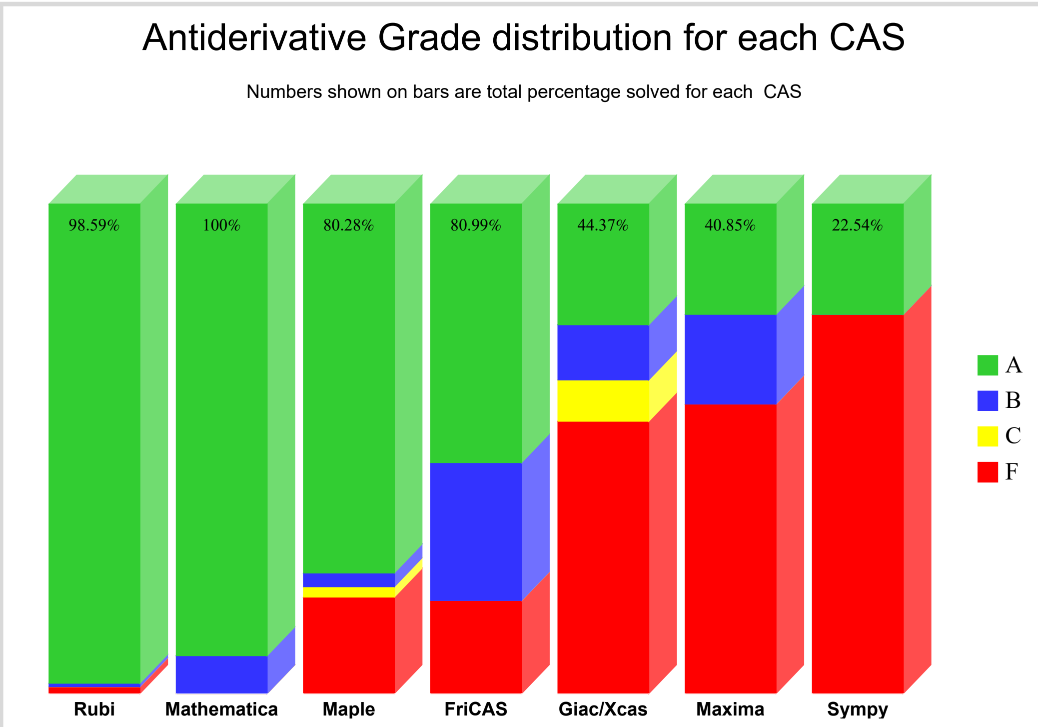
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

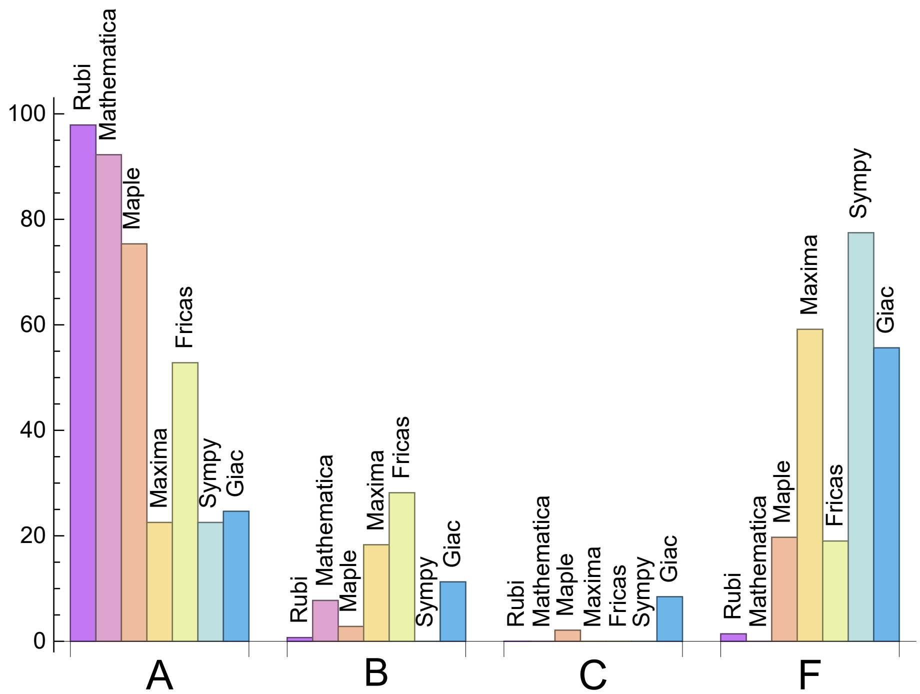
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	97.89	0.7	0.	1.41
Mathematica	92.25	7.75	0.	0.
Maple	75.35	2.82	2.11	19.72
Maxima	22.54	18.31	0.	59.15
Fricas	52.82	28.17	0.	19.01
Sympy	22.54	0.	0.	77.46
Giac	24.65	11.27	8.45	55.63

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	140.37	1.12	129.	1.
Mathematica	1.23	263.82	1.35	117.5	1.
Maple	0.14	150.25	1.67	136.	0.91
Maxima	1.19	310.55	4.88	56.	1.77
Fricas	0.5	528.05	3.34	409.	2.75
Sympy	29.06	228.66	2.31	30.5	1.09
Giac	1.22	768.81	20.	171.	1.49

1.4 list of integrals that has no closed form antiderivative

{29, 30}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {91, 92, 93, 98, 100, 101, 102, 103, 122, 123, 124, 129, 131, 132, 133, 134}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

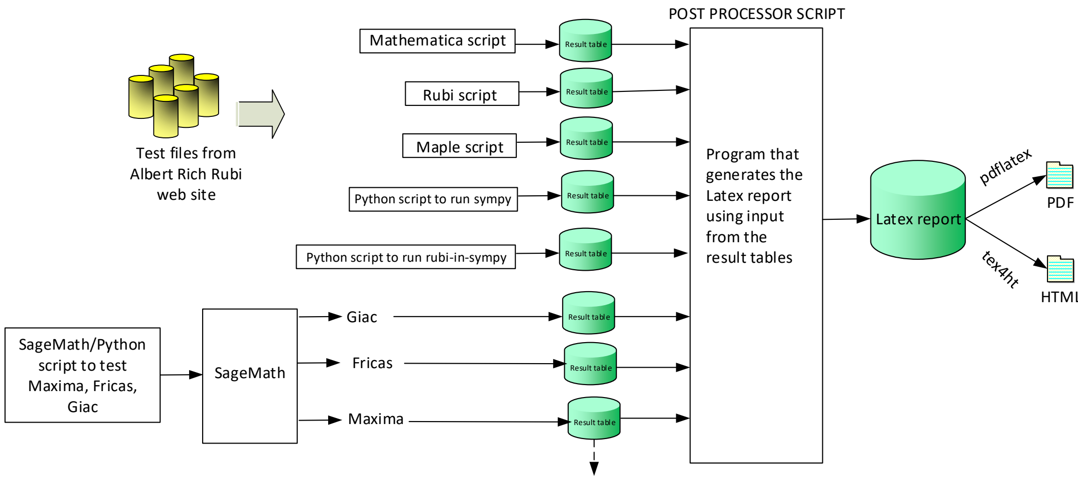
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade: { 34 }

C grade: { }

F grade: { 28, 32}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142 }

B grade: { 7, 21, 22, 63, 99, 101, 102, 124, 130, 132, 133 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 9, 11, 12, 13, 18, 29, 30, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140 }

B grade: { 34, 35, 51, 67 }

C grade: { 31, 32, 33 }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 53, 54, 55, 56, 137, 138, 141, 142 }

2.1.4 Maxima

A grade: { 9, 18, 29, 30, 31, 32, 33, 38, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 76, 104, 107

B grade: { 2, 3, 4, 11, 12, 13, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 63, 70, 74, 75, 105, 106, 135, 136, 139, 140 }

C grade: { }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 36, 37, 53, 54, 55, 56, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 9, 11, 12, 13, 18, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 76, 77, 78, 85, 86, 87, 88, 89, 94, 95, 96, 104, 107, 108, 109, 116, 117, 118, 119, 120, 125, 126, 127, 135, 136, 139, 140 }

B grade: { 63, 70, 74, 75, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 105, 106, 110, 111, 112, 113, 114, 115, 121, 122, 123, 124, 128, 129, 130, 131, 132, 133, 134 }

C grade: { }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 53, 54, 55, 56, 137, 138, 141, 142 }

2.1.6 Sympy

A grade: { 3, 4, 9, 12, 13, 18, 30, 34, 35, 38, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 69, 70, 71, 72, 73, 104, 135, 136, 139, 140 }

B grade: { }

C grade: { }

F grade: { 1, 2, 5, 6, 7, 8, 10, 11, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 63, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.1.7 Giac

A grade: { 9, 18, 29, 30, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 61, 62, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 104, 105, 106 }

B grade: { 31, 32, 63, 70, 79, 80, 81, 82, 83, 84, 110, 111, 112, 113, 114, 115 }

C grade: { 2, 3, 4, 11, 12, 13, 34, 35, 135, 136, 139, 140 }

F grade: { 1, 5, 6, 7, 8, 10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 36, 37, 53, 54, 55, 56, 57, 58, 59, 60, 76, 77, 78, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 141, 142 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.063	0.889	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	154	336	1098	386	0	1770
normalized size	1	1.	0.77	1.69	5.52	1.94	0.	8.89
time (sec)	N/A	0.07	0.685	0.138	1.338	0.508	0.	1.298

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	86	153	481	207	627	1260
normalized size	1	1.	0.67	1.2	3.76	1.62	4.9	9.84
time (sec)	N/A	0.052	0.212	0.049	1.201	0.492	159.735	1.255

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	130	262	115	326	880
normalized size	1	1.	0.66	1.78	3.59	1.58	4.47	12.05
time (sec)	N/A	0.016	0.116	0.015	1.08	0.481	70.008	1.226

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	114	0	0	0	0	0
normalized size	1	1.	1.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	1.775	0.043	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	101	0	0	0	0	0
normalized size	1	1.	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	1.501	0.083	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	334	0	0	0	0	0
normalized size	1	1.	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	7.891	0.117	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	173	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	2.987	0.134	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	33	34	50	115	70	47
normalized size	1	1.	0.61	0.63	0.93	2.13	1.3	0.87
time (sec)	N/A	0.026	0.038	0.01	1.029	0.463	6.416	1.148

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	110	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.055	0.498	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	155	274	1098	331	0	1764
normalized size	1	1.	0.78	1.38	5.52	1.66	0.	8.86
time (sec)	N/A	0.053	0.654	0.11	1.233	0.497	0.	1.32

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	85	153	481	189	627	1260
normalized size	1	1.	0.66	1.2	3.76	1.48	4.9	9.84
time (sec)	N/A	0.037	0.199	0.03	1.079	0.472	127.152	1.222

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	47	133	259	115	352	876
normalized size	1	1.	0.65	1.85	3.6	1.6	4.89	12.17
time (sec)	N/A	0.016	0.097	0.015	1.08	0.476	46.278	1.197

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.019	0.037	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.016	0.109	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.26	0.205	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	111	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.199	0.235	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	33	34	50	113	70	47
normalized size	1	1.	0.61	0.63	0.93	2.09	1.3	0.87
time (sec)	N/A	0.029	0.027	0.007	1.025	0.468	8.329	1.155

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	212	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	2.126	0.102	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	174	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	1.617	0.062	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	166	0	0	0	0	0
normalized size	1	1.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.469	0.046	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	163	0	0	0	0	0
normalized size	1	1.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.256	0.074	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	170	0	0	0	0	0
normalized size	1	1.	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	1.586	0.085	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	210	0	0	0	0	0
normalized size	1	1.	1.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	2.19	0.17	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	133	0	0	0	0	0
normalized size	1	1.	1.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.318	0.087	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	102	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.09	0.477	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.103	0.554	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	139	0	143	0	0	336	0	0
normalized size	1	0.	1.03	0.	0.	2.42	0.	0.
time (sec)	N/A	0.491	0.578	0.366	0.	0.52	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.623	6.411	0.052	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.951	10.633	0.071	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	213	43	65	0	8643
normalized size	1	1.	1.08	8.88	1.79	2.71	0.	360.12
time (sec)	N/A	3.975	1.332	0.251	2.297	0.501	0.	2.324

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	23	0	23	201	41	57	0	6483
normalized size	1	0.	1.	8.74	1.78	2.48	0.	281.87
time (sec)	N/A	2.34	0.898	0.161	2.132	0.495	0.	1.885

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	199	36	51	0	0
normalized size	1	1.	1.	9.05	1.64	2.32	0.	0.
time (sec)	N/A	2.563	0.864	0.15	2.146	0.494	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	327	17	682	1866	43	19	5310
normalized size	1	19.24	1.	40.12	109.76	2.53	1.12	312.35
time (sec)	N/A	0.766	0.387	0.079	1.445	0.471	31.327	1.416

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	268	529	41	17	1710
normalized size	1	1.	1.	16.75	33.06	2.56	1.06	106.88
time (sec)	N/A	0.029	0.027	0.015	1.149	0.473	3.959	1.284

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	40	0	43	0	0
normalized size	1	1.	0.95	2.	0.	2.15	0.	0.
time (sec)	N/A	1.732	0.606	0.069	0.	0.47	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	40	0	46	0	0
normalized size	1	1.	0.95	2.	0.	2.3	0.	0.
time (sec)	N/A	1.95	0.627	0.093	0.	0.471	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	60	59	130	342	74
normalized size	1	1.	0.7	0.95	0.94	2.06	5.43	1.17
time (sec)	N/A	0.047	0.153	0.014	1.045	0.471	70.24	1.111

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	108	726	243	0	132
normalized size	1	1.	0.62	0.91	6.1	2.04	0.	1.11
time (sec)	N/A	0.093	0.654	0.027	1.171	0.481	0.	1.114

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	82	118	743	302	0	150
normalized size	1	1.	0.64	0.91	5.76	2.34	0.	1.16
time (sec)	N/A	0.088	0.931	0.017	1.144	0.487	0.	1.2

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	74	108	726	224	0	135
normalized size	1	1.	0.62	0.91	6.1	1.88	0.	1.13
time (sec)	N/A	0.083	0.665	0.016	1.197	0.486	0.	1.153

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	71	319	208	0	89
normalized size	1	1.	0.72	0.9	4.04	2.63	0.	1.13
time (sec)	N/A	0.076	0.385	0.018	1.046	0.481	0.	1.155

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1550	454	0	209
normalized size	1	1.	0.6	0.91	8.47	2.48	0.	1.14
time (sec)	N/A	0.126	0.913	0.021	1.292	0.512	0.	1.158

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	81	118	743	262	0	150
normalized size	1	1.	0.63	0.91	5.76	2.03	0.	1.16
time (sec)	N/A	0.089	0.678	0.015	1.142	0.488	0.	1.119

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	110	166	1544	444	0	205
normalized size	1	1.	0.6	0.91	8.44	2.43	0.	1.12
time (sec)	N/A	0.125	0.764	0.028	1.287	0.511	0.	1.12

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	118	743	356	0	150
normalized size	1	1.	0.86	0.91	5.76	2.76	0.	1.16
time (sec)	N/A	0.101	0.949	0.022	1.153	0.494	0.	1.147

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	19	19	23	59	27	22
normalized size	1	1.	0.63	0.63	0.77	1.97	0.9	0.73
time (sec)	N/A	0.039	0.038	0.007	1.025	0.456	0.889	1.106

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	27	35	81	48	34
normalized size	1	1.	0.5	0.54	0.7	1.62	0.96	0.68
time (sec)	N/A	0.118	0.035	0.004	1.061	0.46	2.356	1.132

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	23	58	27	20
normalized size	1	1.	0.6	0.67	0.77	1.93	0.9	0.67
time (sec)	N/A	0.04	0.027	0.006	1.02	0.455	0.907	1.126

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	23	28	35	80	48	32
normalized size	1	1.	0.45	0.55	0.69	1.57	0.94	0.63
time (sec)	N/A	0.117	0.031	0.007	1.059	0.464	2.42	1.089

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	103	53	68	27	53
normalized size	1	1.	0.81	3.81	1.96	2.52	1.	1.96
time (sec)	N/A	0.08	0.091	0.017	1.106	0.451	0.328	1.14

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	30	31	84	32	31
normalized size	1	1.	0.8	0.73	0.76	2.05	0.78	0.76
time (sec)	N/A	0.025	0.065	0.007	1.035	0.463	0.6	1.159

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	2.569	0.187	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	2.572	0.175	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.666	0.064	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.627	0.093	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	43	42	50	88	0	0
normalized size	1	1.	0.62	0.61	0.72	1.28	0.	0.
time (sec)	N/A	0.053	0.027	0.099	1.172	0.463	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	44	51	95	0	0
normalized size	1	1.	0.72	0.68	0.78	1.46	0.	0.
time (sec)	N/A	0.047	0.02	0.039	1.026	0.474	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	69	122	0	0
normalized size	1	1.	1.	0.64	0.85	1.51	0.	0.
time (sec)	N/A	0.07	0.08	0.041	1.056	0.474	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	70	127	0	0
normalized size	1	1.	1.06	0.7	0.91	1.65	0.	0.
time (sec)	N/A	0.051	0.081	0.04	1.059	0.477	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	30	39	72	29	28
normalized size	1	1.	0.69	0.86	1.11	2.06	0.83	0.8
time (sec)	N/A	0.077	0.037	0.011	1.007	0.461	13.765	1.141

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	15	5	7
normalized size	1	1.	1.	1.	1.17	2.5	0.83	1.17
time (sec)	N/A	0.008	0.009	0.003	0.98	0.465	0.299	1.165

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	21	5	26	74	0	27
normalized size	1	1.	4.2	1.	5.2	14.8	0.	5.4
time (sec)	N/A	0.022	0.018	0.022	1.073	0.476	0.	1.114

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	14	3	4
normalized size	1	1.	1.	1.	1.	3.5	0.75	1.
time (sec)	N/A	0.008	0.009	0.007	0.984	0.458	0.496	1.133

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	24	7	9
normalized size	1	1.	1.	0.8	0.9	2.4	0.7	0.9
time (sec)	N/A	0.011	0.01	0.005	1.001	0.457	0.502	1.117

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	27	10	9
normalized size	1	1.	1.	0.8	0.9	2.7	1.	0.9
time (sec)	N/A	0.01	0.011	0.007	0.999	0.464	0.492	1.095

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	24	14	34	12	14
normalized size	1	1.	1.	1.85	1.08	2.62	0.92	1.08
time (sec)	N/A	0.017	0.017	0.02	1.01	0.476	11.969	1.129

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	140	0	32
normalized size	1	1.	1.	0.83	1.07	4.67	0.	1.07
time (sec)	N/A	0.036	0.061	0.051	0.987	0.483	0.	1.122

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	5	41	10	9
normalized size	1	1.	1.	1.	0.71	5.86	1.43	1.29
time (sec)	N/A	0.01	0.008	0.002	0.995	0.474	0.643	1.202

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	9	11	65	10	39
normalized size	1	1.	1.	1.8	2.2	13.	2.	7.8
time (sec)	N/A	0.01	0.005	0.002	1.006	0.482	3.565	1.185

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	7	16	3	7
normalized size	1	1.	1.	1.	1.75	4.	0.75	1.75
time (sec)	N/A	0.02	0.01	0.008	1.012	0.465	1.06	1.186

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	9	27	5	9
normalized size	1	1.	1.	1.	1.5	4.5	0.83	1.5
time (sec)	N/A	0.017	0.017	0.054	1.031	0.439	3.251	1.277

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	27	36	38	70	114	47
normalized size	1	1.	0.73	0.97	1.03	1.89	3.08	1.27
time (sec)	N/A	0.013	0.058	0.003	0.988	0.463	1.492	1.281

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	108	88	377	549	0	171
normalized size	1	1.	0.94	0.77	3.28	4.77	0.	1.49
time (sec)	N/A	0.123	0.157	0.124	1.929	0.493	0.	1.14

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	134	117	417	647	0	198
normalized size	1	1.	0.93	0.81	2.9	4.49	0.	1.38
time (sec)	N/A	0.216	0.256	0.109	2.083	0.501	0.	1.169

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	52	69	122	0	0
normalized size	1	1.	1.	0.64	0.85	1.51	0.	0.
time (sec)	N/A	0.055	0.023	0.	1.032	0.471	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	129	62	0	192	0	0
normalized size	1	1.	1.48	0.71	0.	2.21	0.	0.
time (sec)	N/A	0.098	0.218	0.063	0.	0.483	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	165	127	0	427	0	0
normalized size	1	1.	1.06	0.82	0.	2.75	0.	0.
time (sec)	N/A	0.202	0.562	0.208	0.	0.5	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	132	116	0	749	0	405
normalized size	1	1.	0.93	0.82	0.	5.27	0.	2.85
time (sec)	N/A	0.21	0.229	0.267	0.	0.505	0.	1.288

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	156	139	0	770	0	703
normalized size	1	1.	0.99	0.89	0.	4.9	0.	4.48
time (sec)	N/A	0.203	1.072	0.374	0.	0.517	0.	1.388

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	268	239	0	1519	0	803
normalized size	1	1.	0.9	0.8	0.	5.1	0.	2.69
time (sec)	N/A	0.367	0.885	0.481	0.	0.546	0.	1.465

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	152	0	868	0	518
normalized size	1	1.	1.	0.94	0.	5.36	0.	3.2
time (sec)	N/A	0.334	0.385	0.195	0.	0.521	0.	1.317

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	244	175	0	910	0	817
normalized size	1	1.	1.36	0.98	0.	5.08	0.	4.56
time (sec)	N/A	0.352	1.123	0.34	0.	0.519	0.	1.354

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	323	311	0	1773	0	1030
normalized size	1	1.	0.95	0.91	0.	5.21	0.	3.03
time (sec)	N/A	0.602	1.546	0.486	0.	0.556	0.	1.559

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	119	123	0	413	0	0
normalized size	1	1.	0.79	0.81	0.	2.74	0.	0.
time (sec)	N/A	0.207	0.153	0.203	0.	0.497	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	132	145	0	452	0	0
normalized size	1	1.	0.77	0.85	0.	2.64	0.	0.
time (sec)	N/A	0.218	0.25	0.2	0.	0.498	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	224	246	0	826	0	0
normalized size	1	1.	0.74	0.82	0.	2.74	0.	0.
time (sec)	N/A	0.346	0.43	0.335	0.	0.516	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	170	84	0	301	0	0
normalized size	1	1.	1.59	0.79	0.	2.81	0.	0.
time (sec)	N/A	0.198	0.463	0.129	0.	0.499	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	188	107	0	479	0	0
normalized size	1	1.	1.34	0.76	0.	3.42	0.	0.
time (sec)	N/A	0.228	0.789	0.15	0.	0.505	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	386	166	0	871	0	0
normalized size	1	1.	1.81	0.78	0.	4.09	0.	0.
time (sec)	N/A	0.332	2.288	0.271	0.	0.55	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	216	169	0	765	0	0
normalized size	1	1.	1.16	0.9	0.	4.09	0.	0.
time (sec)	N/A	0.367	0.967	0.379	0.	0.549	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	251	191	0	927	0	0
normalized size	1	1.	1.19	0.91	0.	4.39	0.	0.
time (sec)	N/A	0.419	2.253	0.39	0.	0.557	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	377	377	490	338	0	1840	0	0
normalized size	1	1.	1.3	0.9	0.	4.88	0.	0.
time (sec)	N/A	0.655	6.619	0.613	0.	0.668	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	170	0	487	0	0
normalized size	1	1.	0.88	0.97	0.	2.77	0.	0.
time (sec)	N/A	0.335	0.332	0.234	0.	0.498	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	217	0	617	0	0
normalized size	1	1.	0.88	0.94	0.	2.67	0.	0.
time (sec)	N/A	0.383	0.716	0.414	0.	0.506	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	391	338	0	973	0	0
normalized size	1	1.	1.1	0.95	0.	2.75	0.	0.
time (sec)	N/A	0.49	1.004	0.453	0.	0.526	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	230	180	0	778	0	0
normalized size	1	1.	1.19	0.93	0.	4.03	0.	0.
time (sec)	N/A	0.385	0.968	0.378	0.	0.544	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	299	227	0	1025	0	0
normalized size	1	1.	1.22	0.93	0.	4.18	0.	0.
time (sec)	N/A	0.462	3.09	0.424	0.	0.56	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	3291	358	0	1854	0	0
normalized size	1	1.	8.53	0.93	0.	4.8	0.	0.
time (sec)	N/A	0.573	7.012	0.629	0.	0.672	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	347	216	0	938	0	0
normalized size	1	1.	1.64	1.02	0.	4.42	0.	0.
time (sec)	N/A	0.566	2.181	0.289	0.	0.565	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	1120	263	0	1204	0	0
normalized size	1	1.	4.18	0.98	0.	4.49	0.	0.
time (sec)	N/A	0.646	6.725	0.362	0.	0.575	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	3835	430	0	2196	0	0
normalized size	1	1.	8.92	1.	0.	5.11	0.	0.
time (sec)	N/A	0.911	7.248	0.611	0.	0.698	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	324	218	0	938	0	0
normalized size	1	1.	1.52	1.02	0.	4.4	0.	0.
time (sec)	N/A	0.795	1.853	0.454	0.	0.558	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	35	34	69	114	45
normalized size	1	1.	0.72	0.97	0.94	1.92	3.17	1.25
time (sec)	N/A	0.012	0.05	0.008	1.094	0.461	2.475	1.137

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	86	378	549	0	171
normalized size	1	1.	0.95	0.75	3.29	4.77	0.	1.49
time (sec)	N/A	0.099	0.16	0.054	2.107	0.491	0.	1.143

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	135	115	420	647	0	198
normalized size	1	1.	0.94	0.8	2.92	4.49	0.	1.38
time (sec)	N/A	0.169	0.254	0.05	2.361	0.5	0.	1.152

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	82	54	70	127	0	0
normalized size	1	1.	1.06	0.7	0.91	1.65	0.	0.
time (sec)	N/A	0.057	0.083	0.002	1.163	0.475	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	107	60	0	198	0	0
normalized size	1	1.	1.29	0.72	0.	2.39	0.	0.
time (sec)	N/A	0.079	0.194	0.043	0.	0.479	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	166	125	0	431	0	0
normalized size	1	1.	1.1	0.83	0.	2.85	0.	0.
time (sec)	N/A	0.172	0.577	0.066	0.	0.503	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	133	114	0	749	0	405
normalized size	1	1.	0.94	0.8	0.	5.27	0.	2.85
time (sec)	N/A	0.169	0.239	0.066	0.	0.511	0.	1.302

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	158	139	0	768	0	703
normalized size	1	1.	1.01	0.89	0.	4.89	0.	4.48
time (sec)	N/A	0.173	1.092	0.109	0.	0.516	0.	1.347

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	267	235	0	1517	0	803
normalized size	1	1.	0.9	0.79	0.	5.09	0.	2.69
time (sec)	N/A	0.33	0.899	0.217	0.	0.56	0.	1.469

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	163	150	0	868	0	518
normalized size	1	1.	1.01	0.93	0.	5.36	0.	3.2
time (sec)	N/A	0.255	0.39	0.071	0.	0.512	0.	1.326

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	245	175	0	909	0	817
normalized size	1	1.	1.37	0.98	0.	5.08	0.	4.56
time (sec)	N/A	0.284	1.143	0.14	0.	0.522	0.	1.38

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	322	307	0	1771	0	1030
normalized size	1	1.	0.95	0.9	0.	5.21	0.	3.03
time (sec)	N/A	0.52	1.611	0.252	0.	0.559	0.	1.607

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	116	121	0	409	0	0
normalized size	1	1.	0.79	0.82	0.	2.78	0.	0.
time (sec)	N/A	0.168	0.154	0.066	0.	0.492	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	131	145	0	452	0	0
normalized size	1	1.	0.77	0.85	0.	2.64	0.	0.
time (sec)	N/A	0.2	0.257	0.092	0.	0.497	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	218	242	0	815	0	0
normalized size	1	1.	0.74	0.83	0.	2.78	0.	0.
time (sec)	N/A	0.331	0.422	0.202	0.	0.528	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	170	82	0	301	0	0
normalized size	1	1.	1.65	0.8	0.	2.92	0.	0.
time (sec)	N/A	0.154	0.477	0.063	0.	0.504	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	189	107	0	479	0	0
normalized size	1	1.	1.35	0.76	0.	3.42	0.	0.
time (sec)	N/A	0.188	0.827	0.082	0.	0.52	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	389	162	0	869	0	0
normalized size	1	1.	1.9	0.79	0.	4.24	0.	0.
time (sec)	N/A	0.288	2.21	0.195	0.	0.548	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	217	167	0	765	0	0
normalized size	1	1.	1.19	0.91	0.	4.18	0.	0.
time (sec)	N/A	0.304	0.942	0.09	0.	0.548	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	252	191	0	927	0	0
normalized size	1	1.	1.19	0.91	0.	4.39	0.	0.
time (sec)	N/A	0.358	2.298	0.138	0.	0.56	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	369	369	2997	334	0	1839	0	0
normalized size	1	1.	8.12	0.91	0.	4.98	0.	0.
time (sec)	N/A	0.583	7.018	0.31	0.	0.662	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	151	168	0	483	0	0
normalized size	1	1.	0.88	0.98	0.	2.81	0.	0.
time (sec)	N/A	0.258	0.327	0.07	0.	0.496	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	204	217	0	618	0	0
normalized size	1	1.	0.88	0.94	0.	2.68	0.	0.
time (sec)	N/A	0.282	0.639	0.106	0.	0.508	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	386	334	0	964	0	0
normalized size	1	1.	1.12	0.97	0.	2.79	0.	0.
time (sec)	N/A	0.416	0.967	0.237	0.	0.521	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	231	178	0	778	0	0
normalized size	1	1.	1.22	0.94	0.	4.12	0.	0.
time (sec)	N/A	0.265	0.983	0.089	0.	0.548	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	301	227	0	1026	0	0
normalized size	1	1.	1.23	0.93	0.	4.19	0.	0.
time (sec)	N/A	0.406	3.096	0.137	0.	0.56	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	3285	354	0	1852	0	0
normalized size	1	1.	8.69	0.94	0.	4.9	0.	0.
time (sec)	N/A	0.522	6.973	0.316	0.	0.66	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	348	214	0	938	0	0
normalized size	1	1.	1.67	1.03	0.	4.51	0.	0.
time (sec)	N/A	0.395	2.126	0.1	0.	0.561	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	1118	263	0	1206	0	0
normalized size	1	1.	4.17	0.98	0.	4.5	0.	0.
time (sec)	N/A	0.461	6.735	0.176	0.	0.575	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	3829	426	0	2195	0	0
normalized size	1	1.	9.07	1.01	0.	5.2	0.	0.
time (sec)	N/A	0.665	7.183	0.422	0.	0.695	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	325	216	0	938	0	0
normalized size	1	1.	1.56	1.03	0.	4.49	0.	0.
time (sec)	N/A	0.482	1.796	0.099	0.	0.554	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	180	368	784	581	1760	2395
normalized size	1	1.	0.73	1.5	3.2	2.37	7.18	9.78
time (sec)	N/A	0.358	1.604	0.099	1.184	0.511	162.303	1.454

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	183	294	197	408	1270
normalized size	1	1.	0.84	1.85	2.97	1.99	4.12	12.83
time (sec)	N/A	0.159	0.578	0.014	1.059	0.487	12.888	1.263

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	128	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	1.699	0.119	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	240	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	3.23	0.373	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	228	371	780	536	1760	2392
normalized size	1	1.	0.93	1.51	3.18	2.19	7.18	9.76
time (sec)	N/A	0.326	0.629	0.076	1.288	0.499	169.745	1.388

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	82	186	292	188	408	1266
normalized size	1	1.	0.84	1.9	2.98	1.92	4.16	12.92
time (sec)	N/A	0.148	0.235	0.012	1.135	0.479	14.178	1.255

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	80	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.05	0.056	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.393	0.114	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [48] had the largest ratio of [0.5556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.	18	0.111
2	A	2	2	1.	18	0.111
3	A	2	2	1.	18	0.111
4	A	1	1	1.	16	0.062
5	A	1	1	1.	16	0.062
6	A	1	1	1.	18	0.056
7	A	2	2	1.	18	0.111
8	A	2	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	3	2	1.	8	0.25
10	A	2	2	1.	18	0.111
11	A	2	2	1.	18	0.111
12	A	2	2	1.	18	0.111
13	A	1	1	1.	16	0.062
14	A	1	1	1.	16	0.062
15	A	1	1	1.	18	0.056
16	A	2	2	1.	18	0.111
17	A	2	2	1.	18	0.111
18	A	3	2	1.	8	0.25
19	A	6	3	1.	18	0.167
20	A	5	3	1.	18	0.167
21	A	4	3	1.	16	0.188
22	A	4	3	1.	16	0.188
23	A	5	3	1.	18	0.167
24	A	6	3	1.	18	0.167
25	A	5	4	1.	27	0.148
26	A	2	2	1.	18	0.111
27	A	2	2	1.	18	0.111
28	F	0	0	N/A	0	N/A
29	A	0	0	0.	0	0.
30	A	0	0	0.	0	0.
31	A	11	5	1.	44	0.114
32	F	0	0	N/A	0	N/A
33	A	7	4	1.	43	0.093
34	B	14	6	19.24	35	0.171
35	A	1	1	1.	30	0.033
36	A	6	3	1.	38	0.079
37	A	10	4	1.	38	0.105
38	A	3	3	1.	20	0.15
39	A	4	2	1.	22	0.091
40	A	4	2	1.	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	4	2	1.	22	0.091
42	A	4	3	1.	24	0.125
43	A	5	2	1.	24	0.083
44	A	4	2	1.	22	0.091
45	A	5	2	1.	24	0.083
46	A	4	2	1.	24	0.083
47	A	4	3	1.	7	0.429
48	A	11	5	1.	9	0.556
49	A	4	3	1.	7	0.429
50	A	11	5	1.	9	0.556
51	A	4	3	1.	19	0.158
52	A	3	1	1.	15	0.067
53	A	5	4	1.	26	0.154
54	A	5	4	1.	27	0.148
55	A	5	4	1.	26	0.154
56	A	5	4	1.	27	0.148
57	A	6	3	1.	10	0.3
58	A	6	3	1.	10	0.3
59	A	6	3	1.	12	0.25
60	A	6	3	1.	12	0.25
61	A	2	2	1.	15	0.133
62	A	2	2	1.	8	0.25
63	A	3	3	1.	12	0.25
64	A	2	2	1.	8	0.25
65	A	2	2	1.	12	0.167
66	A	2	2	1.	12	0.167
67	A	3	3	1.	10	0.3
68	A	4	3	1.	26	0.115
69	A	2	2	1.	8	0.25
70	A	2	2	1.	8	0.25
71	A	3	3	1.	12	0.25
72	A	3	3	1.	10	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	1	1	1.	10	0.1
74	A	6	4	1.	12	0.333
75	A	6	4	1.	15	0.267
76	A	6	3	1.	12	0.25
77	A	4	2	1.	14	0.143
78	A	6	3	1.	17	0.176
79	A	8	5	1.	16	0.312
80	A	9	6	1.	18	0.333
81	A	14	5	1.	18	0.278
82	A	8	5	1.	19	0.263
83	A	9	6	1.	21	0.286
84	A	14	5	1.	21	0.238
85	A	8	4	1.	16	0.25
86	A	9	4	1.	18	0.222
87	A	14	4	1.	18	0.222
88	A	6	4	1.	18	0.222
89	A	7	4	1.	20	0.2
90	A	10	4	1.	20	0.2
91	A	8	5	1.	21	0.238
92	A	9	5	1.	23	0.217
93	A	14	5	1.	23	0.217
94	A	8	4	1.	19	0.21
95	A	10	4	1.	21	0.19
96	A	14	4	1.	21	0.19
97	A	8	5	1.	21	0.238
98	A	10	5	1.	23	0.217
99	A	14	5	1.	23	0.217
100	A	8	5	1.	24	0.208
101	A	10	5	1.	26	0.192
102	A	14	5	1.	26	0.192
103	A	8	5	1.	24	0.208
104	A	1	1	1.	10	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	6	4	1.	12	0.333
106	A	6	4	1.	15	0.267
107	A	6	3	1.	12	0.25
108	A	4	2	1.	14	0.143
109	A	6	3	1.	17	0.176
110	A	8	5	1.	16	0.312
111	A	9	6	1.	18	0.333
112	A	14	5	1.	18	0.278
113	A	8	5	1.	19	0.263
114	A	9	6	1.	21	0.286
115	A	14	5	1.	21	0.238
116	A	8	4	1.	16	0.25
117	A	9	4	1.	18	0.222
118	A	14	4	1.	18	0.222
119	A	6	4	1.	18	0.222
120	A	7	4	1.	20	0.2
121	A	10	4	1.	20	0.2
122	A	8	5	1.	21	0.238
123	A	9	5	1.	23	0.217
124	A	14	5	1.	23	0.217
125	A	8	4	1.	19	0.21
126	A	10	4	1.	21	0.19
127	A	14	4	1.	21	0.19
128	A	8	5	1.	21	0.238
129	A	10	5	1.	23	0.217
130	A	14	5	1.	23	0.217
131	A	8	5	1.	24	0.208
132	A	10	5	1.	26	0.192
133	A	14	5	1.	26	0.192
134	A	8	5	1.	24	0.208
135	A	8	6	1.	22	0.273
136	A	6	5	1.	20	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	2	2	1.	22	0.091
138	A	3	3	1.	22	0.136
139	A	8	6	1.	22	0.273
140	A	6	5	1.	20	0.25
141	A	2	2	1.	22	0.091
142	A	3	3	1.	22	0.136

Chapter 3

Listing of integrals

3.1 $\int F^{c(a+bx)} \sin^n(d+ex)\,dx$

Optimal. Leaf size=107

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \sin^n(d+ex)\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-\frac{ibc\log(F)}{e}-n+2\right),e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -(e*n + I*b*c*Log[F])/(2*e), (2 -
n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)/((1 - E^((2*I
)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rubi [A] time = 0.13809, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4440, 2259}

$$-\frac{\left(1-e^{2i(d+ex)}\right)^{-n} F^{c(a+bx)} \sin^n(d+ex)\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right);e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

```
[Out] -((F^(c*(a + b*x))*Hypergeometric2F1[-n, -(e*n + I*b*c*Log[F])/(2*e), (2 -
n - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)/((1 - E^((2*I
)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rule 4440

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Dist[(E^(I*n*(d + e*x))*Sin[d + e*x]^n)/(-1 + E^(2*I*(d + e*x)))^n, I
nt[(F^(c*(a + b*x))*(-1 + E^(2*I*(d + e*x)))^n)/E^(I*n*(d + e*x)), x], x] /
; FreeQ[{F, a, b, c, d, e, n}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\sin^n(d+ex)\,dx=\left(e^{in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^{-n}\sin^n(d+ex)\right)\int e^{-in(d+ex)}\left(-1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}\,dx$$
$$=-\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)\sin^n(d+ex)}{ien-bc\log(F)}$$

Mathematica [A] time = 0.0633474, size = 110, normalized size = 1.03

$$\frac{\left(1-e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\sin^n(d+ex)\text{Hypergeometric2F1}\left(-n,-\frac{i(bc\log(F)-ien)}{2e},1-\frac{i(bc\log(F)-ien)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)-ien}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Hypergeometric2F1[-n, ((-I/2)*((-I)*e*n + b*c*Log[F]))/e,
1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, E^((2*I)*(d + e*x))]*Sin[d + e*x]^n)
/((1 - E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F] time = 0.889, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\sin\left(ex+d\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sin(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c} \sin(ex+d)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac} \sin(ex+d)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sin(e*x + d)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sin(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sin(e*x + d)^n, x)
```

3.2 $\int F^{c(a+bx)} \sin^3(d+ex)\,dx$

Optimal. Leaf size=199

$$\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\sin(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{6e^3\cos(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{3e\sin^2}{}$$

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cos[d + e*x])/(9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^
4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(9*e^4 +
10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) - (3*e*F^(c*(a + b*x))*Cos[d +
e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(c*(a + b*x))*Log[
F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0698005, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4434, 4432}

$$\frac{bc\log(F)\sin^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\sin(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{6e^3\cos(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}-\frac{3e\sin^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```


```
[Out] (-6*e^3*F^(c*(a + b*x))*Cos[d + e*x])/(9*e^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^
4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sin[d + e*x])/(9*e^4 +
10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) - (3*e*F^(c*(a + b*x))*Cos[d +
e*x]*Sin[d + e*x]^2)/(9*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(c*(a + b*x))*Log[
F]*Sin[d + e*x]^3)/(9*e^2 + b^2*c^2*Log[F]^2)
```


Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sin^3(d+ex)\,dx = -\frac{3eF^{c(a+bx)}\cos(d+ex)\sin^2(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^3(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{\left(6e^2\right)\int F^{c(a+bx)}\sin(d}{9e^2+b^2c^2\log^2(}$$

$$= -\frac{6e^3F^{c(a+bx)}\cos(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sin(d+ex)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} - \frac{3eF^{c(a+bx)}}{9e}$$

Mathematica [A] time = 0.68481, size = 154, normalized size = 0.77

$$\frac{F^{c(a+bx)}\left(-3e\cos(d+ex)\left(b^2c^2\log^2(F)+9e^2\right)+3\cos(3(d+ex))\left(b^2c^2e\log^2(F)+e^3\right)-2bc\log(F)\sin(d+ex)\left(\cos(2(d+e\right.\right.}{4\left(10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-3*e*Cos[d + e*x]*(9*e^2 + b^2*c^2*Log[F]^2) + 3*Cos[3*(d
 + e*x)]*(e^3 + b^2*c^2*e*Log[F]^2) - 2*b*c*Log[F]*(-13*e^2 - b^2*c^2*Log[F
]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] time = 0.138, size = 336, normalized size = 1.7

$$-\frac{3F^{ac}e\mathrm{e}^{bcx\ln(F)}}{4e^2+4b^2c^2(\ln(F))^2}\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)^{-1} + \frac{3F^{ac}e\mathrm{e}^{bcx\ln(F)}}{4e^2+4b^2c^2(\ln(F))^2}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)^{-1} + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^3,x)
```

```
[Out] -3/4*F^(a*c)/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(
F))+3/4*F^(a*c)/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*
```

```
ln(F))*tan(1/2*d+1/2*e*x)^2+3/2*F^(a*c)/(1+tan(1/2*d+1/2*e*x)^2)*ln(F)*b*c/
(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)+3/4*F^(a*c)/(1+ta
n(3/2*e*x+3/2*d)^2)/(9*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))-3/4*F^(a*c)/
(1+tan(3/2*e*x+3/2*d)^2)/(9*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(3/2
*e*x+3/2*d)^2-1/2*F^(a*c)/(1+tan(3/2*e*x+3/2*d)^2)*ln(F)*b*c/(9*e^2+b^2*c^2
*ln(F)^2)*exp(b*c*x*ln(F))*tan(3/2*e*x+3/2*d)
```

Maxima [B] time = 1.33787, size = 1098, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log
(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)
*cos(3*e*x) - (F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(
3*d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F
^(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*
b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)
*e^3*cos(3*d))*F^(b*c*x)*cos(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3
*d) - F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*
d) - 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*cos(e*x - 2*d) + (F^(a*c)*b^3*c^3*co
s(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)*b*c*e^2*c
os(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x) + (F^(a*c)*b^
3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + F^(a*c)*b
*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*sin(3*e*x + 6*d)
 - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*
d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*
sin(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + F^(a*c)*b^2*c^2*e*l
og(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 9*F^(a*c)*e^3*sin(3*
d))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4 + b^4*c^4*log(F)
^4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2*c^2*cos(3*d)^2*lo
g(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Fricas [A] time = 0.507948, size = 386, normalized size = 1.94

$$\frac{\left(3\,e^3\cos\left(ex+d\right)^3-9\,e^3\cos\left(ex+d\right)+3\left(b^2c^2e\cos\left(ex+d\right)^3-b^2c^2e\cos\left(ex+d\right)\right)\log\left(F\right)^2-\left(\left(b^3c^3\cos\left(ex+d\right)^2-b^3c^3\right)\right.\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (3*e^3*cos(e*x + d)^3 - 9*e^3*cos(e*x + d) + 3*(b^2*c^2*e*cos(e*x + d)^3 -
b^2*c^2*e*cos(e*x + d))*log(F)^2 - ((b^3*c^3*cos(e*x + d)^2 - b^3*c^3)*log(
F)^3 + (b*c*e^2*cos(e*x + d)^2 - 7*b*c*e^2)*log(F))*sin(e*x + d))*F^(b*c*x
+ a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2*log(F)^2 + 9*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29847, size = 1770, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c + 3*x*e + 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 6*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 6*e)*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*x*e + 3*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) + 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F))^
2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(
1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e +
 d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*
log(abs(F)) + a*c*log(abs(F))) - 3/4*(2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sg
n(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*
```

```
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c
 - 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi
*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2
))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*log(abs(F))*sin(1/2
*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*x*e -
3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) - (pi*b*c
*sgn(F) - pi*b*c - 6*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c
*sgn(F) - 1/2*pi*a*c - 3*x*e - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(-2*I*
e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c + 3*I*x*e + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*I*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*
sgn(F) + 1/2*I*pi*a*c - 3*I*x*e - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c +
 16*b*c*log(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/
2*(6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c + I*x*e + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(
F)) + 16*I*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c +
16*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2
*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c - I*x*e - I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(
F)) - 16*I*e) - 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c +
16*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2
*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2
*I*pi*a*c - 3*I*x*e - 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(a
bs(F)) - 48*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*p
i*a*c*sgn(F) + 1/2*I*pi*a*c + 3*I*x*e + 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi
*b*c + 16*b*c*log(abs(F)) + 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
)
```

3.3 $\int F^{c(a+bx)} \sin^2(d+ex)\,dx$

Optimal. Leaf size=128

$$\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^
(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0520328, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4434, 2194}

$$\frac{bc\log(F)\sin^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}-\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) - (2*e*F^(c
*(a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*F^
(c*(a + b*x))*Log[F]*Sin[d + e*x]^2)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int F^{c(a+bx)}\sin^2(d+ex)\,dx = -\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^2(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{(2e^2)\int F^{c(a+bx)}\,dx}{4e^2+b^2c^2\log^2(F)}$$
$$= \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)} - \frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)} + \frac{bcF^{c(a+bx)}\log(F)\sin^2(}{4e^2+b^2c^2\log^2(}$$

Mathematica [A] time = 0.212046, size = 86, normalized size = 0.67

$$\frac{F^{c(a+bx)}\left(-b^2c^2\log^2(F)\cos(2(d+ex))+b^2c^2\log^2(F)-2bce\log(F)\sin(2(d+ex))+4e^2\right)}{2b^3c^3\log^3(F)+8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 - b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 - 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A] time = 0.049, size = 153, normalized size = 1.2

$$\frac{F^{c(bx+a)}}{2\,bc\ln(F)} - \frac{1}{2+2\,(\tan(ex+d))^2}\left(\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{4\,e^2+b^2c^2(\ln(F))^2} + 4\,\frac{e e^{c(bx+a)\ln(F)}\tan(ex+d)}{4\,e^2+b^2c^2(\ln(F))^2} - \frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}(\tan(ex+}{4\,e^2+b^2c^2(\ln(F))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d)^2,x)
```

```
[Out] 1/2/b/c/ln(F)*F^(c*(b*x+a))-1/2*(1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(c*
(b*x+a)*ln(F))+4/(4*e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(e*x+d)-
1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(c*(b*x+a)*ln(F))*tan(e*x+d)^2)/(1+t
an(e*x+d)^2)
```

Maxima [B] time = 1.2013, size = 481, normalized size = 3.76

$$-\frac{\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2+2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex)+\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2-2\,F^{ac}bce\log(F)\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(
2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log
(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*
x))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*cos
(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Fricas [A] time = 0.49243, size = 207, normalized size = 1.62

$$-\frac{\left(2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+\left(b^{2}c^{2}\cos\left(ex+d\right)^{2}-b^{2}c^{2}\right)\log\left(F\right)^{2}-2\,e^{2}\right)F^{bcx+ac}}{b^{3}c^{3}\log\left(F\right)^{3}+4\,bce^{2}\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d) + (b^2*c^2*cos(e*x + d)^2 - b^2*
c^2)*log(F)^2 - 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F)
)
```

Sympy [A] time = 159.735, size = 627, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d +
 e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c)
)**(b*c*x)*sin(d + e*x)**2 + zoo*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(
b*c))**(b*c*x)*sin(d + e*x)*cos(d + e*x) + zoo*e**2*exp(-2*I*e/(b*c))**(a*c
```

```
)*exp(-2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F, exp(-2*I*e/(b*c)))), (z
oo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2 +
 zoo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)*co
s(d + e*x) + zoo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*cos
(d + e*x)**2, Eq(F, exp(2*I*e/(b*c)))), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*
cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d +
 e*x)**2/2 + x*cos(d + e*x)**2/2 - sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0
)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*sin(d + e*x)**2/(b**3*c**3*log
(F)**3 + 4*b*c*e**2*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sin(d + e*
x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(
b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F
**(a*c)*F**(b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*l
og(F)), True))
```

Giac [C] time = 1.25514, size = 1260, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + 2*x*e + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*x*e + 2*d)/(4*b^2*c
^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) - 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/
2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*x*e - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(
F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*
sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2
*x*e - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e
^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) +
1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(a
bs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*
pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2
*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*
b*c + 8*b*c*log(abs(F)) + 16*I*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*p
i*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(-4*I*pi*b*
c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) +
```

```
 a*c*log(abs(F))) - 1/2*I*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(4*I*pi*b*c*sgn(F) -
4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(-4*
I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(ab
s(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*
b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
 + 4*b*c*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/
2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*
log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

3.4 $\int F^{c(a+bx)} \sin(d+ex) \, dx$

Optimal. Leaf size=73

$$\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

```
[Out] -((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0162222, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4432}

$$\frac{bc\log(F)\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}-\frac{e\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] -((e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c*(a
 + b*x))*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\sin(d+ex)\,dx=-\frac{eF^{c(a+bx)}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcF^{c(a+bx)}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.115778, size = 48, normalized size = 0.66

$$\frac{F^{c(a+bx)}(bc\log(F)\sin(d+ex)-e\cos(d+ex))}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sin[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Cos[d + e*x]) + b*c*Log[F]*Sin[d + e*x]))/(e^2 + b^2*
c^2*Log[F]^2)
```

Maple [A] time = 0.015, size = 130, normalized size = 1.8

$$\left(\frac{e\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2-\frac{e\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}+2\frac{bc\ln(F)\mathrm{e}^{c(bx+a)\ln(F)}\tan(d/2+1/2\,ex)}{e^2+b^2c^2(\ln(F))^2}\right)\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sin(e*x+d),x)
```

```
[Out] (1/(e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2-1/(e^2
+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))+2*ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*
exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1/2*e*x)^2)
```

Maxima [B] time = 1.08017, size = 262, normalized size = 3.59

$$-\frac{(F^{ac}bc\log(F)\sin(d)+F^{ac}e\cos(d))F^{bcx}\cos(ex+2\,d)-(F^{ac}bc\log(F)\sin(d)-F^{ac}e\cos(d))F^{bcx}\cos(ex)-(F^{ac}bc\cos(d)}{2\left(b^2c^2\cos(d)^2\log(F)^2+b^2c^2\log(F)^2\sin(d)^2+(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*co
s(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Fricas [A] time = 0.480878, size = 115, normalized size = 1.58

$$\frac{(bc\log(F)\sin(ex+d) - e\cos(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2 + e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*c*log(F)*sin(e*x + d) - e*cos(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [A] time = 70.0082, size = 326, normalized size = 4.47

$$\begin{cases} \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}x\sin(d+ex)}{2} + \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}ix\cos(d+ex)}{2} - \frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}\cos(d+ex)}{2e} & \text{for } F = -1 \wedge b = \frac{e}{\pi c} \\ x\sin(d) & \text{for } F = 1 \wedge e = 0 \\ \tilde{\infty}e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\sin(d+ex) + \tilde{\infty}e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F = e^{-\frac{ie}{bc}} \\ \tilde{\infty}e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\sin(d+ex) + \tilde{\infty}e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F = e^{\frac{ie}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\sin(d+ex)}{b^2c^2\log(F)^2+e^2} - \frac{F^{ac}F^{bcx}e\cos(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sin(e*x+d),x)
```

```
[Out] Piecewise(((-1)**(a*c)*(-1)**(e*x/pi)*x*sin(d + e*x)/2 + (-1)**(a*c)*(-1)**
(e*x/pi)*I*x*cos(d + e*x)/2 - (-1)**(a*c)*(-1)**(e*x/pi)*cos(d + e*x)/(2*e)
, Eq(F, -1) & Eq(b, e/(pi*c))), (x*sin(d), Eq(F, 1) & Eq(e, 0)), (zoo*e*exp
(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e*exp(-I*e/
(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(-I*e/(b*c)))
), (zoo*e*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*
e*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(I*e
/(b*c)))), (F**(a*c)*F**(b*c*x)*b*c*log(F)*sin(d + e*x)/(b**2*c**2*log(F)**
2 + e**2) - F**(a*c)*F**(b*c*x)*e*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2)
, True))
```

Giac [C] time = 1.22642, size = 880, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sin(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(
F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c + 2*e)*cos(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) - (2*b*c*log(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*cos(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) + 1/2*(2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*
pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c
 + 4*b*c*log(abs(F)) + 4*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*
c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-2*I*pi*b*c*sgn(F)
 + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F))) + 1/2*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4
*b*c*log(abs(F)) - 4*I*e) - 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-2*I*pi*b*c*sgn(F) + 2
*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(
F)))
```

3.5 $\int F^{c(a+bx)} \csc(d+ex)\,dx$

Optimal. Leaf size=81

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F])
```

Rubi [A] time = 0.0216913, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4453}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x],x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d + e*x))])/(e - I*b*c*Log[F])
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc(d+ex)\,dx=-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e-ibc\log(F)}$$

Mathematica [A] time = 1.77452, size = 114, normalized size = 1.41

$$\frac{iF^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},-\cos(d+ex)-i\sin(d+ex)\right)-\text{Hypergeometric2F1}\left(1,-\frac{ibc}{}\right.\right.}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x],x]
```

```
[Out] (I*F^(c*(a + b*x))*(Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Lo
g[F])/e, -Cos[d + e*x] - I*Sin[d + e*x]] - Hypergeometric2F1[1, ((-I)*b*c*L
og[F])/e, 1 - (I*b*c*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]))/(b*c*Log[F
])
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\csc(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\csc(ex+d),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\csc(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\csc(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d), x)
```

3.6 $\int F^{c(a+bx)}\csc^2(d+ex)\,dx$

Optimal. Leaf size=78

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

```
[Out] (-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rubi [A] time = 0.0286723, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4453}

$$-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

```
[Out] (-4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc^2(d+ex)\,dx=-\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A] time = 1.50092, size = 101, normalized size = 1.29

$$-\frac{2iF^{c(a+bx)}\left(\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)+\sin(d)\csc(d+ex)(\cos(ex)-i\sin(ex$$
$$\left(-1+e^{2id}\right)e$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^2,x]
```

```
[Out] ((-2*I)*F^(c*(a + b*x))*((-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-I/2)*b*
c*Log[F])/e, 1 - ((I/2)*b*c*Log[F])/e, E^((2*I)*(d + e*x))] + Csc[d + e*x]*
Sin[d]*(Cos[e*x] - I*Sin[e*x])))/(e*(-1 + E^((2*I)*d)))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\csc\left(ex+d\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\csc\left(ex+d\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c} \csc(ex+d)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^2, x)
```

3.7 $\int F^{c(a+bx)} \csc^3(d+ex)\,dx$

Optimal. Leaf size=137

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\csc(d+ex)F^{c}}{2e^2}$$

```
[Out] -(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/(2*e) - (b*c*F^(c*(a + b*x))*C
sc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometr
ic2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d +
e*x))]*(e + I*b*c*Log[F]))/e^2
```

Rubi [A] time = 0.0490738, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4449, 4453}

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\csc(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\cot(d+ex}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] -(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x])/(2*e) - (b*c*F^(c*(a + b*x))*C
sc[d + e*x]*Log[F])/(2*e^2) - (E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometr
ic2F1[1, (e - I*b*c*Log[F])/(2*e), (3 - (I*b*c*Log[F])/e)/2, E^((2*I)*(d +
e*x))]*(e + I*b*c*Log[F]))/e^2
```

Rule 4449

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csc[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a + b*x)
)*Csc[d + e*x]^(n - 1)*Cos[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
```

```
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} + \frac{1}{2}\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int F$$

$$= -\frac{F^{c(a+bx)}\cot(d+ex)\csc(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\csc(d+ex)\log(F)}{2e^2} - \frac{e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1, \frac{e-ibc}{}\right.}{}$$

Mathematica [B] time = 7.89086, size = 334, normalized size = 2.44

$$\frac{F^{c(a+bx)}\left(-\frac{4i\left(b^2c^2\log^2(F)+e^2\right)\left(1+(i\sin(d)+\cos(d)-1)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},\cos(d+ex)+i\sin(d+ex)\right)\right)}{bc\log(F)(i\sin(d)+\cos(d)-1)} - \frac{4i\left(b^2c^2\log^2(F)+e^2\right)\left(1-(i\sin\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[(d + e*x)/2]^2) - 4*b*c*Csc[d]*Log[F] + Csc[d]*((
4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + e*Sec[(d + e*x)/2]^2 - ((4*I)*(e^2 +
b^2*c^2*Log[F]^2)*(1 + Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c
*Log[F])/e, Cos[d + e*x] + I*Sin[d + e*x]]*(-1 + Cos[d] + I*Sin[d])))/(b*c*
Log[F]*(-1 + Cos[d] + I*Sin[d])) - ((4*I)*(e^2 + b^2*c^2*Log[F]^2)*(1 - Hyp
ergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*Log[F])/e, -Cos[d + e*x]
- I*Sin[d + e*x]]*(1 + Cos[d] + I*Sin[d])))/(b*c*Log[F]*(1 + Cos[d] + I*Sin
[d])) + 2*b*c*Csc[d/2]*Csc[(d + e*x)/2]*Log[F]*Sin[(e*x)/2] - 2*b*c*Log[F]*
Sec[d/2]*Sec[(d + e*x)/2]*Sin[(e*x)/2]))/(8*e^2)
```

Maple [F] time = 0.117, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\csc(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\csc\left(ex+d\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c} \csc(ex+d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^3, x)
```

3.8 $\int F^{c(a+bx)} \csc^4(d+ex)\,dx$

Optimal. Leaf size=141

$$\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\csc^2(d+ex)F}{6e^2}$$

```
[Out] -(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x]^2)/(3*e) - (b*c*F^(c*(a + b*x))
*Csc[d + e*x]^2*Log[F])/(6*e^2) + (2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hy
pergeometric2F1[2, 1 - ((I/2)*b*c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((
2*I)*(d + e*x))]*((2*I)*e - b*c*Log[F]))/(3*e^2)
```

Rubi [A] time = 0.0549809, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4449, 4453}

$$\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\csc^2(d+ex)F^{c(a+bx)}}{6e^2}-\frac{\cot(d+}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] -(F^(c*(a + b*x))*Cot[d + e*x]*Csc[d + e*x]^2)/(3*e) - (b*c*F^(c*(a + b*x))
*Csc[d + e*x]^2*Log[F])/(6*e^2) + (2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hy
pergeometric2F1[2, 1 - ((I/2)*b*c*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, E^((
2*I)*(d + e*x))]*((2*I)*e - b*c*Log[F]))/(3*e^2)
```

Rule 4449

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Csc[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Csc[d + e*x]^(n - 2), x], x] - Simp[(F^(c*(a + b*x)
)*Csc[d + e*x]^(n - 1)*Cos[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
```

```
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int F^{c(a+bx)}\csc^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{1}{6}\left(4+\frac{b^2c^2\log^2(F)}{e^2}\right)\int$$

$$= -\frac{F^{c(a+bx)}\cot(d+ex)\csc^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\csc^2(d+ex)\log(F)}{6e^2} + \frac{2e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,\right.}{}$$

Mathematica [A] time = 2.98715, size = 173, normalized size = 1.23

$$\frac{F^{c(a+bx)}\left(-\frac{2i\left(b^2c^2\log^2(F)+4e^2\right)\left(1+\left(-1+e^{2id}\right)\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{2e},1-\frac{ibc\log(F)}{2e},e^{2i(d+ex)}\right)\right)}{-1+e^{2id}}+\csc(d)\sin(ex)\csc(d+ex)\left(b^2c^2\log^2(F\right.\right.}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csc[d + e*x]^2*(2*e*Cot[d] + b*c*Log[F])) - ((2*I)*(1
 + (-1 + E^((2*I)*d))*Hypergeometric2F1[1, ((-I/2)*b*c*Log[F])/e, 1 - ((I/2
)*b*c*Log[F])/e, E^((2*I)*(d + e*x))])*(4*e^2 + b^2*c^2*Log[F]^2))/(-1 + E^
((2*I)*d)) + 2*e^2*Csc[d]*Csc[d + e*x]^3*Sin[e*x] + Csc[d]*Csc[d + e*x]*(4*
e^2 + b^2*c^2*Log[F]^2)*Sin[e*x]))/(6*e^3)
```

Maple [F] time = 0.134, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\csc(ex+d)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\csc\left(ex+d\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\csc\left(ex+d\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^4, x)
```

3.9 $\int e^x \sin^4(x)\,dx$

Optimal. Leaf size=54

$$\frac{24e^x}{85}+\frac{1}{17}e^x\sin^4(x)+\frac{12}{85}e^x\sin^2(x)-\frac{4}{17}e^x\sin^3(x)\cos(x)-\frac{24}{85}e^x\sin(x)\cos(x)$$

```
[Out] (24*E^x)/85 - (24*E^x*Cos[x]*Sin[x])/85 + (12*E^x*Sin[x]^2)/85 - (4*E^x*Cos
[x]*Sin[x]^3)/17 + (E^x*Sin[x]^4)/17
```

Rubi [A] time = 0.0256499, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4434, 2194}

$$\frac{24e^x}{85}+\frac{1}{17}e^x\sin^4(x)+\frac{12}{85}e^x\sin^2(x)-\frac{4}{17}e^x\sin^3(x)\cos(x)-\frac{24}{85}e^x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x]^4,x]
```

```
[Out] (24*E^x)/85 - (24*E^x*Cos[x]*Sin[x])/85 + (12*E^x*Sin[x]^2)/85 - (4*E^x*Cos
[x]*Sin[x]^3)/17 + (E^x*Sin[x]^4)/17
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \sin^4(x)\,dx &= -\frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x) + \frac{12}{17}\int e^x\sin^2(x)\,dx \\ &= -\frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x) + \frac{24\int e^x\,dx}{85} \\ &= \frac{24e^x}{85} - \frac{24}{85}e^x\cos(x)\sin(x) + \frac{12}{85}e^x\sin^2(x) - \frac{4}{17}e^x\cos(x)\sin^3(x) + \frac{1}{17}e^x\sin^4(x)\end{aligned}$$

Mathematica [A] time = 0.0376441, size = 33, normalized size = 0.61

$$\frac{1}{680}e^x(-136\sin(2x) + 20\sin(4x) - 68\cos(2x) + 5\cos(4x) + 255)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x]^4,x]
```

```
[Out] (E^x*(255 - 68*Cos[2*x] + 5*Cos[4*x] - 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A] time = 0.01, size = 34, normalized size = 0.6

$$\frac{(\sin(x) - 4\cos(x))\,e^x(\sin(x))^3}{17} + \frac{(12\sin(x) - 24\cos(x))\,e^x\sin(x)}{85} + \frac{24\,e^x}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x)^4,x)
```

```
[Out] 1/17*(sin(x)-4*cos(x))*exp(x)*sin(x)^3+12/85*(sin(x)-2*cos(x))*exp(x)*sin(x
)+24/85*exp(x)
```

Maxima [A] time = 1.02925, size = 50, normalized size = 0.93

$$\frac{1}{136}\cos(4x)\,e^x - \frac{1}{10}\cos(2x)\,e^x + \frac{1}{34}e^x\sin(4x) - \frac{1}{5}e^x\sin(2x) + \frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/136*cos(4*x)*e^x - 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) - 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Fricas [A] time = 0.462996, size = 115, normalized size = 2.13

$$\frac{4}{85}\left(5\cos(x)^3 - 11\cos(x)\right)e^x\sin(x) + \frac{1}{85}\left(5\cos(x)^4 - 22\cos(x)^2 + 41\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 4/85*(5*cos(x)^3 - 11*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 - 22*cos(x)^2 +
 41)*e^x
```

Sympy [A] time = 6.4161, size = 70, normalized size = 1.3

$$\frac{41e^x\sin^4(x)}{85} - \frac{44e^x\sin^3(x)\cos(x)}{85} + \frac{12e^x\sin^2(x)\cos^2(x)}{17} - \frac{24e^x\sin(x)\cos^3(x)}{85} + \frac{24e^x\cos^4(x)}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)**4,x)
```

```
[Out] 41*exp(x)*sin(x)**4/85 - 44*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 - 24*exp(x)*sin(x)*cos(x)**3/85 + 24*exp(x)*cos(x)**4/85
```

Giac [A] time = 1.14763, size = 47, normalized size = 0.87

$$\frac{1}{136}\left(\cos(4x) + 4\sin(4x)\right)e^x - \frac{1}{10}\left(\cos(2x) + 2\sin(2x)\right)e^x + \frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x - 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

3.10 $\int F^{c(a+bx)}\cos^n(d+ex)\,dx$

Optimal. Leaf size=107

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\text{Hypergeometric2F1}\left(-n,-\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(-\frac{ibc\log(F)}{e}-n+2\right),-e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

```
[Out] -((F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, -(e*n + I*b*c*Log[F
])/(2*e), (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 + E^((2*
I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```

Rubi [A] time = 0.112139, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4441, 2259}

$$-\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\,_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(-n-\frac{ibc\log(F)}{e}+2\right);-e^{2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```


```
[Out] -((F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, -(e*n + I*b*c*Log[F
])/(2*e), (2 - n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/((1 + E^((2*
I)*(d + e*x)))^n*(I*e*n - b*c*Log[F])))
```


Rule 4441

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Dist[(E^(I*n*(d + e*x))*Cos[d + e*x]^n)/(1 + E^(2*I*(d + e*x)))^n, In
t[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e*x)))^n)/E^(I*n*(d + e*x)), x], x] /;
FreeQ[{F, a, b, c, d, e, n}, x] && !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\cos^n(d+ex)\,dx=\left(e^{in(d+ex)}\left(1+e^{2i(d+ex)}\right)^{-n}\cos^n(d+ex)\right)\int e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^nF^{c(a+bx)}\,dx$$
$$=-\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\,{}_2F_1\left(-n,-\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2-n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{ien-bc\log(F)}$$

Mathematica [A] time = 0.0553799, size = 110, normalized size = 1.03

$$\frac{\left(1+e^{2i(d+ex)}\right)^{-n}F^{c(a+bx)}\cos^n(d+ex)\text{Hypergeometric2F1}\left(-n,-\frac{i(bc\log(F)-ien)}{2e},1-\frac{i(bc\log(F)-ien)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)-ien}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^n,x]
```

```
[Out] (F^(c*(a + b*x))*Cos[d + e*x]^n*Hypergeometric2F1[-n, ((-I/2)*((-I)*e*n + b
*c*Log[F]))/e, 1 - ((I/2)*((-I)*e*n + b*c*Log[F]))/e, -E^((2*I)*(d + e*x))]
)/((1 + E^((2*I)*(d + e*x)))^n*((-I)*e*n + b*c*Log[F]))
```

Maple [F] time = 0.498, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\cos(ex+d)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*cos(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\cos(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\cos\left(ex+d\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*cos(e*x + d)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\cos\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*cos(e*x + d)^n, x)
```

3.11 $\int F^{c(a+bx)} \cos^3(d+ex)\,dx$

Optimal. Leaf size=199

$$\frac{6e^3\sin(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\cos(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+\frac{3e\sin(\ldots}{}$$

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (6
*b*c*e^2*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(9*e^4 + 10*b^2*c^2*e^2*Log[F
]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])
/(9*e^2 + b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sin[d + e*x])/(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rubi [A] time = 0.0528869, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4435, 4433}

$$\frac{6e^3\sin(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+\frac{bc\log(F)\cos^3(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+9e^2}+\frac{6bce^2\log(F)\cos(d+ex)F^{c(a+bx)}}{10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4}+\frac{3e\sin(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]^3*Log[F])/(9*e^2 + b^2*c^2*Log[F]^2) + (6
*b*c*e^2*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(9*e^4 + 10*b^2*c^2*e^2*Log[F
]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cos[d + e*x]^2*Sin[d + e*x])
/(9*e^2 + b^2*c^2*Log[F]^2) + (6*e^3*F^(c*(a + b*x))*Sin[d + e*x])/(9*e^4 +
 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4)
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cos^3(d+ex)\,dx = \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{3eF^{c(a+bx)}\cos^2(d+ex)\sin(d+ex)}{9e^2+b^2c^2\log^2(F)} + \frac{\left(6e^2\right)\int F^{c(a+bx)}\cos(\ldots}{9e^2+b^2c^2\log^2\ldots}$$
$$= \frac{bcF^{c(a+bx)}\cos^3(d+ex)\log(F)}{9e^2+b^2c^2\log^2(F)} + \frac{6bce^2F^{c(a+bx)}\cos(d+ex)\log(F)}{9e^4+10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cos^2(d\ldots}{9e^2+b^2c\ldots}$$

Mathematica [A] time = 0.653698, size = 155, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(bc\log(F)\cos(3(d+ex))\left(b^2c^2\log^2(F)+e^2\right)+3bc\log(F)\cos(d+ex)\left(b^2c^2\log^2(F)+9e^2\right)+6e\sin(d+ex)\left(\cos(\ldots\right.\right.}{4\left(10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)+9e^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(b*c*Cos[3*(d + e*x)]*Log[F]*(e^2 + b^2*c^2*Log[F]^2) + 3*
b*c*Cos[d + e*x]*Log[F]*(9*e^2 + b^2*c^2*Log[F]^2) + 6*e*(5*e^2 + b^2*c^2*L
og[F]^2 + Cos[2*(d + e*x)]*(e^2 + b^2*c^2*Log[F]^2))*Sin[d + e*x]))/(4*(9*e
^4 + 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] time = 0.11, size = 274, normalized size = 1.4

$$\frac{1}{4}\left(\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{9e^2+b^2c^2(\ln(F))^2}+6\,\frac{e\,e^{c(bx+a)\ln(F)}\tan(3/2\,ex+3/2\,d)}{9e^2+b^2c^2(\ln(F))^2}-\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{9e^2+b^2c^2(\ln(F))^2}\left(\tan\left(\frac{3ex}{2}+\frac{3d}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{3\ldots}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^3,x)
```

```
[Out] 1/4*(ln(F)*b*c/(9*e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))+6/(9*e^2+b^2*c^
2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(3/2*e*x+3/2*d)-ln(F)*b*c/(9*e^2+b^2*c
```

```
^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))*tan(3/2*e*x+3/2*d)^2)/(1+tan(3/2*e*x+3/2*d
)^2)+3/4*(ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))+2/(e^2+b^2*c
^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-ln(F)*b*c/(e^2+b^2*c^
2*ln(F)^2)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2)/(1+tan(1/2*d+1/2*e*x)
^2)
```

Maxima [B] time = 1.23256, size = 1098, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 + 3*F^(a*c)*b^2*c^2*e*log(F)^2*sin(
3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) + 3*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*
cos(3*e*x) + (F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - 3*F^(a*c)*b^2*c^2*e*log(F
)^2*sin(3*d) + F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 3*F^(a*c)*e^3*sin(3*d))*F^
(b*c*x)*cos(3*e*x + 6*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)^3 - F^(a*c)*b
^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F) - 9*F^(a*c)*
e^3*sin(3*d))*F^(b*c*x)*cos(e*x + 4*d) + 3*(F^(a*c)*b^3*c^3*cos(3*d)*log(F)
^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(3*d) + 9*F^(a*c)*b*c*e^2*cos(3*d)*log(F
) + 9*F^(a*c)*e^3*sin(3*d))*F^(b*c*x)*cos(e*x - 2*d) - (F^(a*c)*b^3*c^3*log
(F)^3*sin(3*d) - 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*b*c*e^2*lo
g(F)*sin(3*d) - 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x) + (F^(a*c)*b^3
*c^3*log(F)^3*sin(3*d) + 3*F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^2 + F^(a*c)*b*
c*e^2*log(F)*sin(3*d) + 3*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*sin(3*e*x + 6*d)
+ 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) + F^(a*c)*b^2*c^2*e*cos(3*d)*log(F)^
2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) + 9*F^(a*c)*e^3*cos(3*d))*F^(b*c*x)*s
in(e*x + 4*d) - 3*(F^(a*c)*b^3*c^3*log(F)^3*sin(3*d) - F^(a*c)*b^2*c^2*e*co
s(3*d)*log(F)^2 + 9*F^(a*c)*b*c*e^2*log(F)*sin(3*d) - 9*F^(a*c)*e^3*cos(3*d
))*F^(b*c*x)*sin(e*x - 2*d))/(b^4*c^4*cos(3*d)^2*log(F)^4 + b^4*c^4*log(F)^
4*sin(3*d)^2 + 9*(cos(3*d)^2 + sin(3*d)^2)*e^4 + 10*(b^2*c^2*cos(3*d)^2*log
(F)^2 + b^2*c^2*log(F)^2*sin(3*d)^2)*e^2)
```

Fricas [A] time = 0.496551, size = 331, normalized size = 1.66

$$\frac{\left(b^3c^3\cos\left(ex+d\right)^3\log\left(F\right)^3+\left(bce^2\cos\left(ex+d\right)^3+6\,bce^2\cos\left(ex+d\right)\right)\log\left(F\right)+3\left(b^2c^2e\cos\left(ex+d\right)^2\log\left(F\right)^2+e^3\cos\left(ex\right.\right.}{b^4c^4\log\left(F\right)^4+10\,b^2c^2e^2\log\left(F\right)^2+9\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] (b^3*c^3*cos(e*x + d)^3*log(F)^3 + (b*c*e^2*cos(e*x + d)^3 + 6*b*c*e^2*cos(
e*x + d))*log(F) + 3*(b^2*c^2*e*cos(e*x + d)^2*log(F)^2 + e^3*cos(e*x + d)^
2 + 2*e^3)*sin(e*x + d))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4 + 10*b^2*c^2*e^2
*log(F)^2 + 9*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.31953, size = 1764, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 3*x*e + 3*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 6*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 6*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 3*x*e + 3*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 6*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1
/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e +
d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*l
og(abs(F)) + a*c*log(abs(F))) + 3/4*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi
*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)*log(abs(F))/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c
- 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
```

```
a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
)*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/4*(2*b*c*cos(1/2*pi*b*c*x*sgn
(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 3*x*e - 3*d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 6*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 6*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*
sgn(F) - 1/2*pi*a*c - 3*x*e - 3*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 6*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I
*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c + 3*I*x*e + 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)
) + 48*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c
*sgn(F) + 1/2*I*pi*a*c - 3*I*x*e - 3*I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c
+ 16*b*c*log(abs(F)) - 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1
/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F)
- 1/2*I*pi*a*c + I*x*e + I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(
abs(F)) + 16*I*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*
pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*
c + 16*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) -
 1/2*I*(-6*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c - I*x*e - I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*lo
g(abs(F)) - 16*I*e) + 6*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*
b*c + 16*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
 - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c - 3*I*x*e - 3*I*d)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*
b*c*log(abs(F)) - 48*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 3*I*x*e + 3*I*d)/(-8*I*pi*b*c*sgn(F)
 + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F)))
```

3.12 $\int F^{c(a+bx)} \cos^2(d+ex)\,dx$

Optimal. Leaf size=128

$$\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c
*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c*(
a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0366937, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4435, 2194}

$$\frac{bc\log(F)\cos^2(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e\sin(d+ex)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+4e^2}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(b^2c^2\log^2(F)+4e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*F^(c
*(a + b*x))*Cos[d + e*x]^2*Log[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*F^(c*(
a + b*x))*Cos[d + e*x]*Sin[d + e*x])/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int F^{c(a+bx)}\cos^2(d+ex)\,dx=\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d+ex)}{4e^2+b^2c^2\log^2(F)}+\frac{(2e^2)\int F^{c(a+bx)}\,dx}{4e^2+b^2c^2\log^2(F)}$$
$$=\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcF^{c(a+bx)}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cos(d+ex)\sin(d}{4e^2+b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.199099, size = 85, normalized size = 0.66

$$\frac{F^{c(a+bx)}\left(b^2c^2\log^2(F)\cos(2(d+ex))+b^2c^2\log^2(F)+2bce\log(F)\sin(2(d+ex))+4e^2\right)}{2b^3c^3\log^3(F)+8bce^2\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[2*(d + e*x)]*Log[F
]^2 + 2*b*c*e*Log[F]*Sin[2*(d + e*x)]))/(8*b*c*e^2*Log[F] + 2*b^3*c^3*Log[F
]^3)
```

Maple [A] time = 0.03, size = 153, normalized size = 1.2

$$\frac{F^{c(bx+a)}}{2\,bc\ln(F)}+\frac{1}{2+2\,(\tan(ex+d))^2}\left(\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{4\,e^2+b^2c^2(\ln(F))^2}+4\,\frac{ee^{c(bx+a)\ln(F)}\tan(ex+d)}{4\,e^2+b^2c^2(\ln(F))^2}-\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}(\tan(ex+d}{4\,e^2+b^2c^2(\ln(F))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d)^2,x)
```

```
[Out] 1/2/b/c/ln(F)*F^(c*(b*x+a))+1/2*(1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(c*
(b*x+a)*ln(F))+4/(4*e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(e*x+d)-
1/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(c*(b*x+a)*ln(F))*tan(e*x+d)^2)/(1+t
an(e*x+d)^2)
```

Maxima [B] time = 1.07897, size = 481, normalized size = 3.76

$$\frac{\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2+2\,F^{ac}bce\log(F)\sin(2\,d)\right)F^{bcx}\cos(2\,ex)+\left(F^{ac}b^2c^2\cos(2\,d)\log(F)^2-2\,F^{ac}bce\log(F)\sin(2\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2
*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log(
F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*x
))/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*cos(
2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2)
```

Fricas [A] time = 0.472239, size = 189, normalized size = 1.48

$$\frac{\left(b^2c^2\cos\left(ex+d\right)^2\log\left(F\right)^2+2\,bce\cos\left(ex+d\right)\log\left(F\right)\sin\left(ex+d\right)+2\,e^2\right)F^{bcx+ac}}{b^3c^3\log\left(F\right)^3+4\,bce^2\log\left(F\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*cos(e*x + d)^2*log(F)^2 + 2*b*c*e*cos(e*x + d)*log(F)*sin(e*x + d)
 + 2*e^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + 4*b*c*e^2*log(F))
```

Sympy [A] time = 127.152, size = 627, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sin(d + e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d +
 e*x)/(2*e), Eq(F, 1)), (zoo*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c)
)**(b*c*x)*sin(d + e*x)**2 + zoo*e**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(
b*c))**(b*c*x)*sin(d + e*x)*cos(d + e*x) + zoo*e**2*exp(-2*I*e/(b*c))**(a*c
)*exp(-2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F, exp(-2*I*e/(b*c)))), (z
```

```
oo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2 +
 zoo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)*co
s(d + e*x) + zoo*e**2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*cos
(d + e*x)**2, Eq(F, exp(2*I*e/(b*c)))), (F**(a*c)*(x*sin(d + e*x)**2/2 + x*
cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e)), Eq(b, 0)), (x*sin(d +
 e*x)**2/2 + x*cos(d + e*x)**2/2 + sin(d + e*x)*cos(d + e*x)/(2*e), Eq(c, 0
)), (F**(a*c)*F**(b*c*x)*b**2*c**2*log(F)**2*cos(d + e*x)**2/(b**3*c**3*log
(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(b*c*x)*b*c*e*log(F)*sin(d + e*
x)*cos(d + e*x)/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F**(a*c)*F**(
b*c*x)*e**2*sin(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*log(F)) + 2*F
**(a*c)*F**(b*c*x)*e**2*cos(d + e*x)**2/(b**3*c**3*log(F)**3 + 4*b*c*e**2*l
og(F)), True))
```

Giac [C] time = 1.2217, size = 1260, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c + 2*x*e + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*x*e + 2*d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) + 1/2*(2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2
*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*x*e - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F
))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*s
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*
x*e - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^
(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1
/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*
log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*
b*c + 8*b*c*log(abs(F)) + 16*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*p
i*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(-4*I*pi*b*
c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
```

```
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(4*I*pi*b*c*sgn(F) -
 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(-4
*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi
*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*
c + 4*b*c*log(abs(F))) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c
*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

3.13 $\int F^{c(a+bx)} \cos(d+ex)\,dx$

Optimal. Leaf size=72

$$\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^(
c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.0161595, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4433}

$$\frac{e\sin(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\cos(d+ex)F^{c(a+bx)}}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Cos[d + e*x],x]
```

```
[Out] (b*c*F^(c*(a + b*x))*Cos[d + e*x]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (e*F^(
c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int F^{c(a+bx)}\cos(d+ex)\,dx=\frac{bcF^{c(a+bx)}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}$$

Mathematica [A] time = 0.0965807, size = 47, normalized size = 0.65

$$\frac{F^{c(a+bx)}(bc\log(F)\cos(d+ex)+e\sin(d+ex))}{b^2c^2\log^2(F)+e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Cos[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(b*c*Cos[d + e*x]*Log[F] + e*Sin[d + e*x]))/(e^2 + b^2*c^2
*Log[F]^2)
```

Maple [A] time = 0.015, size = 133, normalized size = 1.9

$$\left(\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}+2\,\frac{e e^{c(bx+a)\ln(F)}\tan(d/2+1/2\,ex)}{e^2+b^2c^2(\ln(F))^2}-\frac{bc\ln(F)\,e^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*cos(e*x+d),x)
```

```
[Out] (ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))+2/(e^2+b^2*c^2*ln(F)^
2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2
)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2)/(1+tan(1/2*d+1/2*e*x)^2)
```

Maxima [B] time = 1.08046, size = 259, normalized size = 3.6

$$\frac{(F^{ac}bc\cos(d)\log(F)-F^{ac}e\sin(d))F^{bcx}\cos(ex+2\,d)+(F^{ac}bc\cos(d)\log(F)+F^{ac}e\sin(d))F^{bcx}\cos(ex)+(F^{ac}bc\log(F)}{2\left(b^2c^2\cos(d)^2\log(F)^2+b^2c^2\log(F)^2\sin(d)^2+\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*cos
(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Fricas [A] time = 0.476363, size = 115, normalized size = 1.6

$$\frac{(bc\cos(ex+d)\log(F)+e\sin(ex+d))F^{bcx+ac}}{b^2c^2\log(F)^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] (b*c*cos(e*x + d)*log(F) + e*sin(e*x + d))*F^(b*c*x + a*c)/(b^2*c^2*log(F)^
2 + e^2)
```

Sympy [A] time = 46.2778, size = 352, normalized size = 4.89

$$\begin{cases} -\frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}ix\sin(d+ex)}{2}+\frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}x\cos(d+ex)}{2}+\frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}\sin(d+ex)}{e}+\frac{(-1)^{ac}(-1)^{\frac{ex}{\pi}}i\cos(d+ex)}{2e} & \text{for } F=-1\wedge b=\frac{e}{\pi c} \\ x\cos(d) & \text{for } F=1\wedge e=0 \\ \tilde{\infty}e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\sin(d+ex)+\tilde{\infty}e\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F=e^{-\frac{ie}{bc}} \\ \tilde{\infty}e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\sin(d+ex)+\tilde{\infty}e\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F=e^{\frac{ie}{bc}} \\ \frac{F^{ac}F^{bcx}bc\log(F)\cos(d+ex)}{b^2c^2\log(F)^2+e^2}+\frac{F^{ac}F^{bcx}e\sin(d+ex)}{b^2c^2\log(F)^2+e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*cos(e*x+d),x)
```

```
[Out] Piecewise((-(-1)**(a*c)*(-1)**(e*x/pi)*I*x*sin(d + e*x)/2 + (-1)**(a*c)*(-1
)**(e*x/pi)*x*cos(d + e*x)/2 + (-1)**(a*c)*(-1)**(e*x/pi)*sin(d + e*x)/e +
(-1)**(a*c)*(-1)**(e*x/pi)*I*cos(d + e*x)/(2*e), Eq(F, -1) & Eq(b, e/(pi*c)
)), (x*cos(d), Eq(F, 1) & Eq(e, 0)), (zoo*e*exp(-I*e/(b*c))**(a*c)*exp(-I*e
/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c)
)**(b*c*x)*cos(d + e*x), Eq(F, exp(-I*e/(b*c)))), (zoo*e*exp(I*e/(b*c))**(a
*c)*exp(I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e*exp(I*e/(b*c))**(a*c)*exp(
I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(I*e/(b*c)))), (F**(a*c)*F**(b*c
*x)*b*c*log(F)*cos(d + e*x)/(b**2*c**2*log(F)**2 + e**2) + F**(a*c)*F**(b*c
*x)*e*sin(d + e*x)/(b**2*c**2*log(F)**2 + e**2), True))
```

Giac [C] time = 1.19659, size = 876, normalized size = 12.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*cos(e*x+d),x, algorithm="giac")
```

```
[Out] (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c + x*e + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
(F))) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
 1/2*pi*a*c - x*e - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*sgn(
F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*lo
g(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c
*log(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*
b*c + 4*b*c*log(abs(F)) + 4*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi
*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-2*I*pi*b*c*sgn
(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*l
og(abs(F))) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I
*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*
c + 4*b*c*log(abs(F)) - 4*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-2*I*pi*b*c*sgn(F
) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F)))
```

3.14 $\int F^{c(a+bx)} \sec(d+ex)\,dx$

Optimal. Leaf size=84

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F])
```

Rubi [A] time = 0.0178264, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4451}

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x],x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[F])
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec(d+ex)\,dx=\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{ie+bc\log(F)}$$

Mathematica [A] time = 0.0191242, size = 84, normalized size = 1.

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+ie}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x],x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 - ((I/2)*b*c*Lo
g[F])/e, 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/(I*e + b*c*Log[
F])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\sec(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sec(ex+d),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\sec(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d), x)
```

3.15 $\int F^{c(a+bx)}\sec^2(d+ex)\,dx$

Optimal. Leaf size=80

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rubi [A] time = 0.023731, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4451}

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx=\frac{4e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{2ie+bc\log(F)}$$

Mathematica [A] time = 0.0155679, size = 80, normalized size = 1.

$$\frac{4e^{2i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{bc\log(F)+2ie}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^2,x]
```

```
[Out] (4*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c*
Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))])/((2*I)*e + b*c*
Log[F])
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int F^{c(bx+a)} (\sec(ex+d))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sec(ex+d)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\sec^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^2, x)
```

3.16 $\int F^{c(a+bx)} \sec^3(d+ex)\,dx$

Optimal. Leaf size=141

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right),-e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\sec(d+ex)F}{2e^2}$$

```
[Out] -((E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(I*e - b*c*Log[F]))/
e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(2*e^2) + (F^(c*(a + b*x))
*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rubi [A] time = 0.048772, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4448, 4451}

$$-\frac{e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)}{e^2}-\frac{bc\log(F)\sec(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tan(d+e}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] -((E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])/
(2*e), (3 - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*(I*e - b*c*Log[F]))/
e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x])/(2*e^2) + (F^(c*(a + b*x))
*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
```

```
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\log(F)\sec(d+ex)}{2e^2}+\frac{F^{c(a+bx)}\sec(d+ex)\tan(d+ex)}{2e}+\frac{1}{2}\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int$$

$$= -\frac{e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(1,\frac{e-ibc\log(F)}{2e};\frac{1}{2}\left(3-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)(ie-bc\log(F))}{e^2}-\frac{bcF^{c(a+bx)}\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.26007, size = 112, normalized size = 0.79

$$\frac{F^{c(a+bx)}\left(\sec(d+ex)(e\tan(d+ex)-bc\log(F))+2e^{i(d+ex)}(bc\log(F)-ie)\text{Hypergeometric2F1}\left(1,\frac{e-ibc\log(F)}{2e},\frac{3}{2}-\frac{ibc\log(F}{2e}\right.\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(2*E^(I*(d + e*x))*Hypergeometric2F1[1, (e - I*b*c*Log[F])
/(2*e), 3/2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-I)*e + b*c*Log
[F]) + Sec[d + e*x]*(-(b*c*Log[F]) + e*Tan[d + e*x])))/(2*e^2)
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\sec(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sec\left(ex+d\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec\left(ex+d\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^3, x)
```

3.17 $\int F^{c(a+bx)}\sec^4(d+ex)\,dx$

Optimal. Leaf size=143

$$-\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ibc\log(F)}{2e},-e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\sec^2(d+e}{6e^2}$$

```
[Out] (-2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((2*I)*e - b*c*
Log[F]))/(3*e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x]^2)/(6*e^2) + (F
^(c*(a + b*x))*Sec[d + e*x]^2*Tan[d + e*x])/(3*e)
```

Rubi [A] time = 0.0525938, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4448, 4451}

$$-\frac{2e^{2i(d+ex)}F^{c(a+bx)}(-bc\log(F)+2ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)}{3e^2}-\frac{bc\log(F)\sec^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tan(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (-2*E^((2*I)*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((2*I)*e - b*c*
Log[F]))/(3*e^2) - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d + e*x]^2)/(6*e^2) + (F
^(c*(a + b*x))*Sec[d + e*x]^2*Tan[d + e*x])/(3*e)
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
```

```
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^4(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\log(F)\sec^2(d+ex)}{6e^2}+\frac{F^{c(a+bx)}\sec^2(d+ex)\tan(d+ex)}{3e}+\frac{1}{6}\left(4+\frac{b^2c^2\log^2(F)}{e^2}\right)\int$$

$$= -\frac{2e^{2i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{2e};2-\frac{ibc\log(F)}{2e};-e^{2i(d+ex)}\right)(2ie-bc\log(F))}{3e^2}-\frac{bcF^{c(a+bx)}\log}{}$$

Mathematica [A] time = 0.198649, size = 111, normalized size = 0.78

$$\frac{F^{c(a+bx)}\left(\sec^2(d+ex)(2e\tan(d+ex)-bc\log(F))+4e^{2i(d+ex)}(bc\log(F)-2ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{2e},2-\frac{ib}{}\right.\right.}{6e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(4*E^((2*I)*(d + e*x))*Hypergeometric2F1[2, 1 - ((I/2)*b*c
*Log[F])/e, 2 - ((I/2)*b*c*Log[F])/e, -E^((2*I)*(d + e*x))]*((-2*I)*e + b*c
*Log[F]) + Sec[d + e*x]^2*(-(b*c*Log[F]) + 2*e*Tan[d + e*x])))/(6*e^2)
```

Maple [F] time = 0.235, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}(\sec(ex+d))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^4,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sec\left(ex+d\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec\left(ex+d\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^4, x)
```

3.18 $\int e^x \cos^4(x)\,dx$

Optimal. Leaf size=54

$$\frac{24e^x}{85}+\frac{1}{17}e^x\cos^4(x)+\frac{12}{85}e^x\cos^2(x)+\frac{4}{17}e^x\sin(x)\cos^3(x)+\frac{24}{85}e^x\sin(x)\cos(x)$$

```
[Out] (24*E^x)/85 + (12*E^x*Cos[x]^2)/85 + (E^x*Cos[x]^4)/17 + (24*E^x*Cos[x]*Sin
[x])/85 + (4*E^x*Cos[x]^3*Sin[x])/17
```

Rubi [A] time = 0.0289402, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4435, 2194}

$$\frac{24e^x}{85}+\frac{1}{17}e^x\cos^4(x)+\frac{12}{85}e^x\cos^2(x)+\frac{4}{17}e^x\sin(x)\cos^3(x)+\frac{24}{85}e^x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x]^4,x]
```

```
[Out] (24*E^x)/85 + (12*E^x*Cos[x]^2)/85 + (E^x*Cos[x]^4)/17 + (24*E^x*Cos[x]*Sin
[x])/85 + (4*E^x*Cos[x]^3*Sin[x])/17
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \cos^4(x)\,dx &= \frac{1}{17}e^x\cos^4(x) + \frac{4}{17}e^x\cos^3(x)\sin(x) + \frac{12}{17}\int e^x\cos^2(x)\,dx\\ &= \frac{12}{85}e^x\cos^2(x) + \frac{1}{17}e^x\cos^4(x) + \frac{24}{85}e^x\cos(x)\sin(x) + \frac{4}{17}e^x\cos^3(x)\sin(x) + \frac{24\int e^x\,dx}{85}\\ &= \frac{24e^x}{85} + \frac{12}{85}e^x\cos^2(x) + \frac{1}{17}e^x\cos^4(x) + \frac{24}{85}e^x\cos(x)\sin(x) + \frac{4}{17}e^x\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.026743, size = 33, normalized size = 0.61

$$\frac{1}{680}e^x(136\sin(2x) + 20\sin(4x) + 68\cos(2x) + 5\cos(4x) + 255)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x]^4,x]
```

```
[Out] (E^x*(255 + 68*Cos[2*x] + 5*Cos[4*x] + 136*Sin[2*x] + 20*Sin[4*x]))/680
```

Maple [A] time = 0.007, size = 34, normalized size = 0.6

$$\frac{(\cos(x) + 4\sin(x))\,e^x\,(\cos(x))^3}{17} + \frac{(12\cos(x) + 24\sin(x))\,e^x\cos(x)}{85} + \frac{24\,e^x}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x)^4,x)
```

```
[Out] 1/17*(cos(x)+4*sin(x))*exp(x)*cos(x)^3+12/85*(cos(x)+2*sin(x))*exp(x)*cos(x
)+24/85*exp(x)
```

Maxima [A] time = 1.02456, size = 50, normalized size = 0.93

$$\frac{1}{136}\cos(4x)\,e^x + \frac{1}{10}\cos(2x)\,e^x + \frac{1}{34}e^x\sin(4x) + \frac{1}{5}e^x\sin(2x) + \frac{3}{8}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="maxima")
```

```
[Out] 1/136*cos(4*x)*e^x + 1/10*cos(2*x)*e^x + 1/34*e^x*sin(4*x) + 1/5*e^x*sin(2*
x) + 3/8*e^x
```

Fricas [A] time = 0.468187, size = 113, normalized size = 2.09

$$\frac{4}{85}\left(5\cos(x)^{3}+6\cos(x)\right)e^{x}\sin(x)+\frac{1}{85}\left(5\cos(x)^{4}+12\cos(x)^{2}+24\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="fricas")
```

```
[Out] 4/85*(5*cos(x)^3 + 6*cos(x))*e^x*sin(x) + 1/85*(5*cos(x)^4 + 12*cos(x)^2 +
24)*e^x
```

Sympy [A] time = 8.32891, size = 70, normalized size = 1.3

$$\frac{24e^{x}\sin^{4}(x)}{85}+\frac{24e^{x}\sin^{3}(x)\cos(x)}{85}+\frac{12e^{x}\sin^{2}(x)\cos^{2}(x)}{17}+\frac{44e^{x}\sin(x)\cos^{3}(x)}{85}+\frac{41e^{x}\cos^{4}(x)}{85}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)**4,x)
```

```
[Out] 24*exp(x)*sin(x)**4/85 + 24*exp(x)*sin(x)**3*cos(x)/85 + 12*exp(x)*sin(x)**
2*cos(x)**2/17 + 44*exp(x)*sin(x)*cos(x)**3/85 + 41*exp(x)*cos(x)**4/85
```

Giac [A] time = 1.15466, size = 47, normalized size = 0.87

$$\frac{1}{136}\left(\cos(4x)+4\sin(4x)\right)e^{x}+\frac{1}{10}\left(\cos(2x)+2\sin(2x)\right)e^{x}+\frac{3}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/136*(cos(4*x) + 4*sin(4*x))*e^x + 1/10*(cos(2*x) + 2*sin(2*x))*e^x + 3/8*
e^x
```

3.19 $\int e^{c(a+bx)}\tan^3(d+ex)\,dx$

Optimal. Leaf size=194

$$-\frac{6ie^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}$$

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) + ((12*I)*E^(c
*(a + b*x))*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*
I)*(d + e*x))])/(b*c) - ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-I/2)
*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.198565, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4442, 2194, 2251}

$$-\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{8ie^{c(a+bx)}\,{}_2F_1\left(3,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```


```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) + ((12*I)*E^(c
*(a + b*x))*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*
I)*(d + e*x))])/(b*c) - ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-I/2)
*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```


Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tan^3(d+ex)\,dx = -\left(i\int\left(-e^{c(a+bx)}+\frac{8e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}+\frac{6e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx\right)$$

$$= i\int e^{c(a+bx)}\,dx-6i\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx-8i\int\frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^3}\,dx+12i\int\frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}\,dx$$

$$= \frac{ie^{c(a+bx)}}{bc}-\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}+\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+e}\right.}{bc}$$

Mathematica [A] time = 2.12599, size = 212, normalized size = 1.09

$$\frac{1}{2}e^{c(a+bx)}\left(\frac{2e^{2id}\left(b^2c^2-2e^2\right)\left(bce^{2iex}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\text{Hypergeometric2F}\right.}{bc\left(1+e^{2id}\right)e^2(-2e+ibc)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((2*(b^2*c^2 - 2*e^2)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hyper
geometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))]
 - (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e,
-E^((2*I)*(d + e*x))]))/(b*c*(I*b*c - 2*e)*e^2*(1 + E^((2*I)*d))) + Sec[d +
 e*x]^2/e - (b*c*Sec[d]*Sec[d + e*x]*Sin[e*x])/e^2 - (2*Tan[d])/(b*c)))/2
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\tan(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] (4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) - b*c*e^(b*c*x + a*c)*sin(2*e*x + 2
*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 + 2*e*cos(2*e*x + 2*d)*e^(b*c*
x + a*c) + (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)*e^(
b*c*x + a*c))*cos(4*e*x + 4*d) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + (b^
2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a
*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a
*c))*sin(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x
 + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2*e*
x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + 2*(b^2*c^2*e^4*e^(a*c
) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) + 4*(b^2*c^2*e^4*e^(a
*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(e^(b*c*x)*sin(2*e*x + 2*d)
/(e^4*cos(2*e*x + 2*d)^2 + e^4*sin(2*e*x + 2*d)^2 + 2*e^4*cos(2*e*x + 2*d)
+ e^4), x) - (b*c*cos(2*e*x + 2*d)*e^(b*c*x + a*c) + b*c*e^(b*c*x + a*c) -
2*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d))*sin(4*e*x + 4*d))/(e^2*cos(4*e*x + 4*
d)^2 + 4*e^2*cos(2*e*x + 2*d)^2 + e^2*sin(4*e*x + 4*d)^2 + 4*e^2*sin(4*e*x
+ 4*d)*sin(2*e*x + 2*d) + 4*e^2*sin(2*e*x + 2*d)^2 + 4*e^2*cos(2*e*x + 2*d)
 + e^2 + 2*(2*e^2*cos(2*e*x + 2*d) + e^2)*cos(4*e*x + 4*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bcx+ac)}\tan\left(ex+d\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)} \tan(ex+d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^3, x)
```

3.20 $\int e^{c(a+bx)}\tan^2(d+ex)\,dx$

Optimal. Leaf size=130

$$\frac{4e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-$$

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-I/2)*
b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x)
)*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*
x))])/(b*c)
```

Rubi [A] time = 0.125604, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4442, 2194, 2251}

$$\frac{4e^{c(a+bx)}\,_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-I/2)*
b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x)
)*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*
x))])/(b*c)
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\tan^2(d+ex)\,dx = -\int \left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}-\frac{4e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx$$

$$= -\left(4\int \frac{e^{c(a+bx)}}{\left(1+e^{2i(d+ex)}\right)^2}\,dx\right)+4\int \frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx-\int e^{c(a+bx)}\,dx$$

$$= -\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}$$

Mathematica [A] time = 1.61657, size = 174, normalized size = 1.34

$$e^{c(a+bx)}\left(\frac{2ie^{2id}\left(bce^{2iex}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\right.\right.}{\left(1+e^{2id}\right)e(bc+2ie)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) + ((2*I)*E^((2*I)*d)*(b*c*E^((2*I)*e*x)*Hyperge
ometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] -
 (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E
^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(1 + E^((2*I)*d))) + (Sec[d]*Sec[d
 + e*x]*Sin[e*x])/e)
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\tan(ex+d)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bcx+ac)}\tan\left(ex+d\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\tan^{2}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)} \tan(ex+d)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d)^2, x)
```

3.21 $\int e^{c(a+bx)}\tan(d+ex)\,dx$

Optimal. Leaf size=78

$$\frac{2ie^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},-e^{2i(d+ex)}\right)}{bc}-\frac{ie^{c(a+bx)}}{bc}$$

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.0756806, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4442, 2194, 2251}

$$\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}-\frac{ie^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))])/(b*c)
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
```

```
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tan(d+ex)\,dx &= i\int\left(-e^{c(a+bx)}+\frac{2e^{c(a+bx)}}{1+e^{2i(d+ex)}}\right)dx\\ &= -\left(i\int e^{c(a+bx)}\,dx\right)+2i\int\frac{e^{c(a+bx)}}{1+e^{2i(d+ex)}}\,dx\\ &= -\frac{ie^{c(a+bx)}}{bc}+\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};-e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] time = 0.46909, size = 166, normalized size = 2.13

$$\frac{e^{c(a+bx)}\left(2bce^{2i(d+ex)}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},-e^{2i(d+ex)}\right)-(bc+2ie)\left(2e^{2id}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e}\right.\right.\right.}{bc\left(1+e^{2id}\right)(-2e+ibc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Tan[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((I/2)
*b*c)/e, 2 - ((I/2)*b*c)/e, -E^((2*I)*(d + e*x))] - (b*c + (2*I)*e)*(1 - E^
((2*I)*d) + 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-I/2)*b*c)/e, 1 - ((I/2)*b
*c)/e, -E^((2*I)*(d + e*x))])))/(b*c*(I*b*c - 2*e)*(1 + E^((2*I)*d)))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\tan(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*tan(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*tan(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)} \tan(ex+d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(e^{(bcx+ac)} \tan(ex+d), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tan(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac} \int e^{bcx} \tan(d+ex)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*tan(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{((bx+a)c)} \tan(ex+d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*tan(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(e^((b*x + a)*c)*tan(e*x + d), x)
```

3.22 $\int e^{c(a+bx)} \cot(d+ex) \, dx$

Optimal. Leaf size=76

$$\frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)}\text{Hypergeometric2F1}\left(1, -\frac{ibc}{2e}, 1 - \frac{ibc}{2e}, e^{2i(d+ex)}\right)}{bc}$$

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.078103, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4443, 2194, 2251}

$$\frac{ie^{c(a+bx)}}{bc} - \frac{2ie^{c(a+bx)} {}_2F_1\left(1, -\frac{ibc}{2e}; 1 - \frac{ibc}{2e}; e^{2i(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (I*E^(c*(a + b*x)))/(b*c) - ((2*I)*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-
I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
```

```
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\cot(d+ex)\,dx &= -\left(i\int\left(-e^{c(a+bx)}-\frac{2e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\right)\\ &= i\int e^{c(a+bx)}\,dx+2i\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx\\ &= \frac{ie^{c(a+bx)}}{bc}-\frac{2ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] time = 1.25579, size = 163, normalized size = 2.14

$$\frac{e^{c(a+bx)}\left(2ibce^{2i(d+ex)}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+i(bc+2ie)\left(-2e^{2id}\text{Hypergeometric2F1}\left(1,-\frac{i}{2}\right.\right.\right.}{bc\left(-1+e^{2id}\right)(bc+2ie)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*((2*I)*b*c*E^((2*I)*(d + e*x))*Hypergeometric2F1[1, 1 - ((
I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + I*(b*c + (2*I)*e)*(1
 + E^((2*I)*d) - 2*E^((2*I)*d)*Hypergeometric2F1[1, ((-I/2)*b*c)/e, 1 - ((I
/2)*b*c)/e, E^((2*I)*(d + e*x))])))/(b*c*(b*c + (2*I)*e)*(-1 + E^((2*I)*d))
)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\cot(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d),x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cot(ex+d)\,e^{((bx+a)c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cot(ex+d)\,e^{(bcx+ac)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(cot(e*x + d)*e^(b*c*x + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\cot(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot(ex+d)\,e^{((bx+a)c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(cot(e*x + d)*e^((b*x + a)*c), x)
```

3.23 $\int e^{c(a+bx)} \cot^2(d+ex)\,dx$

Optimal. Leaf size=126

$$\frac{4e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-I/2)*
b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x))
*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x)
)])/(b*c)
```

Rubi [A] time = 0.124692, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4443, 2194, 2251}

$$\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{e^{c(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] -(E^(c*(a + b*x))/(b*c)) + (4*E^(c*(a + b*x))*Hypergeometric2F1[1, ((-I/2)*
b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - (4*E^(c*(a + b*x))
*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x)
)])/(b*c)
```

Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\cot^2(d+ex)\,dx &= -\int\left(e^{c(a+bx)}+\frac{4e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}+\frac{4e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx\\ &= -\left(4\int\frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}\,dx\right)-4\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx-\int e^{c(a+bx)}\,dx\\ &= -\frac{e^{c(a+bx)}}{bc}+\frac{4e^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{4e^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [A] time = 1.58555, size = 170, normalized size = 1.35

$$e^{c(a+bx)}\left(-\frac{2ie^{2id}\left((bc+2ie)\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)-bce^{2iex}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\right.\right.}{\left(-1+e^{2id}\right)e(bc+2ie)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))*(-(1/(b*c)) - ((2*I)*E^((2*I)*d)*(-(b*c*E^((2*I)*e*x)*Hyper
geometric2F1[1, 1 - ((I/2)*b*c)/e, 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])
 + (b*c + (2*I)*e)*Hypergeometric2F1[1, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e,
E^((2*I)*(d + e*x))]))/((b*c + (2*I)*e)*e*(-1 + E^((2*I)*d))) + (Csc[d]*Csc
[d + e*x]*Sin[e*x])/e)
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\cot(ex+d)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cot\left(ex+d\right)^2 e^{(bcx+ac)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(e*x + d)^2*e^(b*c*x + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$e^{ac}\int e^{bcx}\cot^2\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**2,x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*cot(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot (ex + d)^2 e^{((bx+a)c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(e*x + d)^2*e^((b*x + a)*c), x)
```

3.24 $\int e^{c(a+bx)}\cot^3(d+ex)\,dx$

Optimal. Leaf size=188

$$\frac{6ie^{c(a+bx)}\text{Hypergeometric2F1}\left(1,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\text{Hypergeometric2F1}\left(2,-\frac{ibc}{2e},1-\frac{ibc}{2e},e^{2i(d+ex)}\right)}{bc}+$$

```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - ((12*I)*E^
(c*(a + b*x))*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2
*I)*(d + e*x))])/(b*c) + ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-I/2
)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```

Rubi [A] time = 0.190527, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4443, 2194, 2251}

$$\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}+\frac{8ie^{c(a+bx)}\,{}_2F_1\left(3,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```


```
[Out] ((-I)*E^(c*(a + b*x)))/(b*c) + ((6*I)*E^(c*(a + b*x))*Hypergeometric2F1[1,
((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c) - ((12*I)*E^
(c*(a + b*x))*Hypergeometric2F1[2, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2
*I)*(d + e*x))])/(b*c) + ((8*I)*E^(c*(a + b*x))*Hypergeometric2F1[3, ((-I/2
)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))])/(b*c)
```


Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{c(a+bx)}\cot^3(d+ex)\,dx = i\int\left(-e^{c(a+bx)}-\frac{8e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}-\frac{12e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^2}-\frac{6e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\right)dx$$
$$=-\left(i\int e^{c(a+bx)}\,dx\right)-6i\int\frac{e^{c(a+bx)}}{-1+e^{2i(d+ex)}}\,dx-8i\int\frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+ex)}\right)^3}\,dx-12i\int\frac{e^{c(a+bx)}}{\left(-1+e^{2i(d+e}\right.}$$
$$=-\frac{ie^{c(a+bx)}}{bc}+\frac{6ie^{c(a+bx)}\,{}_2F_1\left(1,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right)}{bc}-\frac{12ie^{c(a+bx)}\,{}_2F_1\left(2,-\frac{ibc}{2e};1-\frac{ibc}{2e};e^{2i(d+ex)}\right.}{bc}$$

Mathematica [A] time = 2.1905, size = 210, normalized size = 1.12

$$\frac{1}{2}e^{c(a+bx)}\left(\frac{2e^{2id}\left(b^2c^2-2e^2\right)\left(ibce^{2iex}\text{Hypergeometric2F1}\left(1,1-\frac{ibc}{2e},2-\frac{ibc}{2e},e^{2i(d+ex)}\right)+(2e-ibc)\text{Hypergeometric2F}\right.}{bc\left(-1+e^{2id}\right)e^2(bc+2ie)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*Cot[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*((-2*Cot[d])/(b*c) - Csc[d + e*x]^2/e + (2*(b^2*c^2 - 2*e^
2)*E^((2*I)*d)*(I*b*c*E^((2*I)*e*x)*Hypergeometric2F1[1, 1 - ((I/2)*b*c)/e,
 2 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))] + ((-I)*b*c + 2*e)*Hypergeometric2
F1[1, ((-I/2)*b*c)/e, 1 - ((I/2)*b*c)/e, E^((2*I)*(d + e*x))]))/(b*c*(b*c +
 (2*I)*e)*e^2*(-1 + E^((2*I)*d))) + (b*c*Csc[d]*Csc[d + e*x]*Sin[e*x])/e^2)
)/2
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int e^{c(bx+a)}\left(\cot(ex+d)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*cot(e*x+d)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -(4*e*cos(2*e*x + 2*d)^2*e^(b*c*x + a*c) + b*c*e^(b*c*x + a*c)*sin(2*e*x +
2*d) + 4*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d)^2 - 2*e*cos(2*e*x + 2*d)*e^(b*c
*x + a*c) - (b*c*e^(b*c*x + a*c)*sin(2*e*x + 2*d) + 2*e*cos(2*e*x + 2*d)*e^
(b*c*x + a*c))*cos(4*e*x + 4*d) + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) +
(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e
^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e
^(a*c))*sin(4*e*x + 4*d)^2 - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*
e*x + 4*d)*sin(2*e*x + 2*d) + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2
*e*x + 2*d)^2 + 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^2*e^4*e^(
a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*e^4*e
^(a*c) - 2*e^6*e^(a*c))*cos(2*e*x + 2*d))*integrate(1/4*e^(b*c*x)*sin(e*x +
 d)/(e^4*cos(e*x + d)^2 + e^4*sin(e*x + d)^2 + 2*e^4*cos(e*x + d) + e^4), x
) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c) + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e
^(a*c))*cos(4*e*x + 4*d)^2 + 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*
e*x + 2*d)^2 + (b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)^2 - 4
*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) +
4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*sin(2*e*x + 2*d)^2 + 2*(b^2*c^2*e^4
*e^(a*c) - 2*e^6*e^(a*c) - 2*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*e*
x + 2*d))*cos(4*e*x + 4*d) - 4*(b^2*c^2*e^4*e^(a*c) - 2*e^6*e^(a*c))*cos(2*
e*x + 2*d))*integrate(1/4*e^(b*c*x)*sin(e*x + d)/(e^4*cos(e*x + d)^2 + e^4*
sin(e*x + d)^2 - 2*e^4*cos(e*x + d) + e^4), x) + (b*c*cos(2*e*x + 2*d)*e^(b
*c*x + a*c) - b*c*e^(b*c*x + a*c) - 2*e*e^(b*c*x + a*c)*sin(2*e*x + 2*d))*s
in(4*e*x + 4*d))/(e^2*cos(4*e*x + 4*d)^2 + 4*e^2*cos(2*e*x + 2*d)^2 + e^2*s
in(4*e*x + 4*d)^2 - 4*e^2*sin(4*e*x + 4*d)*sin(2*e*x + 2*d) + 4*e^2*sin(2*e
*x + 2*d)^2 - 4*e^2*cos(2*e*x + 2*d) + e^2 - 2*(2*e^2*cos(2*e*x + 2*d) - e^
2)*cos(4*e*x + 4*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cot\left(ex+d\right)^3 e^{(bcx+ac)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(cot(e*x + d)^3*e^(b*c*x + a*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cot\left(ex+d\right)^3 e^{((bx+a)c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*cot(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(cot(e*x + d)^3*e^((b*x + a)*c), x)
```

3.25 $\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)\,dx$

Optimal. Leaf size=76

$$\frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{b\log(F)}$$

```
[Out] (I*F^(a + b*x))/(b*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*Log[F])
```

Rubi [A] time = 0.0935517, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4464, 4442, 2194, 2251}

$$\frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{b\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (I*F^(a + b*x))/(b*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I)*
b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*Log[F])
```

Rule 4464

```
Int[(F_)^((c_.)*(u_))*(G_)[v_]^(n_.), x_Symbol] :> Int[F^(c*ExpandToSum[u,
x])*G[ExpandToSum[v, x]]^n, x] /; FreeQ[{F, c, n}, x] && TrigQ[G] && Linear
Q[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int F^{a+bx}\tan\left(\frac{\pi}{4}+\frac{1}{2}(-c-dx)\right)dx &= -\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)dx\\ &= -\left(i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx\right)\\ &= i\int F^{a+bx}\,dx-2i\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx\\ &= \frac{iF^{a+bx}}{b\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{b\log(F)}\end{aligned}$$

Mathematica [A] time = 0.317999, size = 133, normalized size = 1.75

$$\frac{F^{a+bx}\left(b\log(F)e^{i(c+dx)}\text{Hypergeometric2F1}\left(1,1-\frac{ib\log(F)}{d},2-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)+(d-ib\log(F))\text{Hypergeometric2F1}\right.}{b\log(F)(b\log(F)+id)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(a + b*x)*Tan[Pi/4 + (-c - d*x)/2],x]
```

```
[Out] (F^(a + b*x)*(b*E^(I*(c + d*x))*Hypergeometric2F1[1, 1 - (I*b*Log[F])/d, 2
- (I*b*Log[F])/d, I*E^(I*(c + d*x))]*Log[F] + Hypergeometric2F1[1, ((-I)*b*
Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))]*(d - I*b*Log[F])))/(b*Log
[F]*(I*d + b*Log[F]))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int F^{bx+a} \cot\left(\frac{\pi}{4}+\frac{dx}{2}+\frac{c}{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cot(1/4*Pi+1/2*d*x+1/2*c),x)
```

```
[Out] int(F^(b*x+a)*cot(1/4*Pi+1/2*d*x+1/2*c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{bx+a} \cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="maxima")
```

```
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(F^{bx+a} \cot\left(\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{a+bx} \cot\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x)
```

```
[Out] Integral(F**(a + b*x)*cot(c/2 + d*x/2 + pi/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{bx+a} \cot\left(\frac{1}{4}\,\pi + \frac{1}{2}\,dx + \frac{1}{2}\,c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cot(1/2*c+1/4*pi+1/2*d*x),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*cot(1/4*pi + 1/2*d*x + 1/2*c), x)
```

3.26 $\int F^{c(a+bx)} \sec^n(d+ex)\,dx$

Optimal. Leaf size=100

$$\frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\sec^n(d+ex)\text{Hypergeometric2F1}\left(n,\frac{en-ibc\log(F)}{2e},\frac{1}{2}\left(-\frac{ibc\log(F)}{e}+n+2\right),-e^{2i(d+ex)}\right)}{bc\log(F)+ien}$$

```
[Out] ((1 + E^((2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n - I*
b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*Sec[
d + e*x]^n)/(I*e*n + b*c*Log[F])
```

Rubi [A] time = 0.140473, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4454, 2259}

$$\frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\sec^n(d+ex)\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(n-\frac{ibc\log(F)}{e}+2\right);-e^{2i(d+ex)}\right)}{bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

```
[Out] ((1 + E^((2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Hypergeometric2F1[n, (e*n - I*
b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]*Sec[
d + e*x]^n)/(I*e*n + b*c*Log[F])
```

Rule 4454

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[((1 + E^(2*I*(d + e*x)))^n*Sec[d + e*x]^n)/E^(I*n*(d + e*x)), I
nt[SimplifyIntegrand[(F^(c*(a + b*x))*E^(I*n*(d + e*x)))/(1 + E^(2*I*(d + e
*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int F^{c(a+bx)}\sec^n(d+ex)\,dx = \left(e^{-in(d+ex)}\left(1+e^{2i(d+ex)}\right)^n\sec^n(d+ex)\right)\int e^{idn+ienx}\left(1+e^{2i(d+ex)}\right)^{-n}F^{ac+bcx}\,dx$$
$$= \frac{\left(1+e^{2i(d+ex)}\right)^n F^{ac+bcx}\,{}_2F_1\left(n,\frac{en-ibc\log(F)}{2e};\frac{1}{2}\left(2+n-\frac{ibc\log(F)}{e}\right);-e^{2i(d+ex)}\right)\sec^n(d+ex)}{ien+bc\log(F)}$$

Mathematica [A] time = 0.0896643, size = 102, normalized size = 1.02

$$-\frac{i\left(1+e^{2i(d+ex)}\right)^n F^{c(a+bx)}\sec^n(d+ex)\text{Hypergeometric2F1}\left(n,\frac{en-ibc\log(F)}{2e},\frac{1}{2}\left(-\frac{ibc\log(F)}{e}+n+2\right),-e^{2i(d+ex)}\right)}{en-ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Sec[d + e*x]^n,x]
```

```
[Out] ((-I)*(1 + E^((2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Hypergeometric2F1[n, (e*n
 - I*b*c*Log[F])/(2*e), (2 + n - (I*b*c*Log[F])/e)/2, -E^((2*I)*(d + e*x))]
*Sec[d + e*x]^n)/(e*n - I*b*c*Log[F])
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\sec(ex+d)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*sec(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\sec\left(ex+d\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*sec(e*x + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\sec^{n}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*sec(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*sec(d + e*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\sec\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*sec(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*sec(e*x + d)^n, x)
```

3.27 $\int F^{c(a+bx)} \csc^n(d+ex)\,dx$

Optimal. Leaf size=102

$$-\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(\frac{ibc\log(F)}{e}+n+2\right),e^{-2i(d+ex)}\right)}{-bc\log(F)+ien}$$

```
[Out] -(((1 - E^((-2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(I*e*n - b*c*Log[F]))
```

Rubi [A] time = 0.157018, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4455, 2259}

$$-\frac{\left(1-e^{-2i(d+ex)}\right)^n F^{ac+bcx}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(n+\frac{ibc\log(F)}{e}+2\right);e^{-2i(d+ex)}\right)}{-bc\log(F)+ien}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] -(((1 - E^((-2*I)*(d + e*x)))^n*F^(a*c + b*c*x)*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(I*e*n - b*c*Log[F]))
```

Rule 4455

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[((1 - E^(-2*I*(d + e*x)))^n*Csc[d + e*x]^n)/E^(-(I*n*(d + e*x))
), Int[SimplifyIntegrand[F^(c*(a + b*x))/(E^(I*n*(d + e*x))*(1 - E^(-2*I*(d
 + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x] &&  !IntegerQ[n]
```

Rule 2259

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[(G^(h
*(f + g*x))*H^(t*(r + s*x))*(a + b*F^(e*(c + d*x)))^p*Hypergeometric2F1[-p,
 (g*h*Log[G] + s*t*Log[H])/(d*e*Log[F]), (g*h*Log[G] + s*t*Log[H])/(d*e*Log
[F]) + 1, Simplify[-((b*F^(e*(c + d*x)))/a)]])/((g*h*Log[G] + s*t*Log[H])*(
(a + b*F^(e*(c + d*x)))/a)^p), x] /; FreeQ[{F, G, H, a, b, c, d, e, f, g, h
```

```
, r, s, t, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}\csc^n(d+ex)\,dx&=\left(e^{in(d+ex)}\left(1-e^{-2i(d+ex)}\right)^n\csc^n(d+ex)\right)\int e^{-idn-ienx}\left(1-e^{-2i(d+ex)}\right)^{-n}F^{ac+bcx}\,dx\\&=-\frac{\left(1-e^{-2i(d+ex)}\right)^nF^{ac+bcx}\csc^n(d+ex)\,{}_2F_1\left(n,\frac{en+ibc\log(F)}{2e};\frac{1}{2}\left(2+n+\frac{ibc\log(F)}{e}\right);e^{-2i(d+ex)}\right)}{ien-bc\log(F)}\end{aligned}$$

Mathematica [A] time = 0.102885, size = 102, normalized size = 1.

$$\frac{i\left(1-e^{-2i(d+ex)}\right)^nF^{c(a+bx)}\csc^n(d+ex)\text{Hypergeometric2F1}\left(n,\frac{en+ibc\log(F)}{2e},\frac{1}{2}\left(\frac{ibc\log(F)}{e}+n+2\right),e^{-2i(d+ex)}\right)}{en+ibc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*Csc[d + e*x]^n,x]
```

```
[Out] (I*(1 - E^((-2*I)*(d + e*x)))^n*F^(c*(a + b*x))*Csc[d + e*x]^n*Hypergeometr
ic2F1[n, (e*n + I*b*c*Log[F])/(2*e), (2 + n + (I*b*c*Log[F])/e)/2, E^((-2*I
)*(d + e*x))])/(e*n + I*b*c*Log[F])
```

Maple [F] time = 0.554, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(\csc(ex+d)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

```
[Out] int(F^(c*(b*x+a))*csc(e*x+d)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\csc(ex+d)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="maxima")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(F^{bcx+ac}\csc\left(ex+d\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csc(e*x + d)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int F^{c(a+bx)}\csc^{n}\left(d+ex\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*csc(e*x+d)**n,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csc(d + e*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int F^{(bx+a)c}\csc\left(ex+d\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*csc(e*x+d)^n,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csc(e*x + d)^n, x)
```

3.28 $\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$

Optimal. Leaf size=139

$$-\frac{e^{id}F^{ac}(fx)^m(-x(bc\log(F)+ie))^{-m}\text{Gamma}(m+1,-x(bc\log(F)+ie))}{2(e-ibc\log(F))}-\frac{e^{-id}F^{ac}(fx)^m(x(-bc\log(F)+ie))^{-m}\text{Gamma}(m}{2(e+ibc\log(F))}$$

```
[Out] -(F^(a*c)*(f*x)^m*Gamma[1 + m, x*(I*e - b*c*Log[F])])/(2*E^(I*d)*(x*(I*e -
b*c*Log[F]))^m*(e + I*b*c*Log[F])) - (E^(I*d)*F^(a*c)*(f*x)^m*Gamma[1 + m,
-(x*(I*e + b*c*Log[F]))])/(2*(e - I*b*c*Log[F])*(-(x*(I*e + b*c*Log[F])))^m
)
```

Rubi [F] time = 0.490977, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \sin(d+ex)\,dx = \int F^{ac+bcx}(fx)^m \sin(d+ex)\,dx$$

Mathematica [A] time = 0.578081, size = 143, normalized size = 1.03

$$\frac{1}{2}F^{ac}(fx)^m(-x(bc\log(F)+ie))^{-m}\Big(-ix(\cos(d)-i\sin(d))(-bcx\log(F)-iex)^m(ix(e+ibc\log(F)))^{-m-1}\text{Gamma}(m+1,-bcx$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Sin[d + e*x],x]
```

```
[Out] (F^(a*c)*(f*x)^m*((-I)*x*Gamma[1 + m, I*e*x - b*c*x*Log[F]]*(I*x*(e + I*b*c
*Log[F]))^(-1 - m)*((-I)*e*x - b*c*x*Log[F])^m*(Cos[d] - I*Sin[d]) - (Gamma
[1 + m, -(x*(I*e + b*c*Log[F]))]*(Cos[d] + I*Sin[d]))/(e - I*b*c*Log[F])))/
(2*(-(x*(I*e + b*c*Log[F])))^m)
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int F^{c(bx+a)}\left(fx\right)^{m}\sin\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} F^{(bx+a)c}\sin\left(ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

Fricas [A] time = 0.519904, size = 336, normalized size = 2.42

$$\frac{(i\,bc\log\left(F\right)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)-i\,e}{f}\right)-i\,d\right)}\Gamma\left(m+1,-bcx\log\left(F\right)+i\,ex\right)+(-i\,bc\log\left(F\right)-e)e^{\left(ac\log(F)-m\log\left(-\frac{bc\log(F)+i\,e}{f}\right)+}}{2\left(b^{2}c^{2}\log\left(F\right)^{2}+e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/2*((I*b*c*log(F) - e)*e^(a*c*log(F) - m*log(-(b*c*log(F) - I*e)/f) - I*d)
*gamma(m + 1, -b*c*x*log(F) + I*e*x) + (-I*b*c*log(F) - e)*e^(a*c*log(F) -
```

```
m*log(-(b*c*log(F) + I*e)/f) + I*d)*gamma(m + 1, -b*c*x*log(F) - I*e*x))/(b
^2*c^2*log(F)^2 + e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*sin(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m F^{(bx+a)c} \sin(ex+d) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)*sin(e*x + d), x)
```

3.29 $\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left((fx)^m \csc(d+ex)F^{ac+bcx},x\right)$$

```
[Out] CannotIntegrate[F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x], x]
```

Rubi [A] time = 0.622711, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x], x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx = \int F^{ac+bcx}(fx)^m \csc(d+ex)\,dx$$

Mathematica [A] time = 6.41085, size = 0, normalized size = 0.

$$\int F^{c(a+bx)}(fx)^m \csc(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x],x]
```

```
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x], x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)} \left(fx\right)^{m}}{\sin \left(ex+d\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} F^{(bx+a)c}}{\sin \left(ex+d\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(fx\right)^{m} F^{bcx+ac}}{\sin \left(ex+d\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((f*x)^m*F^(b*c*x + a*c)/sin(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d), x)
```

3.30 $\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left((fx)^m \csc^2(d+ex)F^{ac+bcx},x\right)$$

```
[Out] CannotIntegrate[F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x]^2, x]
```

Rubi [A] time = 0.951491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

```
[Out] Defer[Int][F^(a*c + b*c*x)*(f*x)^m*Csc[d + e*x]^2, x]
```

Rubi steps

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx = \int F^{ac+bcx}(fx)^m \csc^2(d+ex)\,dx$$

Mathematica [A] time = 10.6327, size = 0, normalized size = 0.

$$\int F^{c(a+bx)}(fx)^m \csc^2(d+ex)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2,x]
```

```
[Out] Integrate[F^(c*(a + b*x))*(f*x)^m*Csc[d + e*x]^2, x]
```

Maple [A] time = 0.071, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}\left(fx\right)^{m}}{\left(\sin\left(ex+d\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

```
[Out] int(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m}F^{(bx+a)c}}{\sin\left(ex+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(fx\right)^{m}F^{bcx+ac}}{\cos\left(ex+d\right)^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(-(f*x)^m*F^(b*c*x + a*c)/(cos(e*x + d)^2 - 1), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}\left(fx\right)^{m}}{\sin^{2}\left(d+ex\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m/sin(e*x+d)**2,x)
```

```
[Out] Integral(F**(c*(a + b*x))*(f*x)**m/sin(d + e*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m F^{(bx+a)c}}{\sin(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m/sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*F^((b*x + a)*c)/sin(e*x + d)^2, x)
```

3.31 $\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=24

$$(fx)^{m-1}\sin(d+ex)F^{ac+bcx}$$

```
[Out] F^(a*c + b*c*x)*(f*x)^(-1 + m)*Sin[d + e*x]
```

Rubi [A] time = 3.97525, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {12, 6741, 6742, 4468, 4467}

$$(fx)^{m-1}\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*c*x*Lo
g[F])*Sin[d + e*x]),x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^(-1 + m)*Sin[d + e*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4468

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d + e*x])/(f*(m
```

```
+ 1)), x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 4467

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d + e*x])/(f*(m
+ 1)), x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int fF^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1+m+bcx\log(F))\sin(d+ex))\,dx &= f\int F^{c(a+bx)}(fx)^{-2+m}(ex\cos(d+ex)+(-1\\ &= f\int F^{ac+bcx}(fx)^{-2+m}(ex\cos(d+ex)+(-1\\ &= f\int\left(\frac{eF^{ac+bcx}(fx)^{-1+m}\cos(d+ex)}{f}+F^{ac+}\right.\\ &= e\int F^{ac+bcx}(fx)^{-1+m}\cos(d+ex)\,dx+f\int\\ &= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}+f\int\left(-F^{ac+bc}\right.\\ &= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}-(f(1-m))\int\\ &= \frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{fm}+F^{ac+bcx}(fx)^{-1}\\ &= F^{ac+bcx}(fx)^{-1+m}\sin(d+ex)\end{aligned}$$

Mathematica [A] time = 1.33207, size = 26, normalized size = 1.08

$$fx(fx)^{m-2}\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^(-2 + m)*(e*x*Cos[d + e*x] + (-1 + m + b*
c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(a*c + b*c*x)*x*(f*x)^(-2 + m)*Sin[d + e*x]
```

Maple [C] time = 0.251, size = 213, normalized size = 8.9

$$-\frac{i}{2}F^{c(bx+a)}xf\left(\frac{x^m f^m e^{iex} e^{id} e^{-\frac{i}{2}\pi(\mathrm{csgn}(ifx))^3 m} e^{\frac{i}{2}\pi(\mathrm{csgn}(ifx))^2 \mathrm{csgn}(if)m} e^{\frac{i}{2}\pi(\mathrm{csgn}(ifx))^2 \mathrm{csgn}(ix)m} e^{-\frac{i}{2}\pi\,\mathrm{csgn}(ifx)\mathrm{csgn}(if)\mathrm{csgn}(ix)m}}{x^2f^2} - \frac{x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))*sin(e*x
+d)),x)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*x*f*(1/x^2/f^2*x^m*f^m*exp(I*e*x)*exp(I*d)*exp(-1/2*I*
Pi*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*cs
gn(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-1
/x^2/f^2*x^m*f^m*exp(-I*e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1
/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*
exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [A] time = 2.29703, size = 43, normalized size = 1.79

$$\frac{F^{ac}f^{m-1}e^{(bcx\log(F)+m\log(x))}\sin(ex+d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m - 1)*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)/x
```

Fricas [A] time = 0.500663, size = 65, normalized size = 2.71

$$(fx)^{m-2}F^{bcx+ac}fx\sin(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (f*x)^(m - 2)*F^(b*c*x + a*c)*f*x*sin(e*x + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F**(c*(b*x+a))*(f*x)**(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*ln(F))
*sin(e*x+d)),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.32351, size = 8643, normalized size = 360.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^(-2+m)*(e*x*cos(e*x+d)+(-1+m+b*c*x*log(F))*
sin(e*x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c -
 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*lo
g(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1
/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*
sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e -
```

```
 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^
2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
- 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi
*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs
(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*
pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) -
 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4
*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1
) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
 + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sg
n(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*
pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) -
x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*a
bs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2
*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*
tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x
) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi
*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) +
 m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
+ pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn
(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*p
i*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 +
 x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*
abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e -
2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
```

*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4
*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F))
+ m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b
*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*
b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/
4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*
abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs
(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4
*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*p
i*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*ta
n(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(
-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a
*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m
*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*p
i*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*
pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
- 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x
) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*
sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*ab
s(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*
pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*t
an(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a
*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x))
- 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor
(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi
*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1
/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*lo
g(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b
*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*
m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) -
1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^
2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*l

```
og(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*
sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m
*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4
*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)
^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*
tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x
) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*
pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*s
gn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x)) + x*abs(F)^(a*c)*e
^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*
pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*s
gn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 - x*abs(F)^(a*c)*e^(b*c*x*
log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c + 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*ab
s(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m
*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) -
1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(
f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F
)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))
*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(
x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*flo
or(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*p
i*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F))
+ m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1
/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*lo
g(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1
/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*
sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b
*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*
b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/
4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(
```

```
f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(
f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2
*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x
*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1
/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*a
bs(x)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*
m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) -
 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn
(f) - 1/2*pi*sgn(x))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(
F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs(x))
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
 - 1/2*pi*sgn(x)) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2
*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f)
 - 1/2*pi*sgn(x)) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x
)) - 2*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*
abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)) - 2*log(abs(f)*abs
(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d))*f/(tan(1/4*pi*b*c*x*sgn(
F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn
(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*s
gn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*x*sgn(F) -
1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) +
 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x)
 + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c
 + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x
*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(
x))^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4
*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*p
```

```
i*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*ta
n(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4
*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/
4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c +
1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*s
gn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*
sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/
4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 + tan(1/4*pi*b*c*x*sgn(
F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn
(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*s
gn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*p
i*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi
*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2*pi*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) -
1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor
(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi
*m - 1/2*x*e - 2*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1
/2*pi*sgn(x))^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2
*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e - 2
*pi*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2
+ tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn
(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e - 2*pi*fl
oor(-1/4*sgn(f) - 1/4*sgn(x) + 1) - 1/2*pi*sgn(f) - 1/2*pi*sgn(x))^2 + tan(
1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*
a*c - 1/2*d)^2 + 1)
```

3.32 $\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=23

$$fx(fx)^m\sin(d+ex)F^{c(a+bx)}$$

```
[Out] f*F^(c*(a + b*x))*x*(f*x)^m*Sin[d + e*x]
```

Rubi [F] time = 2.34032, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[F])*Si
n[d + e*x]),x]
```

```
[Out] e*Defer[Int][F^(a*c + b*c*x)*(f*x)^(1 + m)*Cos[d + e*x], x] + f*(1 + m)*Def
er[Int][F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x], x] + b*c*Log[F]*Defer[Int][F^
(a*c + b*c*x)*(f*x)^(1 + m)*Sin[d + e*x], x]
```

Rubi steps

$$\int fF^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bcx\log(F))\sin(d+ex))\,dx = f\int F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(1+m+bc$$

$$= f\int F^{ac+bcx}(fx)^m(ex\cos(d+ex)+(1+m+bc$$

$$= f\int\left(\frac{eF^{ac+bcx}(fx)^{1+m}\cos(d+ex)}{f}+F^{ac+bcx}(fx$$

$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+f\int F^{ac+bc$$

$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+f\int\Big(F^{ac+}$$

$$= e\int F^{ac+bcx}(fx)^{1+m}\cos(d+ex)\,dx+(f(1+m))$$

Mathematica [A] time = 0.897916, size = 23, normalized size = 1.

$$fx(fx)^m \sin(d+ex)F^{c(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f*F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (1 + m + b*c*x*Log[
F])*Sin[d + e*x]),x]
```

```
[Out] f*F^(c*(a + b*x))*x*(f*x)^m*Sin[d + e*x]
```

Maple [C] time = 0.161, size = 201, normalized size = 8.7

$$-\frac{i}{2}F^{c(bx+a)}xf\left(x^m f^m e^{iex}e^{id}e^{-\frac{i}{2}\pi\left(\operatorname{csgn}(ifx)\right)^3 m}e^{\frac{i}{2}\pi\left(\operatorname{csgn}(ifx)\right)^2\operatorname{csgn}(if)m}e^{\frac{i}{2}\pi\left(\operatorname{csgn}(ifx)\right)^2\operatorname{csgn}(ix)m}e^{-\frac{i}{2}\pi\operatorname{csgn}(ifx)\operatorname{csgn}(if)\operatorname{csgn}(ix)m} - x^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e*x+d)),x
)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*x*f*(x^m*f^m*exp(I*e*x)*exp(I*d)*exp(-1/2*I*Pi*csgn(I*
f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^
2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-x^m*f^m*exp
(-I*e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^
2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I
*f*x)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [A] time = 2.13198, size = 41, normalized size = 1.78

$$F^{ac} f^{m+1} x e^{(bcx\log(F)+m\log(x))} \sin(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^(m + 1)*x*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

Fricas [A] time = 0.494808, size = 57, normalized size = 2.48

$$(fx)^m F^{bcx+ac} fx \sin(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="fricas")
```

```
[Out] (f*x)^m*F^(b*c*x + a*c)*f*x*sin(e*x + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(1+m+b*c*x*ln(F))*sin(e
*x+d)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.88481, size = 6483, normalized size = 281.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f*F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(1+m+b*c*x*log(F))*sin(e*
x+d)),x, algorithm="giac")
```

```
[Out] (x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c
*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*p
i*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F
) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(
f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi
*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) - x*abs(F)^(a*c
```

```
)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/
4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1
/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c
*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*
sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log
(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + p
i*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*p
i*m - 1/2*x*e)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c
*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*l
og(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4
*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m +
1/2*x*e)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2
*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*
pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(
x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*
sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan
(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m +
 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f)
 - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*l
og(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
 pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(
x) - 1/2*pi*m + 1/2*x*e)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floo
r(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*p
i*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + x*abs(F)^(
a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) -
 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*
b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi
*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*
d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi
*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1
/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*a*c*sgn
(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) -
```

1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F
)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F
) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(
f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*
pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4
*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2
*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)*tan(1/4*pi*b*c*x*
sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m
*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1
/4*pi*a*c - 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*a
bs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*
a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x))
)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn
(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/
4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c -
1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)*tan(1/4*pi*a*c
*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)
^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi
*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1
/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 - x
*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x
*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*
m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F)
- 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f)
+ 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(
F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*f
loor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/
2*pi*m + 1/2*x*e)*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*
sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1
/2*x*e)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan
(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^
(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi
*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*p

```
i*m*sgn(x) - 1/2*pi*m + 1/2*x*e)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d
)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*p
i*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) +
1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)*tan(1/4*pi*a*c*sgn(
F) - 1/4*pi*a*c + 1/2*d)^2 - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(ab
s(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(
f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x
*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) + x*abs(F)^(a*c)*e^(b*c*x
*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x
 + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sg
n(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d) -
x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)
+ x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*
c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*
pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)*tan(1/4*pi*a*c*sgn(F) -
 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)
*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) -
 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)*
tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*log
(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d
)*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + x*abs(F)^(a*c)*e^(b*c*x*l
og(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x +
 pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(
x) - 1/2*pi*m + 1/2*x*e) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(
f)*abs(x)))*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f)
 - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e
) + x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*log(abs(f)*abs(x)))*tan(1/4*pi*
a*c*sgn(F) - 1/4*pi*a*c + 1/2*d) - x*abs(F)^(a*c)*e^(b*c*x*log(abs(F)) + m*
log(abs(f)*abs(x)))*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d))*f/(tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) -
 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b
*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4
*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F
) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*f
loor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/
2*pi*m + 1/2*x*e)^2*tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/
4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m -
 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*
x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi
```

```
*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) -
 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(
1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) +
1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a
*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*
d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/
4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*t
an(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
 + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2*x*e)^2 + tan(1/4
*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1)
+ 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi*a*c*
sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + p
i*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + ta
n(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x)
+ 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m + 1/2*x*e)^2*tan(1/4*pi
*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*
x + pi*m*floor(-1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*s
gn(x) - 1/2*pi*m - 1/2*x*e)^2*tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c - 1/2*d)^2
 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2*tan(1/4*pi*a*c*sgn(F) - 1/
4*pi*a*c - 1/2*d)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-
1/4*sgn(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m
 + 1/2*x*e)^2 + tan(1/4*pi*b*c*x*sgn(F) - 1/4*pi*b*c*x + pi*m*floor(-1/4*sg
n(f) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(f) + 1/4*pi*m*sgn(x) - 1/2*pi*m - 1/2
*x*e)^2 + tan(1/4*pi*a*c*sgn(F) - 1/4*pi*a*c + 1/2*d)^2 + tan(1/4*pi*a*c*sg
n(F) - 1/4*pi*a*c - 1/2*d)^2 + 1)
```

3.33 $\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx$

Optimal. Leaf size=22

$$(fx)^m\sin(d+ex)F^{ac+bcx}$$

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Rubi [A] time = 2.5633, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {16, 6741, 6742, 4467}

$$(fx)^m\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*Sin[d +
 e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4467

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d + e*x])/(f*(m
+ 1)), x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
```

```
 e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(fx)^m(ex\cos(d+ex)+(m+bcx\log(F))\sin(d+ex))}{x}\,dx = f\int F^{c(a+bx)}(fx)^{-1+m}(ex\cos(d+ex)+(m+bcx\log$$
$$= f\int F^{ac+bcx}(fx)^{-1+m}(ex\cos(d+ex)+(m+bcx\log($$
$$= f\int\left(\frac{eF^{ac+bcx}(fx)^m\cos(d+ex)}{f}+F^{ac+bcx}(fx)^{-1+m}(\right.$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+f\int F^{ac+bcx}(fx)^{-}$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+f\int\left(F^{ac+bcx}m(f\right.$$
$$= e\int F^{ac+bcx}(fx)^m\cos(d+ex)\,dx+(fm)\int F^{ac+bcx}(f$$
$$= F^{ac+bcx}(fx)^m\sin(d+ex)$$

Mathematica [A] time = 0.863564, size = 22, normalized size = 1.

$$(fx)^m\sin(d+ex)F^{ac+bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(f*x)^m*(e*x*Cos[d + e*x] + (m + b*c*x*Log[F])*S
in[d + e*x]))/x,x]
```

```
[Out] F^(a*c + b*c*x)*(f*x)^m*Sin[d + e*x]
```

Maple [C] time = 0.15, size = 199, normalized size = 9.1

$$-\frac{i}{2}F^{c(bx+a)}\left(x^mf^m e^{iex}e^{id}e^{-\frac{i}{2}\pi(\operatorname{csgn}(ifx))^3m}e^{\frac{i}{2}\pi(\operatorname{csgn}(ifx))^2\operatorname{csgn}(if)m}e^{\frac{i}{2}\pi(\operatorname{csgn}(ifx))^2\operatorname{csgn}(ix)m}e^{-\frac{i}{2}\pi\operatorname{csgn}(ifx)\operatorname{csgn}(if)\operatorname{csgn}(ix)m}-x^mf^me\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d))/x,x)
```

```
[Out] -1/2*I*F^(c*(b*x+a))*(x^m*f^m*exp(I*e*x)*exp(I*d)*exp(-1/2*I*Pi*csgn(I*f*x)
^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*cs
gn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x)*csgn(I*f)*csgn(I*x)*m)-x^m*f^m*exp(-I*
e*x)*exp(-I*d)*exp(-1/2*I*Pi*csgn(I*f*x)^3*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*cs
gn(I*f)*m)*exp(1/2*I*Pi*csgn(I*f*x)^2*csgn(I*x)*m)*exp(-1/2*I*Pi*csgn(I*f*x
)*csgn(I*f)*csgn(I*x)*m))
```

Maxima [A] time = 2.14592, size = 36, normalized size = 1.64

$$F^{ac} f^m e^{(bcx \log(F) + m \log(x))} \sin(ex + d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="maxima")
```

```
[Out] F^(a*c)*f^m*e^(b*c*x*log(F) + m*log(x))*sin(e*x + d)
```

Fricas [A] time = 0.494112, size = 51, normalized size = 2.32

$$(fx)^m F^{bcx+ac} \sin(ex + d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="fricas")
```

```
[Out] (f*x)^m*F^(b*c*x + a*c)*sin(e*x + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f*x)**m*(e*x*cos(e*x+d)+(m+b*c*x*ln(F))*sin(e*x+d
))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex\cos(ex+d) + (bcx\log(F) + m)\sin(ex+d))\left(fx\right)^m F^{(bx+a)c}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f*x)^m*(e*x*cos(e*x+d)+(m+b*c*x*log(F))*sin(e*x+d)
)/x,x, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) + m)*sin(e*x + d))*(f*x)^m*F^((
b*x + a)*c)/x, x)
```

3.34 $\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx$

Optimal. Leaf size=17

$$x\sin(d+ex)F^{c(a+bx)}$$

```
[Out] F^(c*(a + b*x))*x*Sin[d + e*x]
```

Rubi [B] time = 0.766368, antiderivative size = 327, normalized size of antiderivative = 19.24, number of steps used = 14, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {6741, 6742, 4433, 4466, 4432, 4465}

$$\frac{b^2c^2x\log^2(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{e^2x\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bc\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{b^3c^3\log^3(F)\sin(d+ex)F^{ac+b}}{\left(b^2c^2\log^2(F)+e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*x]),x]
```

```
[Out] (e^3*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (b^2*c^2*e*
F^(a*c + b*c*x)*Cos[d + e*x]*Log[F]^2)/(e^2 + b^2*c^2*Log[F]^2)^2 - (e*F^(a
*c + b*c*x)*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*e^2*F^(a*c + b*c*
x)*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 - (b^3*c^3*F^(a*c + b*c*
x)*Log[F]^3*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)^2 + (e^2*F^(a*c + b*c*x)
*x*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b*c*F^(a*c + b*c*x)*Log[F]*Sin
[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) + (b^2*c^2*F^(a*c + b*c*x)*x*Log[F]^2*S
in[d + e*x])/(e^2 + b^2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int F^{c(a+bx)}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,dx &= \int F^{ac+bcx}(ex\cos(d+ex)+(1+bcx\log(F))\sin(d+ex))\,d\\
&= \int \left(eF^{ac+bcx}x\cos(d+ex)+F^{ac+bcx}(1+bcx\log(F))\sin(d+\\
&= e\int F^{ac+bcx}x\cos(d+ex)\,dx+\int F^{ac+bcx}(1+bcx\log(F))\,\mathrm{si}\\
&= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\\
&= \frac{bceF^{ac+bcx}x\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{e^2F^{ac+bcx}x\sin(d+ex)}{e^2+b^2c^2\log^2(F)}+\\
&= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\\
&= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-\\
&= \frac{e^3F^{ac+bcx}\cos(d+ex)}{\left(e^2+b^2c^2\log^2(F)\right)^2}+\frac{b^2c^2eF^{ac+bcx}\cos(d+ex)\log^2(F)}{\left(e^2+b^2c^2\log^2(F)\right)^2}-
\end{aligned}
$$

Mathematica [A] time = 0.386972, size = 17, normalized size = 1.

$$x\sin(d+ex)F^{c(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (1 + b*c*x*Log[F])*Sin[d + e*
x]),x]
```

```
[Out] F^(c*(a + b*x))*x*Sin[d + e*x]
```

Maple [B] time = 0.079, size = 682, normalized size = 40.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x)
```

```
[Out] (1/(e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2-1/(e^2
+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))+2*ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*
exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1/2*e*x)^2)+e*((b^2*c
^2*ln(F)^2-e^2)/(e^2+b^2*c^2*ln(F)^2)^2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*
e*x)^2+ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*x*exp(c*(b*x+a)*ln(F))-(b^2*c^2*ln(F
)^2-e^2)/(e^2+b^2*c^2*ln(F)^2)^2*exp(c*(b*x+a)*ln(F))+2/(e^2+b^2*c^2*ln(F)^
2)*e*x*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-4*b*c*ln(F)*e/(e^2+b^2*c^2*l
n(F)^2)^2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-ln(F)*b*c/(e^2+b^2*c^2*ln
(F)^2)*x*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2)/(1+tan(1/2*d+1/2*e*x)^2
)+b*c*ln(F)*(1/(e^2+b^2*c^2*ln(F)^2)*e*x*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2
*e*x)^2-1/(e^2+b^2*c^2*ln(F)^2)*e*x*exp(c*(b*x+a)*ln(F))-2*(b^2*c^2*ln(F)^2
-e^2)/(e^2+b^2*c^2*ln(F)^2)^2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)+2*b*c
*ln(F)*e/(e^2+b^2*c^2*ln(F)^2)^2*exp(c*(b*x+a)*ln(F))-2*b*c*ln(F)*e/(e^2+b^
2*c^2*ln(F)^2)^2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2+2*ln(F)*b*c/(e^2
+b^2*c^2*ln(F)^2)*x*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1
/2*e*x)^2)
```

Maxima [B] time = 1.44494, size = 1866, normalized size = 109.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b^2*c^2*log(F)^2*sin(d) + 2*F^(a*c)*b*c*e*cos(d)*log(F) - F^(
a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) + F^(a*c)*b^2*c^2*e*cos(
d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) + F^(a*c)*e^3*cos(d))*x)*F^(b*c
*x)*cos(e*x + 2*d) - (F^(a*c)*b^2*c^2*log(F)^2*sin(d) - 2*F^(a*c)*b*c*e*cos
(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) - F^(a*c
)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) - F^(a*c)*e^3*c
os(d))*x)*F^(b*c*x)*cos(e*x) - (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*F^(a*c)
*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^
3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) - F^(
a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b^2*c^2*cos(d)*log(
F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^
3*cos(d)*log(F)^3 + F^(a*c)*b^2*c^2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos
(d)*log(F) + F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*sin(e*x))*b*c*log(F)/(b^4*c^4
*cos(d)^2*log(F)^4 + b^4*c^4*log(F)^4*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^4
+ 2*(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2)*e^2) - 1/2*((F^
(a*c)*b^2*c^2*cos(d)*log(F)^2 - 2*F^(a*c)*b*c*e*log(F)*sin(d) - F^(a*c)*e^2
```

```
*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^3 - F^(a*c)*b^2*c^2*e*log(F)^2*sin
(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) - F^(a*c)*e^3*sin(d))*x)*F^(b*c*x)*cos(
e*x + 2*d) + (F^(a*c)*b^2*c^2*cos(d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(
d) - F^(a*c)*e^2*cos(d) - (F^(a*c)*b^3*c^3*cos(d)*log(F)^3 + F^(a*c)*b^2*c^
2*e*log(F)^2*sin(d) + F^(a*c)*b*c*e^2*cos(d)*log(F) + F^(a*c)*e^3*sin(d))*x
)*F^(b*c*x)*cos(e*x) + (F^(a*c)*b^2*c^2*log(F)^2*sin(d) + 2*F^(a*c)*b*c*e*c
os(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)^3*sin(d) + F^(a
*c)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(d) + F^(a*c)*e^3
*cos(d))*x)*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b^2*c^2*log(F)^2*sin(d) - 2
*F^(a*c)*b*c*e*cos(d)*log(F) - F^(a*c)*e^2*sin(d) - (F^(a*c)*b^3*c^3*log(F)
^3*sin(d) - F^(a*c)*b^2*c^2*e*cos(d)*log(F)^2 + F^(a*c)*b*c*e^2*log(F)*sin(
d) - F^(a*c)*e^3*cos(d))*x)*F^(b*c*x)*sin(e*x))*e/(b^4*c^4*cos(d)^2*log(F)^
4 + b^4*c^4*log(F)^4*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^4 + 2*(b^2*c^2*cos(
d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2)*e^2) - 1/2*((F^(a*c)*b*c*log(F)*
sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b*c*log(F)*s
in(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F^(a*c)*b*c*cos(d)*log(F) -
 F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) +
F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*
log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
```

Fricas [A] time = 0.470724, size = 43, normalized size = 2.53

$$F^{bcx+ac}x\sin(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*x*sin(e*x + d)
```

Sympy [A] time = 31.3272, size = 19, normalized size = 1.12

$$F^{ac}F^{bcx}x\sin(d+ex)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*ln(F))*sin(e*x+d)),x)
```

```
[Out] F**(a*c)*F**(b*c*x)*x*sin(d + e*x)
```

Giac [C] time = 1.41614, size = 5310, normalized size = 312.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(1+b*c*x*log(F))*sin(e*x+d)),x, alg
orithm="giac")
```

```
[Out] 1/2*(2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) + 2*b*c*e*l
og(abs(F)))*(pi*b*c*x*sgn(F) - pi*b*c*x + 2*x*e)/((pi^2*b^2*c^2*sgn(F) - pi
^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 2*e
^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) + 2*b*c*e
*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(
F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 2*e^2)*(b*c*x*log(abs(F)) - 1)/((p
i^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sg
n(F) + 2*pi*b*c*e - 2*e^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^
2*log(abs(F)) + 2*b*c*e*log(abs(F)))^2))*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b
*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d) - ((pi^2*b^2*c^2*sgn(F) -
pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 2
*e^2)*(pi*b*c*x*sgn(F) - pi*b*c*x + 2*x*e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2
*c^2 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 2*e^2)^2
+ 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) + 2*b*c*e*log(a
bs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) + 2*
b*c*e*log(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2
*c^2 + 2*b^2*c^2*log(abs(F))^2 - 2*pi*b*c*e*sgn(F) + 2*pi*b*c*e - 2*e^2)^2
+ 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) + 2*b*c*e*log(a
bs(F)))^2))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/
2*pi*a*c + x*e + d))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) + 1/2*(2*(
(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) - 2*b*c*e*log(abs(F
)))*(pi*b*c*x*sgn(F) - pi*b*c*x - 2*x*e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c
^2 + 2*b^2*c^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 2*e^2)^2 +
4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) - 2*b*c*e*log(abs
(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2 +
2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 2*e^2)*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*
c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2
*pi*b*c*e - 2*e^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(ab
s(F)) - 2*b*c*e*log(abs(F)))^2))*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1
/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d) - ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2
*c^2 + 2*b^2*c^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 2*e^2)*(p
```

```
i*b*c*x*sgn(F) - pi*b*c*x - 2*x*e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2
*b^2*c^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 2*e^2)^2 + 4*(pi*
b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) - 2*b*c*e*log(abs(F)))^
2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) - 2*b*c*e*lo
g(abs(F)))*(b*c*x*log(abs(F)) - 1)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2
*b^2*c^2*log(abs(F))^2 + 2*pi*b*c*e*sgn(F) - 2*pi*b*c*e - 2*e^2)^2 + 4*(pi*
b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)) - 2*b*c*e*log(abs(F)))^
2))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 - x*e - d))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*I*((2*pi*b*c
*x*sgn(F) - 2*pi*b*c*x - 4*I*b*c*x*log(abs(F)) + 4*x*e + 4*I)*e^(1/2*I*pi*b
*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e +
 I*d)/(4*pi^2*b^2*c^2*sgn(F) + 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b
^2*c^2 - 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 - 8*pi*b*c*e*
sgn(F) + 8*pi*b*c*e + 16*I*b*c*e*log(abs(F)) - 8*e^2) + (2*pi*b*c*x*sgn(F)
- 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) + 4*x*e - 4*I)*e^(-1/2*I*pi*b*c*x*sgn(
F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(4*
pi^2*b^2*c^2*sgn(F) - 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 +
8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 - 8*pi*b*c*e*sgn(F) +
8*pi*b*c*e - 16*I*b*c*e*log(abs(F)) - 8*e^2))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F)) + 1) - 1/2*I*((2*pi*b*c*x*sgn(F) - 2*pi*b*c*x - 4*I*b*c*x*log(abs
(F)) - 4*x*e + 4*I)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(4*pi^2*b^2*c^2*sgn(F) + 8*I*pi*b^2*
c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 - 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^
2*c^2*log(abs(F))^2 + 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e - 16*I*b*c*e*log(abs(F
)) - 8*e^2) + (2*pi*b*c*x*sgn(F) - 2*pi*b*c*x + 4*I*b*c*x*log(abs(F)) - 4*x
*e - 4*I)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F)
+ 1/2*I*pi*a*c + I*x*e + I*d)/(4*pi^2*b^2*c^2*sgn(F) - 8*I*pi*b^2*c^2*log(a
bs(F))*sgn(F) - 4*pi^2*b^2*c^2 + 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log
(abs(F))^2 + 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e + 16*I*b*c*e*log(abs(F)) - 8*e^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)) + 1) - 1/2*((2*pi*b^2*c^2*x*log(
F)*sgn(F) - 2*pi*b^2*c^2*x*log(F) - 4*I*b^2*c^2*x*log(F)*log(abs(F)) + 4*b*
c*x*e*log(F) + 2*pi*b*c*sgn(F) - 2*pi*b*c + 4*I*b*c*log(F) - 4*I*b*c*log(ab
s(F)) + 4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c + I*x*e + I*d)/(4*pi^2*b^2*c^2*sgn(F) + 8*I*pi*b^2*c^2*log
(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 - 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*l
og(abs(F))^2 - 8*pi*b*c*e*sgn(F) + 8*pi*b*c*e + 16*I*b*c*e*log(abs(F)) - 8*
e^2) - (2*pi*b^2*c^2*x*log(F)*sgn(F) - 2*pi*b^2*c^2*x*log(F) + 4*I*b^2*c^2*
x*log(F)*log(abs(F)) + 4*b*c*x*e*log(F) + 2*pi*b*c*sgn(F) - 2*pi*b*c - 4*I*
b*c*log(F) + 4*I*b*c*log(abs(F)) + 4*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*p
i*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(4*pi^2*b^2*c^2
*sgn(F) - 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 + 8*I*pi*b^2*c
^2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 - 8*pi*b*c*e*sgn(F) + 8*pi*b*c*e -
 16*I*b*c*e*log(abs(F)) - 8*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) -
 1/2*I*((2*I*pi*b^2*c^2*x*log(F)*sgn(F) - 2*I*pi*b^2*c^2*x*log(F) + 4*b^2*c
^2*x*log(F)*log(abs(F)) + 4*I*b*c*x*e*log(F) + 2*I*pi*b*c*sgn(F) - 2*I*pi*b
```

```
*c - 4*b*c*log(F) + 4*b*c*log(abs(F)) + 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1
/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(4*pi^2*b
^2*c^2*sgn(F) + 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 - 8*I*pi
*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 - 8*pi*b*c*e*sgn(F) + 8*pi*b
*c*e + 16*I*b*c*e*log(abs(F)) - 8*e^2) - (-2*I*pi*b^2*c^2*x*log(F)*sgn(F) +
 2*I*pi*b^2*c^2*x*log(F) + 4*b^2*c^2*x*log(F)*log(abs(F)) - 4*I*b*c*x*e*log
(F) - 2*I*pi*b*c*sgn(F) + 2*I*pi*b*c - 4*b*c*log(F) + 4*b*c*log(abs(F)) - 4
*I*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/
2*I*pi*a*c - I*x*e - I*d)/(4*pi^2*b^2*c^2*sgn(F) - 8*I*pi*b^2*c^2*log(abs(F
))*sgn(F) - 4*pi^2*b^2*c^2 + 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs
(F))^2 - 8*pi*b*c*e*sgn(F) + 8*pi*b*c*e - 16*I*b*c*e*log(abs(F)) - 8*e^2))*
e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*((2*pi*b^2*c^2*x*log(F)*sgn(F
) - 2*pi*b^2*c^2*x*log(F) - 4*I*b^2*c^2*x*log(F)*log(abs(F)) - 4*b*c*x*e*lo
g(F) + 2*pi*b*c*sgn(F) - 2*pi*b*c + 4*I*b*c*log(F) - 4*I*b*c*log(abs(F)) -
4*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c - I*x*e - I*d)/(4*pi^2*b^2*c^2*sgn(F) + 8*I*pi*b^2*c^2*log(abs(F))
*sgn(F) - 4*pi^2*b^2*c^2 - 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs(F
))^2 + 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e - 16*I*b*c*e*log(abs(F)) - 8*e^2) - (
2*pi*b^2*c^2*x*log(F)*sgn(F) - 2*pi*b^2*c^2*x*log(F) + 4*I*b^2*c^2*x*log(F)
*log(abs(F)) - 4*b*c*x*e*log(F) + 2*pi*b*c*sgn(F) - 2*pi*b*c - 4*I*b*c*log(
F) + 4*I*b*c*log(abs(F)) - 4*e)*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x
- 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(4*pi^2*b^2*c^2*sgn(F)
- 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 + 8*I*pi*b^2*c^2*log(a
bs(F)) + 8*b^2*c^2*log(abs(F))^2 + 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e + 16*I*b*
c*e*log(abs(F)) - 8*e^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(
(-2*I*pi*b^2*c^2*x*log(F)*sgn(F) + 2*I*pi*b^2*c^2*x*log(F) - 4*b^2*c^2*x*lo
g(F)*log(abs(F)) + 4*I*b*c*x*e*log(F) - 2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*
b*c*log(F) - 4*b*c*log(abs(F)) + 4*I*e)*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi
*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(4*pi^2*b^2*c^2*
sgn(F) + 8*I*pi*b^2*c^2*log(abs(F))*sgn(F) - 4*pi^2*b^2*c^2 - 8*I*pi*b^2*c^
2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 + 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e -
16*I*b*c*e*log(abs(F)) - 8*e^2) - (2*I*pi*b^2*c^2*x*log(F)*sgn(F) - 2*I*pi*
b^2*c^2*x*log(F) - 4*b^2*c^2*x*log(F)*log(abs(F)) - 4*I*b*c*x*e*log(F) + 2*
I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(F) - 4*b*c*log(abs(F)) - 4*I*e)*e^
(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c + I*x*e + I*d)/(4*pi^2*b^2*c^2*sgn(F) - 8*I*pi*b^2*c^2*log(abs(F))*sgn(F
) - 4*pi^2*b^2*c^2 + 8*I*pi*b^2*c^2*log(abs(F)) + 8*b^2*c^2*log(abs(F))^2 +
 8*pi*b*c*e*sgn(F) - 8*pi*b*c*e + 16*I*b*c*e*log(abs(F)) - 8*e^2))*e^(b*c*x
*log(abs(F)) + a*c*log(abs(F)))
```

3.35 $\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx$

Optimal. Leaf size=16

$$\sin(d+ex)F^{c(a+bx)}$$

```
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Rubi [A] time = 0.0293524, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2288}

$$\sin(d+ex)F^{c(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

```
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int F^{c(a+bx)}(e\cos(d+ex)+bc\log(F)\sin(d+ex))\,dx=F^{c(a+bx)}\sin(d+ex)$$

Mathematica [A] time = 0.0267141, size = 16, normalized size = 1.

$$\sin(d+ex)F^{c(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(e*Cos[d + e*x] + b*c*Log[F]*Sin[d + e*x]),x]
```

```
[Out] F^(c*(a + b*x))*Sin[d + e*x]
```

Maple [B] time = 0.015, size = 268, normalized size = 16.8

$$e\left(\frac{bc\ln(F)\,\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}+2\,\frac{e\mathrm{e}^{c(bx+a)\ln(F)}\tan(d/2+1/2\,ex)}{e^2+b^2c^2(\ln(F))^2}-\frac{bc\ln(F)\,\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x)
```

```
[Out] e*(ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))+2/(e^2+b^2*c^2*ln(F
)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-ln(F)*b*c/(e^2+b^2*c^2*ln(F)
^2)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2)/(1+tan(1/2*d+1/2*e*x)^2)+b*c
*ln(F)*(1/(e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2
-1/(e^2+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))+2*ln(F)*b*c/(e^2+b^2*c^2*ln
(F)^2)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x))/(1+tan(1/2*d+1/2*e*x)^2)
```

Maxima [B] time = 1.14917, size = 529, normalized size = 33.06

$$-\frac{\left((F^{ac}bc\log(F)\sin(d)+F^{ac}e\cos(d))F^{bcx}\cos(ex+2\,d)-(F^{ac}bc\log(F)\sin(d)-F^{ac}e\cos(d))F^{bcx}\cos(ex)-(F^{ac}bc\cos(d\right.}{2\left(b^2c^2\cos(d)^2\log(F)^2+b^2c^2\log(F)^2\sin(d)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*b*c*log(F)/
(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)
^2)*e^2) + 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*co
s(e*x + 2*d) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos
(e*x) + (F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x +
2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*e
/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d
```

```
)^2)*e^2)
```

Fricas [A] time = 0.47312, size = 41, normalized size = 2.56

$$F^{bcx+ac}\sin(ex+d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(e*x + d)
```

Sympy [A] time = 3.95878, size = 17, normalized size = 1.06

$$F^{ac}F^{bcx}\sin(d+ex)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*cos(e*x+d)+b*c*ln(F)*sin(e*x+d)),x)
```

```
[Out] F**(a*c)*F**(b*c*x)*sin(d + e*x)
```

Giac [C] time = 1.28385, size = 1710, normalized size = 106.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*cos(e*x+d)+b*c*log(F)*sin(e*x+d)),x, algorithm="
giac")
```

```
[Out] (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c + x*e + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b
*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*sin(1/2*pi*b*c*x*sgn(F) - 1/2
*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs
```

```
(F)) + 1) + (2*b*c*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(
F) - 1/2*pi*a*c - x*e - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*sin(1/2*pi*b*c*x*
sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^
2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) +
 a*c*log(abs(F)) + 1) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c
*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(2*I*pi*b*c*sgn(F) -
 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F) +
1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-2*I*pi
*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F))
 + a*c*log(abs(F)) + 1) - 1/2*I*(-2*I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b
*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(2*I*pi*b*c*sgn(F)
 - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e) + 2*I*e^(-1/2*I*pi*b*c*x*sgn(F)
+ 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-2*I*
pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F)) + 1) - I*(b*c*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*
x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)*log(F)/(2*I*pi*b*c*sg
n(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) - b*c*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)*lo
g(F)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*
x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*I*b*c*e^(1/2*I*pi*b*c*x*sgn(F) -
1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)*log(F)/(
2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e) + 2*I*b*c*e^(-1
/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c
- I*x*e - I*d)*log(F)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(abs(F))
- 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(b*c*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*
d)*log(F)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F)) - 4*I*e) - b*
c*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c + I*x*e + I*d)*log(F)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*c*log(a
bs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(-2*I*b*c*e^
(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*
c - I*x*e - I*d)*log(F)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))
 - 4*I*e) - 2*I*b*c*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a
*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)*log(F)/(-2*I*pi*b*c*sgn(F) + 2*I*pi
*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
```

3.36 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx$

Optimal. Leaf size=20

$$\frac{\sin(d+ex)F^{ac+bcx}}{x}$$

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x
```

Rubi [A] time = 1.73196, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {6741, 6742, 4467}

$$\frac{\sin(d+ex)F^{ac+bcx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d + e*x]))
/x^2,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4467

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d + e*x])/(f*(m
+ 1)), x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}\,dx = \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-1+bcx\log(F))\sin(d+ex))}{x^2}$$
$$= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x} + \frac{F^{ac+bcx}(-1+bcx\log(F))\sin(d}{x^2}\right.$$
$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + \int \frac{F^{ac+bcx}(-1+bcx\log(F))}{x^2}$$
$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + \int \left(-\frac{F^{ac+bcx}\sin(d+ex)}{x^2} + \right.$$
$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x}\,dx + (bc\log(F))\int \frac{F^{ac+bcx}\sin(d}{x}$$
$$= \frac{F^{ac+bcx}\sin(d+ex)}{x}$$

Mathematica [A] time = 0.605656, size = 19, normalized size = 0.95

$$\frac{\sin(d+ex)F^{c(a+bx)}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-1 + b*c*x*Log[F])*Sin[d + 
e*x]))/x^2,x]
```

```
[Out] (F^(c*(a + b*x))*Sin[d + e*x])/x
```

Maple [A] time = 0.069, size = 40, normalized size = 2.

$$2\,\frac{e^{c(bx+a)\ln(F)}\tan(d/2+1/2\,ex)}{\left(1+(\tan(d/2+1/2\,ex))^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x^2,x)
```

```
[Out] 2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)/(1+tan(1/2*d+1/2*e*x)^2)/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.469641, size = 43, normalized size = 2.15

$$\frac{F^{bcx+ac}\sin(ex+d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="fricas")
```

```
[Out] F^(b*c*x + a*c)*sin(e*x + d)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{c(a+bx)}\left(bcx\log(F)\sin(d+ex) + ex\cos(d+ex) - \sin(d+ex)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*ln(F))*sin(e*x+d))/x**2,
x)
```

```
[Out] Integral(F**(c*(a + b*x))*(b*c*x*log(F)*sin(d + e*x) + e*x*cos(d + e*x) - s
in(d + e*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex\cos(ex+d) + (bcx\log(F) - 1)\sin(ex+d))F^{(bx+a)c}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-1+b*c*x*log(F))*sin(e*x+d))/x^2,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 1)*sin(e*x + d))*F^((b*x + a)
*c)/x^2, x)
```

3.37 $\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx$

Optimal. Leaf size=20

$$\frac{\sin(d+ex)F^{ac+bcx}}{x^2}$$

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

Rubi [A] time = 1.94979, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {6741, 6742, 4468, 4467}

$$\frac{\sin(d+ex)F^{ac+bcx}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d + e*x]))
/x^3,x]
```

```
[Out] (F^(a*c + b*c*x)*Sin[d + e*x])/x^2
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4468

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^
(m_), x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d + e*x])/(f*(m
+ 1)), x] + (Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d +
e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b
*x))*Cos[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m,
 -1] || SumSimplerQ[m, 1])
```

Rule 4467

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_)*Sin[(d_.) + (e_.)*(
x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*F^(c*(a + b*x))*Sin[d + e*x])/(f*(m
+ 1)), x] + (-Dist[e/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a + b*x))*Cos[d +
 e*x], x], x] - Dist[(b*c*Log[F])/(f*(m + 1)), Int[(f*x)^(m + 1)*F^(c*(a +
b*x))*Sin[d + e*x], x], x]) /; FreeQ[{F, a, b, c, d, e, f, m}, x] && (LtQ[m
, -1] || SumSimplerQ[m, 1])
```

Rubi steps

$$\int \frac{F^{c(a+bx)}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex))}{x^3}\,dx = \int \frac{F^{ac+bcx}(ex\cos(d+ex)+(-2+bcx\log(F))\sin(d+ex)}{x^3}$$

$$= \int \left(\frac{eF^{ac+bcx}\cos(d+ex)}{x^2}+\frac{F^{ac+bcx}(-2+bcx\log(F))\sin(a}{x^3}\right.$$

$$= e\int \frac{F^{ac+bcx}\cos(d+ex)}{x^2}\,dx+\int \frac{F^{ac+bcx}(-2+bcx\log(F))}{x^3}$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-e^2\int \frac{F^{ac+bcx}\sin(d+ex)}{x}\,dx+(bc$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}-2\int \frac{F^{ac+bcx}\sin(d+ex)}{x^3}\,dx-e^2$$

$$= -\frac{eF^{ac+bcx}\cos(d+ex)}{x}+\frac{F^{ac+bcx}\sin(d+ex)}{x^2}-\frac{bcF^{ac+bcx}\,\mathrm{l}}{}$$

$$= \frac{F^{ac+bcx}\sin(d+ex)}{x^2}$$

Mathematica [A] time = 0.626507, size = 19, normalized size = 0.95

$$\frac{\sin(d+ex)F^{c(a+bx)}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(c*(a + b*x))*(e*x*Cos[d + e*x] + (-2 + b*c*x*Log[F])*Sin[d +
e*x]))/x^3,x]
```

```
[Out] (F^(c*(a + b*x))*Sin[d + e*x])/x^2
```

Maple [A] time = 0.093, size = 40, normalized size = 2.

$$2\,\frac{e^{c(bx+a)\ln(F)}\tan\left(d/2+1/2\,ex\right)}{\left(1+\left(\tan\left(d/2+1/2\,ex\right)\right)^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x^3,x)

[Out] 2*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)/(1+tan(1/2*d+1/2*e*x)^2)/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="maxima")

[Out] Exception raised: IndexError
```

Fricas [A] time = 0.471423, size = 46, normalized size = 2.3

$$\frac{F^{bcx+ac}\sin\left(ex+d\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="fricas")

[Out] F^(b*c*x + a*c)*sin(e*x + d)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*ln(F))*sin(e*x+d))/x**3,
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex\cos(ex+d) + (bcx\log(F) - 2)\sin(ex+d))F^{(bx+a)c}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(e*x*cos(e*x+d)+(-2+b*c*x*log(F))*sin(e*x+d))/x^3,x
, algorithm="giac")
```

```
[Out] integrate((e*x*cos(e*x + d) + (b*c*x*log(F) - 2)*sin(e*x + d))*F^((b*x + a)
*c)/x^3, x)
```

3.38 $\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}$$

```
[Out] -((d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2)) + (b*E^(a + b*x)*Sin[2*c
+ 2*d*x])/(2*(b^2 + 4*d^2))
```

Rubi [A] time = 0.0471897, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4469, 12, 4432}

$$\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] -((d*E^(a + b*x)*Cos[2*c + 2*d*x])/(b^2 + 4*d^2)) + (b*E^(a + b*x)*Sin[2*c
+ 2*d*x])/(2*(b^2 + 4*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cos(c+dx)\sin(c+dx)\,dx &= \int \frac{1}{2}e^{a+bx}\sin(2c+2dx)\,dx\\ &= \frac{1}{2}\int e^{a+bx}\sin(2c+2dx)\,dx\\ &= -\frac{de^{a+bx}\cos(2c+2dx)}{b^2+4d^2}+\frac{be^{a+bx}\sin(2c+2dx)}{2\left(b^2+4d^2\right)}\end{aligned}$$

Mathematica [A] time = 0.152665, size = 44, normalized size = 0.7

$$\frac{e^{a+bx}(b\sin(2(c+dx))-2d\cos(2(c+dx)))}{2\left(b^2+4d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x],x]
```

```
[Out] (E^(a + b*x)*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(2*(b^2 + 4*d^2)
)
```

Maple [A] time = 0.014, size = 60, normalized size = 1.

$$-\frac{de^{bx+a}\cos(2\,dx+2\,c)}{b^2+4\,d^2}+\frac{be^{bx+a}\sin(2\,dx+2\,c)}{2\,b^2+8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] -d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*b*exp(b*x+a)*sin(2*d*x+2*c)/(b
^2+4*d^2)
```

Maxima [A] time = 1.04499, size = 59, normalized size = 0.94

$$-\frac{(2\,d\cos(2\,dx+2\,c)-b\sin(2\,dx+2\,c))e^{(bx+a)}}{2\left(b^2+4\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c) - b*sin(2*d*x + 2*c))*e^(b*x + a)/(b^2 + 4*d^2)
```

Fricas [A] time = 0.471155, size = 130, normalized size = 2.06

$$\frac{b\cos(dx+c)\,e^{(bx+a)}\sin(dx+c)-\left(2\,d\cos(dx+c)^2-d\right)e^{(bx+a)}}{b^2+4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cos(d*x + c)*e^(b*x + a)*sin(d*x + c) - (2*d*cos(d*x + c)^2 - d)*e^(b*x
+ a))/(b^2 + 4*d^2)
```

Sympy [A] time = 70.2396, size = 342, normalized size = 5.43

$$\begin{cases} xe^{a}\sin(c)\cos(c) & \text{for } b=0\wedge d= \\ \frac{ixe^{a}e^{-2idx}\sin^2(c+dx)}{4}+\frac{xe^{a}e^{-2idx}\sin(c+dx)\cos(c+dx)}{2}-\frac{ixe^{a}e^{-2idx}\cos^2(c+dx)}{4}+\frac{e^{a}e^{-2idx}\sin^2(c+dx)}{8d}-\frac{e^{a}e^{-2idx}\cos^2(c+dx)}{8d} & \text{for } b=-2id \\ -\frac{ixe^{a}e^{2idx}\sin^2(c+dx)}{4}+\frac{xe^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{2}+\frac{ixe^{a}e^{2idx}\cos^2(c+dx)}{4}-\frac{ie^{a}e^{2idx}\sin(c+dx)\cos(c+dx)}{4d} & \text{for } b=2id \\ \frac{be^{a}e^{bx}\sin(c+dx)\cos(c+dx)}{b^2+4d^2}+\frac{de^{a}e^{bx}\sin^2(c+dx)}{b^2+4d^2}-\frac{de^{a}e^{bx}\cos^2(c+dx)}{b^2+4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*exp(a)*sin(c)*cos(c), Eq(b, 0) & Eq(d, 0)), (I*x*exp(a)*exp(-2
*I*d*x)*sin(c + d*x)**2/4 + x*exp(a)*exp(-2*I*d*x)*sin(c + d*x)*cos(c + d*x
)/2 - I*x*exp(a)*exp(-2*I*d*x)*cos(c + d*x)**2/4 + exp(a)*exp(-2*I*d*x)*sin
```

```
(c + d*x)**2/(8*d) - exp(a)*exp(-2*I*d*x)*cos(c + d*x)**2/(8*d), Eq(b, -2*I
*d)), (-I*x*exp(a)*exp(2*I*d*x)*sin(c + d*x)**2/4 + x*exp(a)*exp(2*I*d*x)*s
in(c + d*x)*cos(c + d*x)/2 + I*x*exp(a)*exp(2*I*d*x)*cos(c + d*x)**2/4 - I*
exp(a)*exp(2*I*d*x)*sin(c + d*x)*cos(c + d*x)/(4*d), Eq(b, 2*I*d)), (b*exp(
a)*exp(b*x)*sin(c + d*x)*cos(c + d*x)/(b**2 + 4*d**2) + d*exp(a)*exp(b*x)*s
in(c + d*x)**2/(b**2 + 4*d**2) - d*exp(a)*exp(b*x)*cos(c + d*x)**2/(b**2 +
4*d**2), True))
```

Giac [A] time = 1.11114, size = 74, normalized size = 1.17

$$-\frac{1}{2}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*c)/(b^2 + 4*d^2)
)*e^(b*x + a)
```

3.39 $\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}+\frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(4*(b^2 + 9*d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) - (3*
d*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rubi [A] time = 0.0929448, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4469, 4433}

$$\frac{de^{a+bx}\sin(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}+\frac{be^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(4*(b^2 + 9*d^2)) + (d*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) - (3*
d*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos(c+dx)\sin^2(c+dx)\,dx = \int\left(\frac{1}{4}e^{a+bx}\cos(c+dx)-\frac{1}{4}e^{a+bx}\cos(3c+3dx)\right)dx$$
$$=\frac{1}{4}\int e^{a+bx}\cos(c+dx)\,dx-\frac{1}{4}\int e^{a+bx}\cos(3c+3dx)\,dx$$
$$=\frac{be^{a+bx}\cos(c+dx)}{4\left(b^2+d^2\right)}-\frac{be^{a+bx}\cos(3c+3dx)}{4\left(b^2+9d^2\right)}+\frac{de^{a+bx}\sin(c+dx)}{4\left(b^2+d^2\right)}-\frac{3de^{a+bx}\sin(3c+}{4\left(b^2+9d^2\right)}$$

Mathematica [A] time = 0.654124, size = 74, normalized size = 0.62

$$\frac{1}{4}e^{a+bx}\left(\frac{b\cos(c+dx)+d\sin(c+dx)}{b^2+d^2}-\frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((b*Cos[c + d*x] + d*Sin[c + d*x])/(b^2 + d^2) - (b*Cos[3*(c +
 d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A] time = 0.027, size = 108, normalized size = 0.9

$$\frac{be^{bx+a}\cos(dx+c)}{4b^2+4d^2}-\frac{be^{bx+a}\cos(3dx+3c)}{4b^2+36d^2}+\frac{de^{bx+a}\sin(dx+c)}{4b^2+4d^2}-\frac{3de^{bx+a}\sin(3dx+3c)}{4b^2+36d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x)
```

```
[Out] 1/4*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/4*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2+
9*d^2)+1/4*d*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*sin(3*d*x+3*c
)/(b^2+9*d^2)
```

Maxima [B] time = 1.17087, size = 726, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a
- 3*b^2*d*e^a*sin(3*c) - 3*d^3*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) - (b^
3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(
3*c))*cos(d*x + 4*c)*e^(b*x) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b
^2*d*e^a*sin(3*c) + 9*d^3*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) + (3*b^2*d*c
os(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a + b^3*e^a*si
n(3*c) + b*d^2*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^2*d*cos(3*c)*e^a
 + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c)
- 9*b*d^2*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Fricas [A] time = 0.481363, size = 243, normalized size = 2.04

$$\frac{\left(b^{2} d + 3 \, d^{3} - 3 \, \left(b^{2} d + d^{3}\right) \cos\left(dx + c\right)^{2}\right) e^{(bx+a)} \sin\left(dx + c\right) - \left(\left(b^{3} + b d^{2}\right) \cos\left(dx + c\right)^{3} - \left(b^{3} + 3 \, b d^{2}\right) \cos\left(dx + c\right)\right) e^{(bx+a)}}{b^{4} + 10 \, b^{2} d^{2} + 9 \, d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d + 3*d^3 - 3*(b^2*d + d^3)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c)
- ((b^3 + b*d^2)*cos(d*x + c)^3 - (b^3 + 3*b*d^2)*cos(d*x + c))*e^(b*x + a)
)/(b^4 + 10*b^2*d^2 + 9*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11371, size = 132, normalized size = 1.11

$$-\frac{1}{4}\left(\frac{b\cos\left(3\,dx+3\,c\right)}{b^{2}+9\,d^{2}}+\frac{3\,d\sin\left(3\,dx+3\,c\right)}{b^{2}+9\,d^{2}}\right)e^{(bx+a)}+\frac{1}{4}\left(\frac{b\cos\left(dx+c\right)}{b^{2}+d^{2}}+\frac{d\sin\left(dx+c\right)}{b^{2}+d^{2}}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) + 1/4*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

3.40 $\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=129

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}+\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

```
[Out] -(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(2*(b^2 + 4*d^2)) + (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) - (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rubi [A] time = 0.0877769, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4469, 4432}

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}+\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] -(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(2*(b^2 + 4*d^2)) + (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) - (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos(c+dx)\sin^3(c+dx)\,dx = \int \left(\frac{1}{4}e^{a+bx}\sin(2c+2dx) - \frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx$$
$$= -\left(\frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx\right) + \frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx$$
$$= -\frac{de^{a+bx}\cos(2c+2dx)}{2(b^2+4d^2)} + \frac{de^{a+bx}\cos(4c+4dx)}{2(b^2+16d^2)} + \frac{be^{a+bx}\sin(2c+2dx)}{4(b^2+4d^2)} - \frac{be^{a+bx}\sin(\ldots}{8(b^2+\ldots}$$

Mathematica [A] time = 0.931011, size = 82, normalized size = 0.64

$$\frac{1}{8}e^{a+bx}\left(\frac{2(b\sin(2(c+dx)) - 2d\cos(2(c+dx)))}{b^2+4d^2} + \frac{4d\cos(4(c+dx)) - b\sin(4(c+dx))}{b^2+16d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (4*d*Cos[4*(c + d*x)] - b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A] time = 0.017, size = 118, normalized size = 0.9

$$-\frac{de^{bx+a}\cos(2dx+2c)}{2b^2+8d^2} + \frac{de^{bx+a}\cos(4dx+4c)}{2b^2+32d^2} + \frac{be^{bx+a}\sin(2dx+2c)}{4b^2+16d^2} - \frac{be^{bx+a}\sin(4dx+4c)}{8b^2+128d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x)
```

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] time = 1.14362, size = 743, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b*
d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4
*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x)
 - 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b*
d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) - 2*(2*b^2*d*cos(4*c)*e^a + 32*d
^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*c
)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c)
 + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*co
s(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
+ 8*c) + 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c)
 - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) + 2*(b^3*cos(4*c)*e^a + 16
*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*s
in(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4*
c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Fricas [A] time = 0.487336, size = 302, normalized size = 2.34

$$-\frac{\left(\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3-\left(b^3+10\,bd^2\right)\cos\left(dx+c\right)\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(4\left(b^2d+4\,d^3\right)\cos\left(dx+c\right)^4+b^2d+10\,d^3\right.}{b^4+20\,b^2d^2+64\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^3 - (b^3 + 10*b*d^2)*cos(d*x + c))*e^(b*x +
 a)*sin(d*x + c) - (4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + b^2*d + 10*d^3 - (5*
b^2*d + 32*d^3)*cos(d*x + c)^2)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20013, size = 150, normalized size = 1.16

$$\frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^2
))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2*
c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.41 $\int e^{a+bx} \cos^2(c+dx) \sin(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

```
[Out] -(d*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(4*(b^2 + 9*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) +
(b*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rubi [A] time = 0.0825251, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4469, 4432}

$$\frac{be^{a+bx}\sin(c+dx)}{4(b^2+d^2)}+\frac{be^{a+bx}\sin(3c+3dx)}{4(b^2+9d^2)}-\frac{de^{a+bx}\cos(c+dx)}{4(b^2+d^2)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4(b^2+9d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

```
[Out] -(d*E^(a + b*x)*Cos[c + d*x])/(4*(b^2 + d^2)) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(4*(b^2 + 9*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(4*(b^2 + d^2)) +
(b*E^(a + b*x)*Sin[3*c + 3*d*x])/(4*(b^2 + 9*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^2(c+dx)\sin(c+dx)\,dx = \int \left(\frac{1}{4}e^{a+bx}\sin(c+dx)+\frac{1}{4}e^{a+bx}\sin(3c+3dx)\right)dx$$
$$= \frac{1}{4}\int e^{a+bx}\sin(c+dx)\,dx+\frac{1}{4}\int e^{a+bx}\sin(3c+3dx)\,dx$$
$$= -\frac{de^{a+bx}\cos(c+dx)}{4\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{4\left(b^2+9d^2\right)}+\frac{be^{a+bx}\sin(c+dx)}{4\left(b^2+d^2\right)}+\frac{be^{a+bx}\sin(3c+}{4\left(b^2+9d\right.}$$

Mathematica [A] time = 0.664618, size = 74, normalized size = 0.62

$$\frac{1}{4}e^{a+bx}\left(\frac{b\sin(c+dx)-d\cos(c+dx)}{b^2+d^2}+\frac{b\sin(3(c+dx))-3d\cos(3(c+dx))}{b^2+9d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x],x]
```

```
[Out] (E^(a + b*x)*((-(d*Cos[c + d*x]) + b*Sin[c + d*x])/(b^2 + d^2) + (-3*d*Cos[
3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2)))/4
```

Maple [A] time = 0.016, size = 108, normalized size = 0.9

$$-\frac{de^{bx+a}\cos(dx+c)}{4\,b^2+4\,d^2}-\frac{3\,de^{bx+a}\cos(3\,dx+3\,c)}{4\,b^2+36\,d^2}+\frac{be^{bx+a}\sin(dx+c)}{4\,b^2+4\,d^2}+\frac{be^{bx+a}\sin(3\,dx+3\,c)}{4\,b^2+36\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x)
```

```
[Out] -1/4*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/4*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)+1/4*b*exp(b*x+a)*sin(d*x+c)/(b^2+d^2)+1/4*b*exp(b*x+a)*sin(3*d*x+3*
c)/(b^2+9*d^2)
```

Maxima [B] time = 1.19727, size = 726, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*e^a - b^3*e^a*sin(3*c) - b*d^2
*e^a*sin(3*c))*cos(3*d*x)*e^(b*x) + (3*b^2*d*cos(3*c)*e^a + 3*d^3*cos(3*c)*
e^a + b^3*e^a*sin(3*c) + b*d^2*e^a*sin(3*c))*cos(3*d*x + 6*c)*e^(b*x) + (b^
2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a + b^3*e^a*sin(3*c) + 9*b*d^2*e^a*sin(
3*c))*cos(d*x + 4*c)*e^(b*x) + (b^2*d*cos(3*c)*e^a + 9*d^3*cos(3*c)*e^a - b
^3*e^a*sin(3*c) - 9*b*d^2*e^a*sin(3*c))*cos(d*x - 2*c)*e^(b*x) - (b^3*cos(3
*c)*e^a + b*d^2*cos(3*c)*e^a + 3*b^2*d*e^a*sin(3*c) + 3*d^3*e^a*sin(3*c))*e
^(b*x)*sin(3*d*x) - (b^3*cos(3*c)*e^a + b*d^2*cos(3*c)*e^a - 3*b^2*d*e^a*si
n(3*c) - 3*d^3*e^a*sin(3*c))*e^(b*x)*sin(3*d*x + 6*c) - (b^3*cos(3*c)*e^a +
 9*b*d^2*cos(3*c)*e^a - b^2*d*e^a*sin(3*c) - 9*d^3*e^a*sin(3*c))*e^(b*x)*si
n(d*x + 4*c) - (b^3*cos(3*c)*e^a + 9*b*d^2*cos(3*c)*e^a + b^2*d*e^a*sin(3*c
) + 9*d^3*e^a*sin(3*c))*e^(b*x)*sin(d*x - 2*c))/(b^4*cos(3*c)^2 + b^4*sin(3
*c)^2 + 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 + 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)
^2)*d^2)
```

Fricas [A] time = 0.486486, size = 224, normalized size = 1.88

$$\frac{\left(2\,bd^2+\left(b^3+bd^2\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(2\,b^2d\cos\left(dx+c\right)-3\left(b^2d+d^3\right)\cos\left(dx+c\right)^3\right)e^{(bx+a)}}{b^4+10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((2*b*d^2 + (b^3 + b*d^2)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (2*b^2
*d*cos(d*x + c) - 3*(b^2*d + d^3)*cos(d*x + c)^3)*e^(b*x + a))/(b^4 + 10*b^
2*d^2 + 9*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15325, size = 135, normalized size = 1.13

$$-\frac{1}{4}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{d\cos\left(dx+c\right)}{b^2+d^2}-\frac{b\sin\left(dx+c\right)}{b^2+d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/4*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x + 3*c)/(b^2 + 9*d^2)
)*e^(b*x + a) - 1/4*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d*x + c)/(b^2 + d^2
))*e^(b*x + a)
```

3.42 $\int e^{a+bx} \cos^2(c+dx) \sin^2(c+dx) \, dx$

Optimal. Leaf size=79

$$-\frac{de^{a+bx}\sin(4c+4dx)}{2\left(b^2+16d^2\right)}-\frac{be^{a+bx}\cos(4c+4dx)}{8\left(b^2+16d^2\right)}+\frac{e^{a+bx}}{8b}$$

```
[Out] E^(a + b*x)/(8*b) - (b*E^(a + b*x)*Cos[4*c + 4*d*x])/(8*(b^2 + 16*d^2)) - (
d*E^(a + b*x)*Sin[4*c + 4*d*x])/(2*(b^2 + 16*d^2))
```

Rubi [A] time = 0.0756521, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4469, 2194, 4433}

$$-\frac{de^{a+bx}\sin(4c+4dx)}{2\left(b^2+16d^2\right)}-\frac{be^{a+bx}\cos(4c+4dx)}{8\left(b^2+16d^2\right)}+\frac{e^{a+bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] E^(a + b*x)/(8*b) - (b*E^(a + b*x)*Cos[4*c + 4*d*x])/(8*(b^2 + 16*d^2)) - (
d*E^(a + b*x)*Sin[4*c + 4*d*x])/(2*(b^2 + 16*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cos^2(c+dx)\sin^2(c+dx)\,dx &= \int\left(\frac{1}{8}e^{a+bx}-\frac{1}{8}e^{a+bx}\cos(4c+4dx)\right)dx\\ &= \frac{1}{8}\int e^{a+bx}\,dx-\frac{1}{8}\int e^{a+bx}\cos(4c+4dx)\,dx\\ &= \frac{e^{a+bx}}{8b}-\frac{be^{a+bx}\cos(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\sin(4c+4dx)}{2\left(b^2+16d^2\right)}\end{aligned}$$

Mathematica [A] time = 0.385389, size = 57, normalized size = 0.72

$$\frac{e^{a+bx}\left(b^2(-\cos(4(c+dx)))+b^2-4bd\sin(4(c+dx))+16d^2\right)}{8\left(b^3+16bd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*(b^2 + 16*d^2 - b^2*Cos[4*(c + d*x)] - 4*b*d*Sin[4*(c + d*x)])
)/(8*(b^3 + 16*b*d^2))
```

Maple [A] time = 0.018, size = 71, normalized size = 0.9

$$\frac{e^{bx+a}}{8\,b}-\frac{be^{bx+a}\cos(4\,dx+4\,c)}{8\,b^2+128\,d^2}-\frac{de^{bx+a}\sin(4\,dx+4\,c)}{2\,b^2+32\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x)
```

```
[Out] 1/8*exp(b*x+a)/b-1/8*b*exp(b*x+a)*cos(4*d*x+4*c)/(b^2+16*d^2)-1/2*d*exp(b*x
+a)*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] time = 1.04606, size = 319, normalized size = 4.04

$$-\frac{\left(b^{2} \cos\left(4 c\right) e^{a} + 4 b d e^{a} \sin\left(4 c\right)\right) \cos\left(4 d x\right) e^{\left(b x\right)} + \left(b^{2} \cos\left(4 c\right) e^{a} - 4 b d e^{a} \sin\left(4 c\right)\right) \cos\left(4 d x + 8 c\right) e^{\left(b x\right)} + \left(4 b d \cos\left(4 c\right.\right.}{16 \left(b^{3} \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*((b^2*cos(4*c)*e^a + 4*b*d*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (b^2*co
s(4*c)*e^a - 4*b*d*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x) + (4*b*d*cos(4*c)
*e^a - b^2*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) + (4*b*d*cos(4*c)*e^a + b^2*e^a
*sin(4*c))*e^(b*x)*sin(4*d*x + 8*c) - 2*(b^2*cos(4*c)^2*e^a + b^2*e^a*sin(4
*c)^2 + 16*(cos(4*c)^2*e^a + e^a*sin(4*c)^2)*d^2)*e^(b*x))/(b^3*cos(4*c)^2
+ b^3*sin(4*c)^2 + 16*(b*cos(4*c)^2 + b*sin(4*c)^2)*d^2)
```

Fricas [A] time = 0.481435, size = 208, normalized size = 2.63

$$-\frac{2 \left(2 b d \cos\left(d x + c\right)^{3} - b d \cos\left(d x + c\right)\right) e^{\left(b x + a\right)} \sin\left(d x + c\right) + \left(b^{2} \cos\left(d x + c\right)^{4} - b^{2} \cos\left(d x + c\right)^{2} - 2 d^{2}\right) e^{\left(b x + a\right)}}{b^{3} + 16 b d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*(2*b*d*cos(d*x + c)^3 - b*d*cos(d*x + c))*e^(b*x + a)*sin(d*x + c) + (b
^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*d^2)*e^(b*x + a))/(b^3 + 16*b*d^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15453, size = 89, normalized size = 1.13

$$-\frac{1}{8}\left(\frac{b\cos\left(4\,dx+4\,c\right)}{b^2+16\,d^2}+\frac{4\,d\sin\left(4\,dx+4\,c\right)}{b^2+16\,d^2}\right)e^{(bx+a)}+\frac{e^{(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(b*cos(4*d*x + 4*c)/(b^2 + 16*d^2) + 4*d*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) + 1/8*e^(b*x + a)/b
```

3.43 $\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=183

$$\frac{be^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}+\frac{be^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\sin(5c+5dx)}{16\left(b^2+25d^2\right)}-\frac{de^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}+\frac{5de^{a+}}{16}$$

```
[Out] -(d*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(16*(b^2 + 9*d^2)) + (5*d*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 +
25*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) + (b*E^(a + b*x)*Si
n[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(
b^2 + 25*d^2))
```

Rubi [A] time = 0.125889, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4469, 4432}

$$\frac{be^{a+bx}\sin(c+dx)}{8\left(b^2+d^2\right)}+\frac{be^{a+bx}\sin(3c+3dx)}{16\left(b^2+9d^2\right)}-\frac{be^{a+bx}\sin(5c+5dx)}{16\left(b^2+25d^2\right)}-\frac{de^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}+\frac{5de^{a+}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] -(d*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Cos[3*c +
3*d*x])/(16*(b^2 + 9*d^2)) + (5*d*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 +
25*d^2)) + (b*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) + (b*E^(a + b*x)*Si
n[3*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(
b^2 + 25*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
```

```
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^2(c+dx)\sin^3(c+dx)\,dx = \int\left(\frac{1}{8}e^{a+bx}\sin(c+dx)+\frac{1}{16}e^{a+bx}\sin(3c+3dx)-\frac{1}{16}e^{a+bx}\sin(5c+5dx)\right)dx$$
$$= \frac{1}{16}\int e^{a+bx}\sin(3c+3dx)\,dx-\frac{1}{16}\int e^{a+bx}\sin(5c+5dx)\,dx+\frac{1}{8}\int e^{a+bx}\sin(c+dx$$
$$= -\frac{de^{a+bx}\cos(c+dx)}{8\left(b^2+d^2\right)}-\frac{3de^{a+bx}\cos(3c+3dx)}{16\left(b^2+9d^2\right)}+\frac{5de^{a+bx}\cos(5c+5dx)}{16\left(b^2+25d^2\right)}+\frac{be^{a+bx}\sin}{8\left(b^2\right.}$$

Mathematica [A] time = 0.913075, size = 110, normalized size = 0.6

$$\frac{1}{16}e^{a+bx}\left(\frac{2(b\sin(c+dx)-d\cos(c+dx))}{b^2+d^2}+\frac{b\sin(3(c+dx))-3d\cos(3(c+dx))}{b^2+9d^2}+\frac{5d\cos(5(c+dx))-b\sin(5(c+dx))}{b^2+25d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*((2*(-(d*Cos[c + d*x]) + b*Sin[c + d*x]))/(b^2 + d^2) + (-3*d*
Cos[3*(c + d*x)] + b*Sin[3*(c + d*x)])/(b^2 + 9*d^2) + (5*d*Cos[5*(c + d*x)
] - b*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A] time = 0.021, size = 166, normalized size = 0.9

$$-\frac{de^{bx+a}\cos(dx+c)}{8b^2+8d^2}-\frac{3de^{bx+a}\cos(3dx+3c)}{16b^2+144d^2}+\frac{5de^{bx+a}\cos(5dx+5c)}{16b^2+400d^2}+\frac{be^{bx+a}\sin(dx+c)}{8b^2+8d^2}+\frac{be^{bx+a}\sin(3dx+3c)}{16b^2+144d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x)
```

```
[Out] -1/8*d*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*cos(3*d*x+3*c)/(b^
2+9*d^2)+5/16*d*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*b*exp(b*x+a)*sin
(d*x+c)/(b^2+d^2)+1/16*b*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-1/16*b*exp(b
*x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Maxima [B] time = 1.29158, size = 1550, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*cos(5*c)*e^a
 - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^4*e^a*sin(5*c))*cos(5
*d*x)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a + 45*d^5*co
s(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c) + 9*b*d^4*e^a*sin(5
*c))*cos(5*d*x + 10*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c
)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^3*d^2*e^a*sin(5*c) +
25*b*d^4*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) - (3*b^4*d*cos(5*c)*e^a + 7
8*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 26*b^3*d^
2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^4*d
*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)*e^a + b^5*e^a*si
n(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5*c))*cos(d*x + 6*c)*e
^(b*x) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(5*c)
*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 225*b*d^4*e^a*sin(5*c))
*cos(d*x - 4*c)*e^(b*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b
*d^4*cos(5*c)*e^a + 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5
*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) - (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)
*e^a + 9*b*d^4*cos(5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c
) - 45*d^5*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (b^5*cos(5*c)*e^a + 26
*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b
^2*d^3*e^a*sin(5*c) - 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (b^5*
cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^
a*sin(5*c) + 78*b^2*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*e^(b*x)*sin(3*d
*x - 2*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5
*c)*e^a - b^4*d*e^a*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*
c))*e^(b*x)*sin(d*x + 6*c) + 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a
+ 225*b*d^4*cos(5*c)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 2
25*d^5*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c)
^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c)
^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

Fricas [A] time = 0.511893, size = 454, normalized size = 2.48

$$\frac{\left(2\,b^3d^2+26\,bd^4-\left(b^5+10\,b^3d^2+9\,bd^4\right)\cos\left(dx+c\right)^4+\left(b^5+14\,b^3d^2+13\,bd^4\right)\cos\left(dx+c\right)^2\right)e^{(bx+a)}\sin\left(dx+c\right)+\left(5\left(\right.\right.}{b^6+35\,b^4d^2+259}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] ((2*b^3*d^2 + 26*b*d^4 - (b^5 + 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^4 + (b^5
 + 14*b^3*d^2 + 13*b*d^4)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) + (5*(b^
4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^5 - (7*b^4*d + 82*b^2*d^3 + 75*d^5)*
cos(d*x + c)^3 + 2*(b^4*d + 13*b^2*d^3)*cos(d*x + c))*e^(b*x + a))/(b^6 + 3
5*b^4*d^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**2*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15763, size = 209, normalized size = 1.14

$$\frac{1}{16}\left(\frac{5\,d\cos\left(5\,dx+5\,c\right)}{b^2+25\,d^2}-\frac{b\sin\left(5\,dx+5\,c\right)}{b^2+25\,d^2}\right)e^{(bx+a)}-\frac{1}{16}\left(\frac{3\,d\cos\left(3\,dx+3\,c\right)}{b^2+9\,d^2}-\frac{b\sin\left(3\,dx+3\,c\right)}{b^2+9\,d^2}\right)e^{(bx+a)}-\frac{1}{8}\left(\frac{d\cos\left(dx+\right.}{b^2+d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*d*cos(5*d*x + 5*c)/(b^2 + 25*d^2) - b*sin(5*d*x + 5*c)/(b^2 + 25*d^
2))*e^(b*x + a) - 1/16*(3*d*cos(3*d*x + 3*c)/(b^2 + 9*d^2) - b*sin(3*d*x +
3*c)/(b^2 + 9*d^2))*e^(b*x + a) - 1/8*(d*cos(d*x + c)/(b^2 + d^2) - b*sin(d
*x + c)/(b^2 + d^2))*e^(b*x + a)
```

3.44 $\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx$

Optimal. Leaf size=129

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}+\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

```
[Out] -(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(2*(b^2 + 4*d^2)) - (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) + (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rubi [A] time = 0.0894424, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4469, 4432}

$$\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}+\frac{be^{a+bx}\sin(4c+4dx)}{8\left(b^2+16d^2\right)}-\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

```
[Out] -(d*E^(a + b*x)*Cos[2*c + 2*d*x])/(2*(b^2 + 4*d^2)) - (d*E^(a + b*x)*Cos[4*
c + 4*d*x])/(2*(b^2 + 16*d^2)) + (b*E^(a + b*x)*Sin[2*c + 2*d*x])/(4*(b^2 +
 4*d^2)) + (b*E^(a + b*x)*Sin[4*c + 4*d*x])/(8*(b^2 + 16*d^2))
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\cos^3(c+dx)\sin(c+dx)\,dx &= \int\left(\frac{1}{4}e^{a+bx}\sin(2c+2dx)+\frac{1}{8}e^{a+bx}\sin(4c+4dx)\right)dx\\ &= \frac{1}{8}\int e^{a+bx}\sin(4c+4dx)\,dx+\frac{1}{4}\int e^{a+bx}\sin(2c+2dx)\,dx\\ &= -\frac{de^{a+bx}\cos(2c+2dx)}{2\left(b^2+4d^2\right)}-\frac{de^{a+bx}\cos(4c+4dx)}{2\left(b^2+16d^2\right)}+\frac{be^{a+bx}\sin(2c+2dx)}{4\left(b^2+4d^2\right)}+\frac{be^{a+bx}\sin}{8\left(b^2+\right.}\end{aligned}$$

Mathematica [A] time = 0.678169, size = 81, normalized size = 0.63

$$\frac{1}{8}e^{a+bx}\left(\frac{2(b\sin(2(c+dx))-2d\cos(2(c+dx)))}{b^2+4d^2}+\frac{b\sin(4(c+dx))-4d\cos(4(c+dx))}{b^2+16d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x],x]
```

```
[Out] (E^(a + b*x)*((2*(-2*d*Cos[2*(c + d*x)] + b*Sin[2*(c + d*x)]))/(b^2 + 4*d^2
) + (-4*d*Cos[4*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2 + 16*d^2)))/8
```

Maple [A] time = 0.015, size = 118, normalized size = 0.9

$$-\frac{d\mathrm{e}^{bx+a}\cos(2\,dx+2\,c)}{2\,b^2+8\,d^2}-\frac{d\mathrm{e}^{bx+a}\cos(4\,dx+4\,c)}{2\,b^2+32\,d^2}+\frac{b\mathrm{e}^{bx+a}\sin(2\,dx+2\,c)}{4\,b^2+16\,d^2}+\frac{b\mathrm{e}^{bx+a}\sin(4\,dx+4\,c)}{8\,b^2+128\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x)
```

```
[Out] -1/2*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)-1/2*d*exp(b*x+a)*cos(4*d*x+4*c
)/(b^2+16*d^2)+1/4*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)+1/8*b*exp(b*x+a)
*sin(4*d*x+4*c)/(b^2+16*d^2)
```

Maxima [B] time = 1.14217, size = 743, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 4*b
*d^2*e^a*sin(4*c))*cos(4*d*x)*e^(b*x) + (4*b^2*d*cos(4*c)*e^a + 16*d^3*cos(
4*c)*e^a + b^3*e^a*sin(4*c) + 4*b*d^2*e^a*sin(4*c))*cos(4*d*x + 8*c)*e^(b*x
) + 2*(2*b^2*d*cos(4*c)*e^a + 32*d^3*cos(4*c)*e^a + b^3*e^a*sin(4*c) + 16*b
*d^2*e^a*sin(4*c))*cos(2*d*x + 6*c)*e^(b*x) + 2*(2*b^2*d*cos(4*c)*e^a + 32*
d^3*cos(4*c)*e^a - b^3*e^a*sin(4*c) - 16*b*d^2*e^a*sin(4*c))*cos(2*d*x - 2*
c)*e^(b*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*cos(4*c)*e^a + 4*b^2*d*e^a*sin(4*c
) + 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x) - (b^3*cos(4*c)*e^a + 4*b*d^2*c
os(4*c)*e^a - 4*b^2*d*e^a*sin(4*c) - 16*d^3*e^a*sin(4*c))*e^(b*x)*sin(4*d*x
 + 8*c) - 2*(b^3*cos(4*c)*e^a + 16*b*d^2*cos(4*c)*e^a - 2*b^2*d*e^a*sin(4*c
) - 32*d^3*e^a*sin(4*c))*e^(b*x)*sin(2*d*x + 6*c) - 2*(b^3*cos(4*c)*e^a + 1
6*b*d^2*cos(4*c)*e^a + 2*b^2*d*e^a*sin(4*c) + 32*d^3*e^a*sin(4*c))*e^(b*x)*
sin(2*d*x - 2*c))/(b^4*cos(4*c)^2 + b^4*sin(4*c)^2 + 64*(cos(4*c)^2 + sin(4
*c)^2)*d^4 + 20*(b^2*cos(4*c)^2 + b^2*sin(4*c)^2)*d^2)
```

Fricas [A] time = 0.487606, size = 262, normalized size = 2.03

$$\frac{\left(6\,bd^2\cos\left(dx+c\right) + \left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3\right)e^{(bx+a)}\sin\left(dx+c\right) + \left(3\,b^2d\cos\left(dx+c\right)^2 - 4\left(b^2d+4\,d^3\right)\cos\left(dx+c\right)^4 + \cdots}{b^4+20\,b^2d^2+64\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((6*b*d^2*cos(d*x + c) + (b^3 + 4*b*d^2)*cos(d*x + c)^3)*e^(b*x + a)*sin(d*
x + c) + (3*b^2*d*cos(d*x + c)^2 - 4*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 6*d^3
)*e^(b*x + a))/(b^4 + 20*b^2*d^2 + 64*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11925, size = 150, normalized size = 1.16

$$-\frac{1}{8}\left(\frac{4\,d\cos\left(4\,dx+4\,c\right)}{b^{2}+16\,d^{2}}-\frac{b\sin\left(4\,dx+4\,c\right)}{b^{2}+16\,d^{2}}\right)e^{(bx+a)}-\frac{1}{4}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^{2}+4\,d^{2}}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^{2}+4\,d^{2}}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(4*d*cos(4*d*x + 4*c)/(b^2 + 16*d^2) - b*sin(4*d*x + 4*c)/(b^2 + 16*d^
2))*e^(b*x + a) - 1/4*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x + 2
*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.45 $\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=183

$$\frac{de^{a+bx}\sin(c+dx)}{8(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)}-\frac{5de^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}+\frac{be^{a+bx}\cos(c+dx)}{8(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)}-\frac{be^{a+}}{1}$$

```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 + 25*d^
2)) + (d*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Sin[3
*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (5*d*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(b
^2 + 25*d^2))
```

Rubi [A] time = 0.125023, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4469, 4433}

$$\frac{de^{a+bx}\sin(c+dx)}{8(b^2+d^2)}-\frac{3de^{a+bx}\sin(3c+3dx)}{16(b^2+9d^2)}-\frac{5de^{a+bx}\sin(5c+5dx)}{16(b^2+25d^2)}+\frac{be^{a+bx}\cos(c+dx)}{8(b^2+d^2)}-\frac{be^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)}-\frac{be^{a+}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```


```
[Out] (b*E^(a + b*x)*Cos[c + d*x])/(8*(b^2 + d^2)) - (b*E^(a + b*x)*Cos[3*c + 3*d
*x])/(16*(b^2 + 9*d^2)) - (b*E^(a + b*x)*Cos[5*c + 5*d*x])/(16*(b^2 + 25*d^
2)) + (d*E^(a + b*x)*Sin[c + d*x])/(8*(b^2 + d^2)) - (3*d*E^(a + b*x)*Sin[3
*c + 3*d*x])/(16*(b^2 + 9*d^2)) - (5*d*E^(a + b*x)*Sin[5*c + 5*d*x])/(16*(b
^2 + 25*d^2))
```


Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
```

```
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^3(c+dx)\sin^2(c+dx)\,dx = \int \left(\frac{1}{8}e^{a+bx}\cos(c+dx) - \frac{1}{16}e^{a+bx}\cos(3c+3dx) - \frac{1}{16}e^{a+bx}\cos(5c+5dx)\right)dx$$
$$= -\left(\frac{1}{16}\int e^{a+bx}\cos(3c+3dx)\,dx\right) - \frac{1}{16}\int e^{a+bx}\cos(5c+5dx)\,dx + \frac{1}{8}\int e^{a+bx}\cos(c$$
$$= \frac{be^{a+bx}\cos(c+dx)}{8(b^2+d^2)} - \frac{be^{a+bx}\cos(3c+3dx)}{16(b^2+9d^2)} - \frac{be^{a+bx}\cos(5c+5dx)}{16(b^2+25d^2)} + \frac{de^{a+bx}\sin(c+}{8(b^2+d^2}$$

Mathematica [A] time = 0.764136, size = 110, normalized size = 0.6

$$\frac{1}{16}e^{a+bx}\left(\frac{2(b\cos(c+dx)+d\sin(c+dx))}{b^2+d^2} - \frac{b\cos(3(c+dx))+3d\sin(3(c+dx))}{b^2+9d^2} - \frac{b\cos(5(c+dx))+5d\sin(5(c+dx))}{b^2+25d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (E^(a + b*x)*((2*(b*Cos[c + d*x] + d*Sin[c + d*x]))/(b^2 + d^2) - (b*Cos[3*
(c + d*x)] + 3*d*Sin[3*(c + d*x)])/(b^2 + 9*d^2) - (b*Cos[5*(c + d*x)] + 5*
d*Sin[5*(c + d*x)])/(b^2 + 25*d^2)))/16
```

Maple [A] time = 0.028, size = 166, normalized size = 0.9

$$\frac{be^{bx+a}\cos(dx+c)}{8b^2+8d^2} - \frac{be^{bx+a}\cos(3dx+3c)}{16b^2+144d^2} - \frac{be^{bx+a}\cos(5dx+5c)}{16b^2+400d^2} + \frac{de^{bx+a}\sin(dx+c)}{8b^2+8d^2} - \frac{3de^{bx+a}\sin(3dx+3c)}{16b^2+144d^2} - 5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x)
```

```
[Out] 1/8*b*exp(b*x+a)*cos(d*x+c)/(b^2+d^2)-1/16*b*exp(b*x+a)*cos(3*d*x+3*c)/(b^2
+9*d^2)-1/16*b*exp(b*x+a)*cos(5*d*x+5*c)/(b^2+25*d^2)+1/8*d*exp(b*x+a)*sin(
d*x+c)/(b^2+d^2)-3/16*d*exp(b*x+a)*sin(3*d*x+3*c)/(b^2+9*d^2)-5/16*d*exp(b*
```

```
x+a)*sin(5*d*x+5*c)/(b^2+25*d^2)
```

Maxima [B] time = 1.28723, size = 1544, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/32*((b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(5*c)*e^a +
 5*b^4*d*e^a*sin(5*c) + 50*b^2*d^3*e^a*sin(5*c) + 45*d^5*e^a*sin(5*c))*cos(
5*d*x)*e^(b*x) + (b^5*cos(5*c)*e^a + 10*b^3*d^2*cos(5*c)*e^a + 9*b*d^4*cos(
5*c)*e^a - 5*b^4*d*e^a*sin(5*c) - 50*b^2*d^3*e^a*sin(5*c) - 45*d^5*e^a*sin(
5*c))*cos(5*d*x + 10*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b^3*d^2*cos(5*c)*e
^a + 25*b*d^4*cos(5*c)*e^a - 3*b^4*d*e^a*sin(5*c) - 78*b^2*d^3*e^a*sin(5*c)
 - 75*d^5*e^a*sin(5*c))*cos(3*d*x + 8*c)*e^(b*x) + (b^5*cos(5*c)*e^a + 26*b
^3*d^2*cos(5*c)*e^a + 25*b*d^4*cos(5*c)*e^a + 3*b^4*d*e^a*sin(5*c) + 78*b^2
*d^3*e^a*sin(5*c) + 75*d^5*e^a*sin(5*c))*cos(3*d*x - 2*c)*e^(b*x) - 2*(b^5*
cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c)*e^a - b^4*d*e^a
*sin(5*c) - 34*b^2*d^3*e^a*sin(5*c) - 225*d^5*e^a*sin(5*c))*cos(d*x + 6*c)*
e^(b*x) - 2*(b^5*cos(5*c)*e^a + 34*b^3*d^2*cos(5*c)*e^a + 225*b*d^4*cos(5*c
)*e^a + b^4*d*e^a*sin(5*c) + 34*b^2*d^3*e^a*sin(5*c) + 225*d^5*e^a*sin(5*c)
)*cos(d*x - 4*c)*e^(b*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos(5*c)*e^a
+ 45*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 10*b^3*d^2*e^a*sin(5*c) - 9*b*d^
4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x) + (5*b^4*d*cos(5*c)*e^a + 50*b^2*d^3*cos
(5*c)*e^a + 45*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 10*b^3*d^2*e^a*sin(5*c
) + 9*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(5*d*x + 10*c) + (3*b^4*d*cos(5*c)*e^a
 + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a + b^5*e^a*sin(5*c) + 26*b^
3*d^2*e^a*sin(5*c) + 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*d*x + 8*c) + (3*b
^4*d*cos(5*c)*e^a + 78*b^2*d^3*cos(5*c)*e^a + 75*d^5*cos(5*c)*e^a - b^5*e^a
*sin(5*c) - 26*b^3*d^2*e^a*sin(5*c) - 25*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(3*
d*x - 2*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e^a + 225*d^5*cos(
5*c)*e^a + b^5*e^a*sin(5*c) + 34*b^3*d^2*e^a*sin(5*c) + 225*b*d^4*e^a*sin(5
*c))*e^(b*x)*sin(d*x + 6*c) - 2*(b^4*d*cos(5*c)*e^a + 34*b^2*d^3*cos(5*c)*e
^a + 225*d^5*cos(5*c)*e^a - b^5*e^a*sin(5*c) - 34*b^3*d^2*e^a*sin(5*c) - 22
5*b*d^4*e^a*sin(5*c))*e^(b*x)*sin(d*x - 4*c))/(b^6*cos(5*c)^2 + b^6*sin(5*c
)^2 + 225*(cos(5*c)^2 + sin(5*c)^2)*d^6 + 259*(b^2*cos(5*c)^2 + b^2*sin(5*c
)^2)*d^4 + 35*(b^4*cos(5*c)^2 + b^4*sin(5*c)^2)*d^2)
```

Fricas [A] time = 0.510938, size = 444, normalized size = 2.43

$$\frac{\left(6\,b^{2}d^{3}+30\,d^{5}-5\left(b^{4}d+10\,b^{2}d^{3}+9\,d^{5}\right)\cos\left(dx+c\right)^{4}+3\left(b^{4}d+6\,b^{2}d^{3}+5\,d^{5}\right)\cos\left(dx+c\right)^{2}\right)e^{(bx+a)}\sin\left(dx+c\right)-\left(\left(b^{5}+\right.\right.}{b^{6}+35\,b^{4}d^{2}+259\,b^{2}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*d^3 + 30*d^5 - 5*(b^4*d + 10*b^2*d^3 + 9*d^5)*cos(d*x + c)^4 + 3*(b
^4*d + 6*b^2*d^3 + 5*d^5)*cos(d*x + c)^2)*e^(b*x + a)*sin(d*x + c) - ((b^5
+ 10*b^3*d^2 + 9*b*d^4)*cos(d*x + c)^5 - (b^5 + 6*b^3*d^2 + 5*b*d^4)*cos(d*
x + c)^3 - 6*(b^3*d^2 + 5*b*d^4)*cos(d*x + c))*e^(b*x + a))/(b^6 + 35*b^4*d
^2 + 259*b^2*d^4 + 225*d^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11975, size = 205, normalized size = 1.12

$$-\frac{1}{16}\left(\frac{b\cos\left(5\,dx+5\,c\right)}{b^{2}+25\,d^{2}}+\frac{5\,d\sin\left(5\,dx+5\,c\right)}{b^{2}+25\,d^{2}}\right)e^{(bx+a)}-\frac{1}{16}\left(\frac{b\cos\left(3\,dx+3\,c\right)}{b^{2}+9\,d^{2}}+\frac{3\,d\sin\left(3\,dx+3\,c\right)}{b^{2}+9\,d^{2}}\right)e^{(bx+a)}+\frac{1}{8}\left(\frac{b\cos\left(dx\right.}{b^{2}+d^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*(b*cos(5*d*x + 5*c)/(b^2 + 25*d^2) + 5*d*sin(5*d*x + 5*c)/(b^2 + 25*d
^2))*e^(b*x + a) - 1/16*(b*cos(3*d*x + 3*c)/(b^2 + 9*d^2) + 3*d*sin(3*d*x +
 3*c)/(b^2 + 9*d^2))*e^(b*x + a) + 1/8*(b*cos(d*x + c)/(b^2 + d^2) + d*sin(
d*x + c)/(b^2 + d^2))*e^(b*x + a)
```

3.46 $\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=129

$$\frac{3be^{a+bx}\sin(2c+2dx)}{32\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(6c+6dx)}{32\left(b^2+36d^2\right)}-\frac{3de^{a+bx}\cos(2c+2dx)}{16\left(b^2+4d^2\right)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16\left(b^2+36d^2\right)}$$

```
[Out] (-3*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(16*(b^2 + 4*d^2)) + (3*d*E^(a + b*x)*C
os[6*c + 6*d*x])/(16*(b^2 + 36*d^2)) + (3*b*E^(a + b*x)*Sin[2*c + 2*d*x])/(
32*(b^2 + 4*d^2)) - (b*E^(a + b*x)*Sin[6*c + 6*d*x])/(32*(b^2 + 36*d^2))
```

Rubi [A] time = 0.100633, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4469, 4432}

$$\frac{3be^{a+bx}\sin(2c+2dx)}{32\left(b^2+4d^2\right)}-\frac{be^{a+bx}\sin(6c+6dx)}{32\left(b^2+36d^2\right)}-\frac{3de^{a+bx}\cos(2c+2dx)}{16\left(b^2+4d^2\right)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16\left(b^2+36d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```


```
[Out] (-3*d*E^(a + b*x)*Cos[2*c + 2*d*x])/(16*(b^2 + 4*d^2)) + (3*d*E^(a + b*x)*C
os[6*c + 6*d*x])/(16*(b^2 + 36*d^2)) + (3*b*E^(a + b*x)*Sin[2*c + 2*d*x])/(
32*(b^2 + 4*d^2)) - (b*E^(a + b*x)*Sin[6*c + 6*d*x])/(32*(b^2 + 36*d^2))
```


Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{a+bx}\cos^3(c+dx)\sin^3(c+dx)\,dx = \int\left(\frac{3}{32}e^{a+bx}\sin(2c+2dx)-\frac{1}{32}e^{a+bx}\sin(6c+6dx)\right)dx$$
$$= -\left(\frac{1}{32}\int e^{a+bx}\sin(6c+6dx)\,dx\right)+\frac{3}{32}\int e^{a+bx}\sin(2c+2dx)\,dx$$
$$= -\frac{3de^{a+bx}\cos(2c+2dx)}{16\left(b^2+4d^2\right)}+\frac{3de^{a+bx}\cos(6c+6dx)}{16\left(b^2+36d^2\right)}+\frac{3be^{a+bx}\sin(2c+2dx)}{32\left(b^2+4d^2\right)}-\frac{be^{a+b\ldots}}{32\ldots}$$

Mathematica [A] time = 0.948675, size = 111, normalized size = 0.86

$$\frac{e^{a+bx}\left(-6d\left(b^2+36d^2\right)\cos(2(c+dx))+6d\left(b^2+4d^2\right)\cos(6(c+dx))-2b\sin(2(c+dx))\left(\left(b^2+4d^2\right)\cos(4(c+dx))-b^2-\ldots\right.\right.}{32\left(40b^2d^2+b^4+144d^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Cos[c + d*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (E^(a + b*x)*(-6*d*(b^2 + 36*d^2)*Cos[2*(c + d*x)] + 6*d*(b^2 + 4*d^2)*Cos[
6*(c + d*x)] - 2*b*(-b^2 - 52*d^2 + (b^2 + 4*d^2)*Cos[4*(c + d*x)])*Sin[2*(
c + d*x)]))/(32*(b^4 + 40*b^2*d^2 + 144*d^4))
```

Maple [A] time = 0.022, size = 118, normalized size = 0.9

$$-\frac{3\,d{e}^{bx+a}\cos\left(2\,dx+2\,c\right)}{16\,{b}^{2}+64\,{d}^{2}}+\frac{3\,d{e}^{bx+a}\cos\left(6\,dx+6\,c\right)}{16\,{b}^{2}+576\,{d}^{2}}+\frac{3\,b{e}^{bx+a}\sin\left(2\,dx+2\,c\right)}{32\,{b}^{2}+128\,{d}^{2}}-\frac{b{e}^{bx+a}\sin\left(6\,dx+6\,c\right)}{32\,{b}^{2}+1152\,{d}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x)
```

```
[Out] -3/16*d*exp(b*x+a)*cos(2*d*x+2*c)/(b^2+4*d^2)+3/16*d*exp(b*x+a)*cos(6*d*x+6
*c)/(b^2+36*d^2)+3/32*b*exp(b*x+a)*sin(2*d*x+2*c)/(b^2+4*d^2)-1/32*b*exp(b*
x+a)*sin(6*d*x+6*c)/(b^2+36*d^2)
```

Maxima [B] time = 1.1529, size = 743, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 4*b*
d^2*e^a*sin(6*c))*cos(6*d*x)*e^(b*x) + (6*b^2*d*cos(6*c)*e^a + 24*d^3*cos(6
*c)*e^a + b^3*e^a*sin(6*c) + 4*b*d^2*e^a*sin(6*c))*cos(6*d*x + 12*c)*e^(b*x
) - 3*(2*b^2*d*cos(6*c)*e^a + 72*d^3*cos(6*c)*e^a + b^3*e^a*sin(6*c) + 36*b
*d^2*e^a*sin(6*c))*cos(2*d*x + 8*c)*e^(b*x) - 3*(2*b^2*d*cos(6*c)*e^a + 72*
d^3*cos(6*c)*e^a - b^3*e^a*sin(6*c) - 36*b*d^2*e^a*sin(6*c))*cos(2*d*x - 4*
c)*e^(b*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*cos(6*c)*e^a + 6*b^2*d*e^a*sin(6*c
) + 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x) - (b^3*cos(6*c)*e^a + 4*b*d^2*c
os(6*c)*e^a - 6*b^2*d*e^a*sin(6*c) - 24*d^3*e^a*sin(6*c))*e^(b*x)*sin(6*d*x
 + 12*c) + 3*(b^3*cos(6*c)*e^a + 36*b*d^2*cos(6*c)*e^a - 2*b^2*d*e^a*sin(6*
c) - 72*d^3*e^a*sin(6*c))*e^(b*x)*sin(2*d*x + 8*c) + 3*(b^3*cos(6*c)*e^a +
36*b*d^2*cos(6*c)*e^a + 2*b^2*d*e^a*sin(6*c) + 72*d^3*e^a*sin(6*c))*e^(b*x)
*sin(2*d*x - 4*c))/(b^4*cos(6*c)^2 + b^4*sin(6*c)^2 + 144*(cos(6*c)^2 + sin
(6*c)^2)*d^4 + 40*(b^2*cos(6*c)^2 + b^2*sin(6*c)^2)*d^2)
```

Fricas [A] time = 0.493848, size = 356, normalized size = 2.76

$$-\frac{\left(\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^5-6\,bd^2\cos\left(dx+c\right)-\left(b^3+4\,bd^2\right)\cos\left(dx+c\right)^3\right)e^{(bx+a)}\sin\left(dx+c\right)-3\,(2\,\left(b^2d+4\,d^3\right)\cos(}{b^4+40\,b^2d^2+144\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(((b^3 + 4*b*d^2)*cos(d*x + c)^5 - 6*b*d^2*cos(d*x + c) - (b^3 + 4*b*d^2)*
cos(d*x + c)^3)*e^(b*x + a)*sin(d*x + c) - 3*(2*(b^2*d + 4*d^3)*cos(d*x + c
)^6 + b^2*d*cos(d*x + c)^2 - 3*(b^2*d + 4*d^3)*cos(d*x + c)^4 + 2*d^3)*e^(b
*x + a))/(b^4 + 40*b^2*d^2 + 144*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)**3*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14695, size = 150, normalized size = 1.16

$$\frac{1}{32}\left(\frac{6\,d\cos\left(6\,dx+6\,c\right)}{b^2+36\,d^2}-\frac{b\sin\left(6\,dx+6\,c\right)}{b^2+36\,d^2}\right)e^{(bx+a)}-\frac{3}{32}\left(\frac{2\,d\cos\left(2\,dx+2\,c\right)}{b^2+4\,d^2}-\frac{b\sin\left(2\,dx+2\,c\right)}{b^2+4\,d^2}\right)e^{(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*cos(d*x+c)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(6*d*cos(6*d*x + 6*c)/(b^2 + 36*d^2) - b*sin(6*d*x + 6*c)/(b^2 + 36*d^
2))*e^(b*x + a) - 3/32*(2*d*cos(2*d*x + 2*c)/(b^2 + 4*d^2) - b*sin(2*d*x +
2*c)/(b^2 + 4*d^2))*e^(b*x + a)
```

3.47 $\int e^x x \sin(x)\, dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x \cos(x) - \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*Cos[x])/2 - (E^x*x*Cos[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0393673, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4432, 4465, 4433}

$$\frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x \cos(x) - \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Sin[x],x]
```

```
[Out] (E^x*Cos[x])/2 - (E^x*x*Cos[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^x x\sin(x)\,dx &= -\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\ &= -\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)+\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\ &= \frac{1}{2}e^x\cos(x)-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0379688, size = 19, normalized size = 0.63

$$\frac{1}{2}e^x(x\sin(x)-x\cos(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Sin[x],x]
```

```
[Out] (E^x*(Cos[x] - x*Cos[x] + x*Sin[x]))/2
```

Maple [A] time = 0.007, size = 19, normalized size = 0.6

$$\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\cos(x)+\frac{e^x x\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*sin(x),x)
```

```
[Out] (-1/2*x+1/2)*exp(x)*cos(x)+1/2*exp(x)*x*sin(x)
```

Maxima [A] time = 1.02485, size = 23, normalized size = 0.77

$$-\frac{1}{2}(x-1)\cos(x)e^x+\frac{1}{2}xe^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Fricas [A] time = 0.455893, size = 59, normalized size = 1.97

$$-\frac{1}{2}(x-1)\cos(x)e^{x}+\frac{1}{2}xe^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x - 1)*cos(x)*e^x + 1/2*x*e^x*sin(x)
```

Sympy [A] time = 0.889217, size = 27, normalized size = 0.9

$$\frac{xe^{x}\sin(x)}{2}-\frac{xe^{x}\cos(x)}{2}+\frac{e^{x}\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 - x*exp(x)*cos(x)/2 + exp(x)*cos(x)/2
```

Giac [A] time = 1.10606, size = 22, normalized size = 0.73

$$-\frac{1}{2}((x-1)\cos(x)-x\sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x - 1)*cos(x) - x*sin(x))*e^x
```

3.48 $\int e^x x^2 \sin(x)\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x
*x^2*Sin[x])/2
```

Rubi [A] time = 0.118287, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x
*x^2*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \sin(x)\,dx &= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int x\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\int\left(-\frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)\right)dx\\
&= -\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)+\int e^x x\cos(x)\,dx-\int e^x x\sin(x)\,dx\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)+\int\left(-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e\right.\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{2}\int e^x\cos(x)\,dx\right)\\
&= e^x x\cos(x)-\frac{1}{2}e^x x^2\cos(x)+\frac{1}{2}e^x x^2\sin(x)-2\left(\frac{1}{4}e^x\cos(x)+\frac{1}{4}e^x\sin(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0348365, size = 25, normalized size = 0.5

$$\frac{1}{2}e^x\left(\left(x^2-1\right)\sin(x)-(x-1)^2\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Sin[x],x]
```

```
[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A] time = 0.004, size = 27, normalized size = 0.5

$$\left(-\frac{x^2}{2}+x-\frac{1}{2}\right)\mathrm{e}^x\cos(x)+\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*sin(x),x)
```

```
[Out] (-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.06062, size = 35, normalized size = 0.7

$$-\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Fricas [A] time = 0.46049, size = 81, normalized size = 1.62

$$-\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A] time = 2.35648, size = 48, normalized size = 0.96

$$\frac{x^2e^x\sin(x)}{2} - \frac{x^2e^x\cos(x)}{2} + xe^x\cos(x) - \frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.1317, size = 34, normalized size = 0.68

$$-\frac{1}{2}\left(\left(x^2 - 2x + 1\right)\cos(x) - \left(x^2 - 1\right)\sin(x)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

3.49 $\int e^x x \cos(x)\, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0401881, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^x x\cos(x)\,dx &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\ &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\ &= \frac{1}{2}e^x x\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0272623, size = 18, normalized size = 0.6

$$\frac{1}{2}e^x((x-1)\sin(x)+x\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]

[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0.006, size = 20, normalized size = 0.7

$$\frac{e^x x\cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x)

[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.01969, size = 23, normalized size = 0.77

$$\frac{1}{2}x\cos(x)e^x+\frac{1}{2}(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A] time = 0.454764, size = 58, normalized size = 1.93

$$\frac{1}{2}\,x\cos\left(x\right)e^{x}+\frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] time = 0.907299, size = 27, normalized size = 0.9

$$\frac{xe^{x}\sin\left(x\right)}{2}+\frac{xe^{x}\cos\left(x\right)}{2}-\frac{e^{x}\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Giac [A] time = 1.12599, size = 20, normalized size = 0.67

$$\frac{1}{2}\,(x\cos\left(x\right)+(x-1)\sin\left(x\right))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.50 $\int e^x x^2 \cos(x)\, dx$

Optimal. Leaf size=51

$$\frac{1}{2}e^x x^2 \sin(x) + \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*x^2*Cos[x])/2 + (E^x*Sin[x])/2 - E^x*x*Sin[x] + (E^x
*x^2*Sin[x])/2
```

Rubi [A] time = 0.117484, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4433, 4466, 14, 4432, 4465}

$$\frac{1}{2}e^x x^2 \sin(x) + \frac{1}{2}e^x x^2 \cos(x) - e^x x \sin(x) + \frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Cos[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*x^2*Cos[x])/2 + (E^x*Sin[x])/2 - E^x*x*Sin[x] + (E^x
*x^2*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^x x^2 \cos(x)\,dx &= \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int x\left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx \\
&= \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int\left(\frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x)\right)dx \\
&= \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - \int e^x x\cos(x)\,dx - \int e^x x\sin(x)\,dx \\
&= \frac{1}{2}e^x x^2\cos(x) - e^x x\sin(x) + \frac{1}{2}e^x x^2\sin(x) + \int\left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx + \int\left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x \right. \\
&= \frac{1}{2}e^x x^2\cos(x) - e^x x\sin(x) + \frac{1}{2}e^x x^2\sin(x) + 2\left(\frac{1}{2}\int e^x\sin(x)\,dx\right) \\
&= \frac{1}{2}e^x x^2\cos(x) - e^x x\sin(x) + \frac{1}{2}e^x x^2\sin(x) + 2\left(-\frac{1}{4}e^x\cos(x) + \frac{1}{4}e^x\sin(x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0307821, size = 23, normalized size = 0.45

$$\frac{1}{2}e^x(x-1)((x-1)\sin(x) + (x+1)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Cos[x],x]
```

```
[Out] (E^x*(-1 + x)*((1 + x)*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0.007, size = 28, normalized size = 0.6

$$\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^x\cos(x)-\left(-\frac{x^2}{2}+x-\frac{1}{2}\right)\mathrm{e}^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*cos(x),x)
```

```
[Out] (1/2*x^2-1/2)*exp(x)*cos(x)-(-1/2*x^2+x-1/2)*exp(x)*sin(x)
```

Maxima [A] time = 1.05925, size = 35, normalized size = 0.69

$$\frac{1}{2}\left(x^2-1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-2x+1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Fricas [A] time = 0.463794, size = 80, normalized size = 1.57

$$\frac{1}{2}\left(x^2-1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-2x+1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*cos(x)*e^x + 1/2*(x^2 - 2*x + 1)*e^x*sin(x)
```

Sympy [A] time = 2.41969, size = 48, normalized size = 0.94

$$\frac{x^2e^x\sin(x)}{2}+\frac{x^2e^x\cos(x)}{2}-xe^x\sin(x)+\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*cos(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 + x**2*exp(x)*cos(x)/2 - x*exp(x)*sin(x) + exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.0891, size = 32, normalized size = 0.63

$$\frac{1}{2}\left(\left(x^2-1\right)\cos(x)+\left(x^2-2\,x+1\right)\sin(x)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*((x^2 - 1)*cos(x) + (x^2 - 2*x + 1)*sin(x))*e^x
```

3.51 $\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx$

Optimal. Leaf size=27

$$-\frac{14}{25}e^{3x}\sin(4x)-\frac{23}{25}e^{3x}\cos(4x)$$

```
[Out] (-23*E^(3*x)*Cos[4*x])/25 - (14*E^(3*x)*Sin[4*x])/25
```

Rubi [A] time = 0.0801718, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6742, 4433, 4432}

$$-\frac{14}{25}e^{3x}\sin(4x)-\frac{23}{25}e^{3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

```
[Out] (-23*E^(3*x)*Cos[4*x])/25 - (14*E^(3*x)*Sin[4*x])/25
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{3x}(-5\cos(4x)+2\sin(4x))\,dx &= \int \left(-5e^{3x}\cos(4x)+2e^{3x}\sin(4x)\right)\,dx\\ &= 2\int e^{3x}\sin(4x)\,dx-5\int e^{3x}\cos(4x)\,dx\\ &= -\frac{23}{25}e^{3x}\cos(4x)-\frac{14}{25}e^{3x}\sin(4x)\end{aligned}$$

Mathematica [A] time = 0.0912628, size = 22, normalized size = 0.81

$$-\frac{1}{25}e^{3x}(14\sin(4x)+23\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*(-5*Cos[4*x] + 2*Sin[4*x]),x]
```

```
[Out] -(E^(3*x)*(23*Cos[4*x] + 14*Sin[4*x]))/25
```

Maple [B] time = 0.017, size = 103, normalized size = 3.8

$$-\frac{(24\cos(x)+32\sin(x))\mathrm{e}^{3x}(\cos(x))^3}{5}+\frac{(24\cos(x)+16\sin(x))\mathrm{e}^{3x}\cos(x)}{5}-\frac{3(\mathrm{e}^{x})^3}{5}-\frac{8\mathrm{e}^{3x}\cos(4x)}{25}+\frac{6\mathrm{e}^{3x}\sin(4}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x)
```

```
[Out] -8/5*(3*cos(x)+4*sin(x))*exp(3*x)*cos(x)^3+8/5*(3*cos(x)+2*sin(x))*exp(3*x)
*cos(x)-3/5*exp(x)^3-8/25*exp(3*x)*cos(4*x)+6/25*exp(3*x)*sin(4*x)-8/13*exp
(3*x)*cos(2*x)+12/13*exp(3*x)*sin(2*x)-4/13*exp(3*x)*(3*sin(2*x)-2*cos(2*x)
)
```

Maxima [A] time = 1.10554, size = 53, normalized size = 1.96

$$-\frac{2}{25}(4\cos(4x)-3\sin(4x))e^{(3x)}-\frac{1}{5}(3\cos(4x)+4\sin(4x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="maxima")
```

```
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

Fricas [A] time = 0.451213, size = 68, normalized size = 2.52

$$-\frac{23}{25}\cos(4x)e^{(3x)} - \frac{14}{25}e^{(3x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="fricas")
```

```
[Out] -23/25*cos(4*x)*e^(3*x) - 14/25*e^(3*x)*sin(4*x)
```

Sympy [A] time = 0.328428, size = 27, normalized size = 1.

$$-\frac{14e^{3x}\sin(4x)}{25} - \frac{23e^{3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x)
```

```
[Out] -14*exp(3*x)*sin(4*x)/25 - 23*exp(3*x)*cos(4*x)/25
```

Giac [A] time = 1.14005, size = 53, normalized size = 1.96

$$-\frac{2}{25}(4\cos(4x) - 3\sin(4x))e^{(3x)} - \frac{1}{5}(3\cos(4x) + 4\sin(4x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*(-5*cos(4*x)+2*sin(4*x)),x, algorithm="giac")
```

```
[Out] -2/25*(4*cos(4*x) - 3*sin(4*x))*e^(3*x) - 1/5*(3*cos(4*x) + 4*sin(4*x))*e^(
3*x)
```

3.52 $\int (e^{-x}\sin(x)+e^x\sin(x))\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^x\cos(x)$$

```
[Out] -Cos[x]/(2*E^x) - (E^x*Cos[x])/2 - Sin[x]/(2*E^x) + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0252271, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4432}

$$-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/E^x + E^x*Sin[x],x]
```

```
[Out] -Cos[x]/(2*E^x) - (E^x*Cos[x])/2 - Sin[x]/(2*E^x) + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int (e^{-x}\sin(x)+e^x\sin(x))\,dx &= \int e^{-x}\sin(x)\,dx+\int e^x\sin(x)\,dx\\ &= -\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^x\cos(x)-\frac{1}{2}e^{-x}\sin(x)+\frac{1}{2}e^x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0646951, size = 33, normalized size = 0.8

$$-\frac{1}{2}e^x\left(e^{-2x}-1\right)\sin(x)-\frac{1}{2}e^x\left(e^{-2x}+1\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/E^x + E^x*Sin[x],x]
```

```
[Out] -(E^x*(1 + E^(-2*x))*Cos[x])/2 - (E^x*(-1 + E^(-2*x))*Sin[x])/2
```

Maple [A] time = 0.007, size = 30, normalized size = 0.7

$$-\frac{e^{x}\cos(x)}{2}+\frac{e^{x}\sin(x)}{2}-\frac{e^{-x}\cos(x)}{2}-\frac{e^{-x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/exp(x)+exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)-1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)
```

Maxima [A] time = 1.03471, size = 31, normalized size = 0.76

$$-\frac{1}{2}(\cos(x)+\sin(x))e^{(-x)}-\frac{1}{2}(\cos(x)-\sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.463393, size = 84, normalized size = 2.05

$$-\frac{1}{2}\left(\cos(x)e^{(2x)}-\left(e^{(2x)}-1\right)\sin(x)+\cos(x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*e^(2*x) - (e^(2*x) - 1)*sin(x) + cos(x))*e^(-x)
```

Sympy [A] time = 0.60042, size = 32, normalized size = 0.78

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2} - \frac{e^{-x} \sin(x)}{2} - \frac{e^{-x} \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2 - exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Giac [A] time = 1.1592, size = 31, normalized size = 0.76

$$-\frac{1}{2} (\cos(x) + \sin(x))e^{(-x)} - \frac{1}{2} (\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x)+exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x) - 1/2*(cos(x) - sin(x))*e^x
```

3.53 $\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rubi [A] time = 0.135502, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4459, 4442, 2194, 2251}

$$\frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```


```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, I*E^(I*(c + d*x))])/(b*e*Log[F])
```


Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}\cos(c+dx)}{e+e\sin(c+dx)}\,dx &= -\frac{\int F^{a+bx}\tan\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)dx}{e}\\ &= -\frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx}{e}\\ &= \frac{i\int F^{a+bx}\,dx}{e}-\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}-\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e}\\ &= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};ie^{i(c+dx)}\right)}{be\log(F)}\end{aligned}$$

Mathematica [A] time = 2.56852, size = 64, normalized size = 0.78

$$-\frac{iF^{a+bx}\left(-1+2\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e + e*Sin[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, I*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F] time = 0.187, size = 0, normalized size = 0.

$$\int \frac{F^{bx+a} \cos(dx+c)}{e + e \sin(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

```
[Out] int(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,F^{bx}F^{a}bd\cos(dx+c)\log(F) + 2\,F^{bx}F^{a}d^{2}\sin(dx+c) + \left(F^{a}b^{2}\log(F)^{2} + F^{a}d^{2}\right)F^{bx}\cos(dx+c)^{2} + \left(F^{a}b^{2}\log(F)^{2} + F^{a}c\right.}{\left(b^{3}\log(F)^{3} + bd^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) +
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 + (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 - (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) - 2*(
(F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*log(F
)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 + 2*(F^a*b^3*d*log(F)^3 + F^a*b*d^
3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)*integ
rate((2*F^(b*x)*b*cos(d*x + c)*log(F) + F^(b*x)*b*log(F)*sin(2*d*x + 2*c) -
 F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) + F^(b*x)*d)/((b^2*l
og(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)
^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b^2*log(F)^2
 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 + 4*
(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e - 2*(2*(b^2*lo
g(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)), x
))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*log(F)^3 + b*d^2*
log(F))*e*sin(d*x + c)^2 + 2*(b^3*log(F)^3 + b*d^2*log(F))*e*sin(d*x + c) +
 (b^3*log(F)^3 + b*d^2*log(F))*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{F^aF^{bx}\cos(c+dx)}{\sin(c+dx)+1}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x)
```

```
[Out] Integral(F**a*F**(b*x)*cos(c + d*x)/(sin(c + d*x) + 1), x)/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{F^{bx+a}\cos(dx+c)}{e\sin(dx+c)+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e+e*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) + e), x)
```

3.54 $\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2iF^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, (-I)*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rubi [A] time = 0.127621, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4459, 4442, 2194, 2251}

$$\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, (-I)*E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}\cos(c+dx)}{e-e\sin(c+dx)}\,dx &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)dx}{e}\\ &= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\right)dx}{e}\\ &= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{\pi}{4}+\frac{dx}{2}\right)}}\,dx}{e}\\ &= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-ie^{i(c+dx)}\right)}{be\log(F)}\end{aligned}$$

Mathematica [A] time = 2.57243, size = 64, normalized size = 0.78

$$\frac{iF^{a+bx}\left(-1+2\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-ie^{i(c+dx)}\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Cos[c + d*x])/(e - e*Sin[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, (-I)*E^(I*(c + d*x))]))/(b*e*Log[F])
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}\cos(dx+c)}{e-e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

```
[Out] int(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,F^{bx}F^{a}bd\cos(dx+c)\log(F)+2\,F^{bx}F^{a}d^{2}\sin(dx+c)-\left(F^{a}b^{2}\log(F)^{2}+F^{a}d^{2}\right)F^{bx}\cos(dx+c)^{2}-\left(F^{a}b^{2}\log(F)^{2}+F^{a}d\right.}{\left(b^{3}\log(F)^{3}+bd^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*F^(b*x)*F^a*b*d*cos(d*x + c)*log(F) + 2*F^(b*x)*F^a*d^2*sin(d*x + c) -
(F^a*b^2*log(F)^2 + F^a*d^2)*F^(b*x)*cos(d*x + c)^2 - (F^a*b^2*log(F)^2 + F
^a*d^2)*F^(b*x)*sin(d*x + c)^2 + (F^a*b^2*log(F)^2 - F^a*d^2)*F^(b*x) + 2*(
(F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e*cos(d*x + c)^2 + (F^a*b^3*d*log(F
)^3 + F^a*b*d^3*log(F))*e*sin(d*x + c)^2 - 2*(F^a*b^3*d*log(F)^3 + F^a*b*d^
3*log(F))*e*sin(d*x + c) + (F^a*b^3*d*log(F)^3 + F^a*b*d^3*log(F))*e)*integ
rate(-(2*F^(b*x)*b*cos(d*x + c)*log(F) - F^(b*x)*b*log(F)*sin(2*d*x + 2*c)
+ F^(b*x)*d*cos(2*d*x + 2*c) + 2*F^(b*x)*d*sin(d*x + c) - F^(b*x)*d)/((b^2*
log(F)^2 + d^2)*e*cos(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c
)^2 - 4*(b^2*log(F)^2 + d^2)*e*cos(d*x + c)*sin(2*d*x + 2*c) + (b^2*log(F)^
2 + d^2)*e*sin(2*d*x + 2*c)^2 + 4*(b^2*log(F)^2 + d^2)*e*sin(d*x + c)^2 - 4
*(b^2*log(F)^2 + d^2)*e*sin(d*x + c) + (b^2*log(F)^2 + d^2)*e + 2*(2*(b^2*l
og(F)^2 + d^2)*e*sin(d*x + c) - (b^2*log(F)^2 + d^2)*e)*cos(2*d*x + 2*c)),
x))/((b^3*log(F)^3 + b*d^2*log(F))*e*cos(d*x + c)^2 + (b^3*log(F)^3 + b*d^2
*log(F))*e*sin(d*x + c)^2 - 2*(b^3*log(F)^3 + b*d^2*log(F))*e*sin(d*x + c)
+ (b^3*log(F)^3 + b*d^2*log(F))*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{F^{bx+a}\cos\left(dx+c\right)}{e\sin\left(dx+c\right)-e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{F^{a}F^{bx}\cos\left(c+dx\right)}{\sin\left(c+dx\right)-1}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x)
```

```
[Out] -Integral(F**a*F**(b*x)*cos(c + d*x)/(sin(c + d*x) - 1), x)/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{F^{bx+a}\cos\left(dx+c\right)}{e\sin\left(dx+c\right)-e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*cos(d*x+c)/(e-e*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(-F^(b*x + a)*cos(d*x + c)/(e*sin(d*x + c) - e), x)
```

3.55 $\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2iF^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-e^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, -E^(I*(c + d*x))])/(b*e*Log[F])
```

Rubi [A] time = 0.116375, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4460, 4442, 2194, 2251}

$$\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-e^{i(c+dx)}\right)}{be\log(F)}-\frac{iF^{a+bx}}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```



```
[Out] ((-I)*F^(a + b*x))/(b*e*Log[F]) + ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, (
(-I)*b*Log[F])/d, 1 - (I*b*Log[F])/d, -E^(I*(c + d*x))])/(b*e*Log[F])
```


Rule 4460

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{F^{a+bx}\sin(c+dx)}{e+e\cos(c+dx)}\,dx &= \frac{\int F^{a+bx}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{e}\\ &= \frac{i\int\left(-F^{a+bx}+\frac{2F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)dx}{e}\\ &= -\frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e}\\ &= -\frac{iF^{a+bx}}{be\log(F)}+\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};-e^{i(c+dx)}\right)}{be\log(F)}\end{aligned}$$

Mathematica [A] time = 0.665764, size = 68, normalized size = 0.85

$$\frac{iF^{a+bx}\left(-1+2\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},-\cos(c+dx)-i\sin(c+dx)\right)\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e + e*Cos[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*Log
[F])/d, -Cos[c + d*x] - I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{F^{bx+a} \sin (dx+c)}{e+e \cos (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

```
[Out] int(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{F^{bx+a} \sin (dx+c)}{e \cos (dx+c)+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{a} F^{bx} \sin (c+dx)}{\cos (c+dx)+1}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x)
```

```
[Out] Integral(F**a*F**(b*x)*sin(c + d*x)/(cos(c + d*x) + 1), x)/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{bx+a}\sin(dx+c)}{e\cos(dx+c)+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e+e*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) + e), x)
```

3.56 $\int \frac{F^{a+bx} \sin(c+dx)}{e-e\cos(c+dx)} dx$

Optimal. Leaf size=78

$$\frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},e^{i(c+dx)}\right)}{be\log(F)}$$

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^(I*(c + d*x))])/(b*e*Log[F])
```

Rubi [A] time = 0.117861, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4461, 4443, 2194, 2251}

$$\frac{iF^{a+bx}}{be\log(F)} - \frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};e^{i(c+dx)}\right)}{be\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

```
[Out] (I*F^(a + b*x))/(b*e*Log[F]) - ((2*I)*F^(a + b*x)*Hypergeometric2F1[1, ((-I
)*b*Log[F])/d, 1 - (I*b*Log[F])/d, E^(I*(c + d*x))])/(b*e*Log[F])
```

Rule 4461

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{F^{a+bx}\sin(c+dx)}{e-e\cos(c+dx)}\,dx &= \frac{\int F^{a+bx}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{e}\\
&= -\frac{i\int\left(-F^{a+bx}-\frac{2F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\right)dx}{e}\\
&= \frac{i\int F^{a+bx}\,dx}{e}+\frac{(2i)\int\frac{F^{a+bx}}{-1+e^{2i\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,dx}{e}\\
&= \frac{iF^{a+bx}}{be\log(F)}-\frac{2iF^{a+bx}\,{}_2F_1\left(1,-\frac{ib\log(F)}{d};1-\frac{ib\log(F)}{d};e^{i(c+dx)}\right)}{be\log(F)}
\end{aligned}
$$

Mathematica [A] time = 0.626621, size = 66, normalized size = 0.85

$$
-\frac{iF^{a+bx}\left(-1+2\text{Hypergeometric2F1}\left(1,-\frac{ib\log(F)}{d},1-\frac{ib\log(F)}{d},\cos(c+dx)+i\sin(c+dx)\right)\right)}{be\log(F)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(F^(a + b*x)*Sin[c + d*x])/(e - e*Cos[c + d*x]),x]
```

```
[Out] ((-I)*F^(a + b*x)*(-1 + 2*Hypergeometric2F1[1, ((-I)*b*Log[F])/d, 1 - (I*b*
Log[F])/d, Cos[c + d*x] + I*Sin[c + d*x]]))/(b*e*Log[F])
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \frac{F^{bx+a} \sin (dx + c)}{e - e \cos (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

```
[Out] int(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{F^{bx+a} \sin (dx + c)}{e \cos (dx + c) - e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{F^{a} F^{bx} \sin (c+dx)}{\cos (c+dx)-1}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x)
```

```
[Out] -Integral(F**a*F**(b*x)*sin(c + d*x)/(cos(c + d*x) - 1), x)/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{F^{bx+a}\sin\left(dx+c\right)}{e\cos\left(dx+c\right)-e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(b*x+a)*sin(d*x+c)/(e-e*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(-F^(b*x + a)*sin(d*x + c)/(e*cos(d*x + c) - e), x)
```

3.57 $\int e^{x^2} \sin(bx)\, dx$

Optimal. Leaf size=69

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[Out] (I/4)*E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(b^2/4)*Sqrt[Pi]*
Erfi[(I*b + 2*x)/2]
```

Rubi [A] time = 0.0526159, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4472, 2234, 2204}

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[b*x],x]
```

```
[Out] (I/4)*E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(b^2/4)*Sqrt[Pi]*
Erfi[(I*b + 2*x)/2]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\sin(bx)\,dx &= \int\left(\frac{1}{2}ie^{-ibx+x^2}-\frac{1}{2}ie^{ibx+x^2}\right)dx\\ &= \frac{1}{2}i\int e^{-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ibx+x^2}\,dx\\ &= \frac{1}{2}\left(ie^{\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}ie^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A] time = 0.0266167, size = 43, normalized size = 0.62

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\mathrm{Erf}\left(\frac{b}{2}+ix\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erf[b/2 - I*x] + Erf[b/2 + I*x]))/4
```

Maple [A] time = 0.099, size = 42, normalized size = 0.6

$$\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(-ix+\frac{b}{2}\right)+\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(ix+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2)*erf(I*x
+1/2*b)
```

Maxima [A] time = 1.17189, size = 50, normalized size = 0.72

$$\frac{1}{4}\sqrt{\pi}\left(\mathrm{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}-\mathrm{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x)*e^(1/4*b^2) - erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A] time = 0.462975, size = 88, normalized size = 1.28

$$\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)-\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(erf(1/2*b + I*x) - erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sin(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x),x)
```

```
[Out] Integral(exp(x**2)*sin(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(x^2\right)}\sin(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(b*x), x)
```

3.58 $\int e^{x^2} \cos(bx)\, dx$

Optimal. Leaf size=65

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[Out] (E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(b^2/4)*Sqrt[Pi]*Erfi[(I
*b + 2*x)/2])/4
```

Rubi [A] time = 0.0473719, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4473, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\operatorname{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(b^2/4)*Sqrt[Pi]*Erfi[(I
*b + 2*x)/2])/4
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\cos(bx)\,dx &= \int \left(\frac{1}{2}e^{-ibx+x^2}+\frac{1}{2}e^{ibx+x^2}\right)dx\\
&= \frac{1}{2}\int e^{-ibx+x^2}\,dx+\frac{1}{2}\int e^{ibx+x^2}\,dx\\
&= \frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\
&= \frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0203367, size = 47, normalized size = 0.72

$$
\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Erfi[((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2]))/4
```

Maple [A] time = 0.039, size = 44, normalized size = 0.7

$$
-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{Erf}\left(ix+\frac{b}{2}\right)+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\,\mathrm{Erf}\left(-ix+\frac{b}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/4*b^2)*erf
(-I*x+1/2*b)
```

Maxima [A] time = 1.0258, size = 51, normalized size = 0.78

$$
-\frac{1}{4}\sqrt{\pi}\left(i\,\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}+i\,\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*(I*erf(1/2*b + I*x)*e^(1/4*b^2) + I*erf(-1/2*b + I*x)*e^(1/4*
b^2))
```

Fricas [A] time = 0.473565, size = 95, normalized size = 1.46

$$\frac{1}{4}\sqrt{\pi}\left(-i\,\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)-i\,\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)\right)e^{\left(\frac{1}{4}\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(1/2*b + I*x) - I*erf(-1/2*b + I*x))*e^(1/4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cos(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x),x)
```

```
[Out] Integral(exp(x**2)*cos(b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx)\,e^{\left(x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x)*e^(x^2), x)
```

3.59 $\int e^{x^2} \sin(a + bx)\, dx$

Optimal. Leaf size=81

$$\frac{1}{4} i \sqrt{\pi} e^{\frac{b^2}{4} - ia} \operatorname{Erfi}\left(\frac{1}{2}(2x - ib)\right) - \frac{1}{4} i \sqrt{\pi} e^{\frac{b^2}{4} + ia} \operatorname{Erfi}\left(\frac{1}{2}(2x + ib)\right)$$

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rubi [A] time = 0.0703958, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4472, 2234, 2204}

$$\frac{1}{4} i \sqrt{\pi} e^{\frac{b^2}{4} - ia} \operatorname{Erfi}\left(\frac{1}{2}(2x - ib)\right) - \frac{1}{4} i \sqrt{\pi} e^{\frac{b^2}{4} + ia} \operatorname{Erfi}\left(\frac{1}{2}(2x + ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sin(a+bx)\,dx &= \int\left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx\\
&=\frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+x^2}\,dx\\
&=\frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\
&=\frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0802278, size = 81, normalized size = 1.

$$
\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(\cos(a)\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\cos(a)\mathrm{Erf}\left(\frac{b}{2}+ix\right)+\sin(a)\left(\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A] time = 0.041, size = 52, normalized size = 0.6

$$
\frac{\sqrt{\pi}\mathrm{e}^{ia}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(-ix+\frac{b}{2}\right)+\frac{\sqrt{\pi}\mathrm{e}^{-ia}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(ix+\frac{b}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Maxima [A] time = 1.05629, size = 69, normalized size = 0.85

$$\frac{1}{4}\sqrt{\pi}\left((\cos(a) - i\sin(a))\operatorname{erf}\left(\frac{1}{2}b + ix\right)e^{\left(\frac{1}{4}b^2\right)} - (\cos(a) + i\sin(a))\operatorname{erf}\left(-\frac{1}{2}b + ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A] time = 0.474402, size = 122, normalized size = 1.51

$$-\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(-\frac{1}{2}b + ix\right)e^{\left(\frac{1}{4}b^2 + ia\right)} - \operatorname{erf}\left(\frac{1}{2}b + ix\right)e^{\left(\frac{1}{4}b^2 - ia\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sin(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(x^2\right)}\sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

3.60 $\int e^{x^2} \cos(a + bx)\, dx$

Optimal. Leaf size=77

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x - ib)\right) + \frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x + ib)\right)$$

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rubi [A] time = 0.0508698, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4473, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x - ib)\right) + \frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x + ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cos(a+bx)\,dx &= \int \left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx\\ &= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A] time = 0.0814511, size = 82, normalized size = 1.06

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(-\sin(a)\left(\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\mathrm{Erf}\left(\frac{b}{2}+ix\right)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A] time = 0.04, size = 54, normalized size = 0.7

$$-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{-ia}\mathrm{Erf}\left(ix+\frac{b}{2}\right)+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{ia}\mathrm{Erf}\left(-ix+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Maxima [A] time = 1.05873, size = 70, normalized size = 0.91

$$-\frac{1}{4}\sqrt{\pi}\left(\left(i\cos(a)+\sin(a)\right)\mathrm{erf}\left(\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}+\left(i\cos(a)-\sin(a)\right)\mathrm{erf}\left(-\frac{1}{2}b+ix\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A] time = 0.476709, size = 127, normalized size = 1.65

$$\frac{1}{4}\sqrt{\pi}\left(-i\,\operatorname{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-i\,\operatorname{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(bx+a)\,e^{\left(x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

3.61 $\int e^{2x^2} x \cos\left(2x^2\right) dx$

Optimal. Leaf size=35

$$\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)+\frac{1}{8}e^{2x^2}\cos\left(2x^2\right)$$

```
[Out] (E^(2*x^2)*Cos[2*x^2])/8 + (E^(2*x^2)*Sin[2*x^2])/8
```

Rubi [A] time = 0.0774638, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6715, 4433}

$$\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)+\frac{1}{8}e^{2x^2}\cos\left(2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x^2)*x*Cos[2*x^2],x]
```

```
[Out] (E^(2*x^2)*Cos[2*x^2])/8 + (E^(2*x^2)*Sin[2*x^2])/8
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2x^2}x\cos\left(2x^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int e^{2x}\cos(2x)\,dx,x,x^2\right)\\&=\frac{1}{8}e^{2x^2}\cos\left(2x^2\right)+\frac{1}{8}e^{2x^2}\sin\left(2x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0370865, size = 24, normalized size = 0.69

$$\frac{1}{8}e^{2x^2}\left(\sin\left(2x^2\right)+\cos\left(2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x^2)*x*Cos[2*x^2],x]
```

```
[Out] (E^(2*x^2)*(Cos[2*x^2] + Sin[2*x^2]))/8
```

Maple [A] time = 0.011, size = 30, normalized size = 0.9

$$\frac{e^{2x^2}\cos\left(2x^2\right)}{8}+\frac{e^{2x^2}\sin\left(2x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x^2)*x*cos(2*x^2),x)
```

```
[Out] 1/8*exp(2*x^2)*cos(2*x^2)+1/8*exp(2*x^2)*sin(2*x^2)
```

Maxima [A] time = 1.00692, size = 39, normalized size = 1.11

$$\frac{1}{8}\cos\left(2x^2\right)e^{\left(2x^2\right)}+\frac{1}{8}e^{\left(2x^2\right)}\sin\left(2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Fricas [A] time = 0.460905, size = 72, normalized size = 2.06

$$\frac{1}{8}\cos\left(2x^2\right)e^{\left(2x^2\right)}+\frac{1}{8}e^{\left(2x^2\right)}\sin\left(2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*cos(2*x^2)*e^(2*x^2) + 1/8*e^(2*x^2)*sin(2*x^2)
```

Sympy [A] time = 13.7648, size = 29, normalized size = 0.83

$$\frac{e^{2x^2}\sin\left(2x^2\right)}{8}+\frac{e^{2x^2}\cos\left(2x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x**2)*x*cos(2*x**2),x)
```

```
[Out] exp(2*x**2)*sin(2*x**2)/8 + exp(2*x**2)*cos(2*x**2)/8
```

Giac [A] time = 1.14067, size = 28, normalized size = 0.8

$$\frac{1}{8}\left(\cos\left(2x^2\right)+\sin\left(2x^2\right)\right)e^{\left(2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x^2)*x*cos(2*x^2),x, algorithm="giac")
```

```
[Out] 1/8*(cos(2*x^2) + sin(2*x^2))*e^(2*x^2)
```

3.62 $\int e^x \sin(e^x) \, dx$

Optimal. Leaf size=6

$$-\cos(e^x)$$

```
[Out] -Cos[E^x]
```

Rubi [A] time = 0.0083315, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 2638}

$$-\cos(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[E^x],x]
```

```
[Out] -Cos[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int e^x \sin(e^x) \, dx &= \text{Subst}\left(\int \sin(x) \, dx, x, e^x\right) \\ &= -\cos(e^x) \end{aligned}$$

Mathematica [A] time = 0.0093841, size = 6, normalized size = 1.

$$-\cos(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[E^x],x]
```

```
[Out] -Cos[E^x]
```

Maple [A] time = 0.003, size = 6, normalized size = 1.

$$-\cos(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))
```

Maxima [A] time = 0.980391, size = 7, normalized size = 1.17

$$-\cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -cos(e^x)
```

Fricas [A] time = 0.464562, size = 15, normalized size = 2.5

$$-\cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="fricas")
```

```
[Out] -cos(e^x)
```

Sympy [A] time = 0.298982, size = 5, normalized size = 0.83

$$-\cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))
```

Giac [A] time = 1.16469, size = 7, normalized size = 1.17

$$-\cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(exp(x)),x, algorithm="giac")
```

```
[Out] -cos(e^x)
```

3.63 $\int e^x \csc(e^x) \sec(e^x)\, dx$

Optimal. Leaf size=5

$$\log(\tan(e^x))$$

```
[Out] Log[Tan[E^x]]
```

Rubi [A] time = 0.0215424, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 2620, 29}

$$\log(\tan(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Csc[E^x]*Sec[E^x],x]
```

```
[Out] Log[Tan[E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int e^x \csc(e^x) \sec(e^x)\, dx = \text{Subst}\left(\int \csc(x)\sec(x)\, dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \frac{1}{x}\, dx, x, \tan(e^x)\right)$$
$$= \log(\tan(e^x))$$

Mathematica [B] time = 0.0177249, size = 21, normalized size = 4.2

$$2\left(\frac{1}{2}\log(\sin(e^x)) - \frac{1}{2}\log(\cos(e^x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csc[E^x]*Sec[E^x],x]
```

```
[Out] 2*(-Log[Cos[E^x]]/2 + Log[Sin[E^x]]/2)
```

Maple [A] time = 0.022, size = 5, normalized size = 1.

$$\ln(\tan(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csc(exp(x))*sec(exp(x)),x)
```

```
[Out] ln(tan(exp(x)))
```

Maxima [B] time = 1.07253, size = 26, normalized size = 5.2

$$-\frac{1}{2}\log\left(\sin(e^x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(e^x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(sin(e^x)^2 - 1) + 1/2*log(sin(e^x)^2)
```

Fricas [B] time = 0.475521, size = 74, normalized size = 14.8

$$-\frac{1}{2}\log\left(\cos\left(e^{x}\right)^{2}\right)+\frac{1}{2}\log\left(-\frac{1}{4}\cos\left(e^{x}\right)^{2}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(cos(e^x)^2) + 1/2*log(-1/4*cos(e^x)^2 + 1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\csc\left(e^{x}\right)\sec\left(e^{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x)
```

```
[Out] Integral(exp(x)*csc(exp(x))*sec(exp(x)), x)
```

Giac [B] time = 1.11425, size = 27, normalized size = 5.4

$$\frac{1}{2}\log\left(\sin\left(e^{x}\right)^{2}\right)-\frac{1}{2}\log\left(\left|\sin\left(e^{x}\right)^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))*sec(exp(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(e^x)^2) - 1/2*log(abs(sin(e^x)^2 - 1))
```

3.64 $\int e^x \cos(e^x)\,dx$

Optimal. Leaf size=4

$$\sin(e^x)$$

```
[Out] Sin[E^x]
```

Rubi [A] time = 0.0084126, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 2637}

$$\sin(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x],x]
```

```
[Out] Sin[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \cos(e^x)\,dx &= \text{Subst}\left(\int \cos(x)\,dx, x, e^x\right)\\ &= \sin(e^x)\end{aligned}$$

Mathematica [A] time = 0.0094151, size = 4, normalized size = 1.

$$\sin(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x],x]
```

```
[Out] Sin[E^x]
```

Maple [A] time = 0.007, size = 4, normalized size = 1.

$$\sin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x)),x)
```

```
[Out] sin(exp(x))
```

Maxima [A] time = 0.983681, size = 4, normalized size = 1.

$$\sin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="maxima")
```

```
[Out] sin(e^x)
```

Fricas [A] time = 0.457546, size = 14, normalized size = 3.5

$$\sin(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="fricas")
```

```
[Out] sin(e^x)
```

Sympy [A] time = 0.495608, size = 3, normalized size = 0.75

$$\sin\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x)
```

```
[Out] sin(exp(x))
```

Giac [A] time = 1.13345, size = 4, normalized size = 1.

$$\sin\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x)),x, algorithm="giac")
```

```
[Out] sin(e^x)
```

3.65 $\int e^{2x} \cos\left(e^{2x}\right) dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

```
[Out] Sin[E^(2*x)]/2
```

Rubi [A] time = 0.0106718, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 2637}

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[E^(2*x)],x]
```

```
[Out] Sin[E^(2*x)]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\cos\left(e^{2x}\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int \cos(x)\,dx, x, e^{2x}\right)\\ &= \frac{1}{2}\sin\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0102269, size = 10, normalized size = 1.

$$\frac{1}{2}\sin\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[E^(2*x)],x]
```

```
[Out] Sin[E^(2*x)]/2
```

Maple [A] time = 0.005, size = 8, normalized size = 0.8

$$\frac{\sin\left(e^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(exp(2*x)),x)
```

```
[Out] 1/2*sin(exp(2*x))
```

Maxima [A] time = 1.00125, size = 9, normalized size = 0.9

$$\frac{1}{2}\sin\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*sin(e^(2*x))
```

Fricas [A] time = 0.457448, size = 24, normalized size = 2.4

$$\frac{1}{2}\sin\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*sin(e^(2*x))
```

Sympy [A] time = 0.502239, size = 7, normalized size = 0.7

$$\frac{\sin\left(e^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x)
```

```
[Out] sin(exp(2*x))/2
```

Giac [A] time = 1.11665, size = 9, normalized size = 0.9

$$\frac{1}{2}\sin\left(e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*sin(e^(2*x))
```

3.66 $\int e^{-2x}\cos\left(e^{-2x}\right)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

```
[Out] -Sin[E^(-2*x)]/2
```

Rubi [A] time = 0.0104177, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2282, 2637}

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[E^(-2*x)]/E^(2*x),x]

[Out] -Sin[E^(-2*x)]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}\cos\left(e^{-2x}\right)\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \cos(x)\,dx, x, e^{-2x}\right)\right)\\ &= -\frac{1}{2}\sin\left(e^{-2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0112636, size = 10, normalized size = 1.

$$-\frac{1}{2}\sin\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[E^(-2*x)]/E^(2*x),x]
```

```
[Out] -Sin[E^(-2*x)]/2
```

Maple [A] time = 0.007, size = 8, normalized size = 0.8

$$-\frac{\sin\left(\mathrm{e}^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(-2*x))/exp(2*x),x)
```

```
[Out] -1/2*sin(exp(-2*x))
```

Maxima [A] time = 0.998854, size = 9, normalized size = 0.9

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(e^(-2*x))
```

Fricas [A] time = 0.463653, size = 27, normalized size = 2.7

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*sin(e^(-2*x))
```

Sympy [A] time = 0.492375, size = 10, normalized size = 1.

$$-\frac{\sin\left(e^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x)
```

```
[Out] -sin(exp(-2*x))/2
```

Giac [A] time = 1.09451, size = 9, normalized size = 0.9

$$-\frac{1}{2}\sin\left(e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

```
[Out] -1/2*sin(e^(-2*x))
```

3.67 $\int e^{2x} \cos(e^x) \, dx$

Optimal. Leaf size=13

$$e^x \sin(e^x) + \cos(e^x)$$

```
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Rubi [A] time = 0.0165981, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 3296, 2638}

$$e^x \sin(e^x) + \cos(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[E^x],x]
```

```
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int e^{2x}\cos\left(e^x\right)\,dx = \text{Subst}\left(\int x\cos(x)\,dx, x, e^x\right)$$
$$= e^x\sin\left(e^x\right) - \text{Subst}\left(\int \sin(x)\,dx, x, e^x\right)$$
$$= \cos\left(e^x\right) + e^x\sin\left(e^x\right)$$

Mathematica [A] time = 0.0166958, size = 13, normalized size = 1.

$$e^x\sin\left(e^x\right) + \cos\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[E^x],x]
```

```
[Out] Cos[E^x] + E^x*Sin[E^x]
```

Maple [B] time = 0.02, size = 24, normalized size = 1.9

$$\left(2\,e^x\tan\left(1/2\,e^x\right) + 2\right)\left(1 + \left(\tan\left(\frac{e^x}{2}\right)\right)^2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(exp(x)),x)
```

```
[Out] (2*exp(x)*tan(1/2*exp(x))+2)/(1+tan(1/2*exp(x))^2)
```

Maxima [A] time = 1.00955, size = 14, normalized size = 1.08

$$e^x\sin\left(e^x\right) + \cos\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="maxima")
```

```
[Out] e^x*sin(e^x) + cos(e^x)
```

Fricas [A] time = 0.47621, size = 34, normalized size = 2.62

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="fricas")
```

```
[Out] e^x*sin(e^x) + cos(e^x)
```

Sympy [A] time = 11.9689, size = 12, normalized size = 0.92

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x)
```

```
[Out] exp(x)*sin(exp(x)) + cos(exp(x))
```

Giac [A] time = 1.12902, size = 14, normalized size = 1.08

$$e^x \sin(e^x) + \cos(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(exp(x)),x, algorithm="giac")
```

```
[Out] e^x*sin(e^x) + cos(e^x)
```

3.68 $\int e^{-1+3x} \cos\left(e^{-1+3x}\right) \sin\left(1+e^{-1+3x}\right) dx$

Optimal. Leaf size=30

$$\frac{1}{6}e^{3x-1}\sin(1)-\frac{1}{12}\cos\left(2e^{3x-1}+1\right)$$

```
[Out] -Cos[1 + 2*E^(-1 + 3*x)]/12 + (E^(-1 + 3*x)*Sin[1])/6
```

Rubi [A] time = 0.0355204, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2282, 4574, 2638}

$$\frac{1}{6}e^{3x-1}\sin(1)-\frac{1}{12}\cos\left(2e^{3x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

```
[Out] -Cos[1 + 2*E^(-1 + 3*x)]/12 + (E^(-1 + 3*x)*Sin[1])/6
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-1+3x}\cos\left(e^{-1+3x}\right)\sin\left(1+e^{-1+3x}\right)dx &= \frac{1}{3}\operatorname{Subst}\left(\int\cos(x)\sin(1+x)\,dx,x,e^{-1+3x}\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{\sin(1)}{2}+\frac{1}{2}\sin(1+2x)\right)dx,x,e^{-1+3x}\right)\\ &= \frac{1}{6}e^{-1+3x}\sin(1)+\frac{1}{6}\operatorname{Subst}\left(\int\sin(1+2x)\,dx,x,e^{-1+3x}\right)\\ &= -\frac{1}{12}\cos\left(1+2e^{-1+3x}\right)+\frac{1}{6}e^{-1+3x}\sin(1)\end{aligned}$$

Mathematica [A] time = 0.0608572, size = 30, normalized size = 1.

$$\frac{1}{6}e^{3x-1}\sin(1)-\frac{1}{12}\cos\left(2e^{3x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-1 + 3*x)*Cos[E^(-1 + 3*x)]*Sin[1 + E^(-1 + 3*x)],x]
```

```
[Out] -Cos[1 + 2*E^(-1 + 3*x)]/12 + (E^(-1 + 3*x)*Sin[1])/6
```

Maple [A] time = 0.051, size = 25, normalized size = 0.8

$$-\frac{\cos\left(1+2\,e^{-1+3\,x}\right)}{12}+\frac{e^{-1+3\,x}\sin(1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x)
```

```
[Out] -1/12*cos(1+2*exp(-1+3*x))+1/6*exp(-1+3*x)*sin(1)
```

Maxima [A] time = 0.986635, size = 32, normalized size = 1.07

$$\frac{1}{6}\,e^{(3\,x-1)}\sin(1)-\frac{1}{12}\,\cos\left(2\,e^{(3\,x-1)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="max
ima")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

Fricas [A] time = 0.483324, size = 140, normalized size = 4.67

$$-\frac{1}{6}\cos(1)\cos\left(e^{(3x-1)}\right)^{2}+\frac{1}{6}\cos\left(e^{(3x-1)}\right)\sin(1)\sin\left(e^{(3x-1)}\right)+\frac{1}{6}e^{(3x-1)}\sin(1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="fri
cas")
```

```
[Out] -1/6*cos(1)*cos(e^(3*x - 1))^2 + 1/6*cos(e^(3*x - 1))*sin(1)*sin(e^(3*x - 1
)) + 1/6*e^(3*x - 1)*sin(1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12193, size = 32, normalized size = 1.07

$$\frac{1}{6}e^{(3x-1)}\sin(1)-\frac{1}{12}\cos\left(2e^{(3x-1)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1+3*x)*cos(exp(-1+3*x))*sin(1+exp(-1+3*x)),x, algorithm="gia
c")
```

```
[Out] 1/6*e^(3*x - 1)*sin(1) - 1/12*cos(2*e^(3*x - 1) + 1)
```

3.69 $\int e^x \tan(e^x)\, dx$

Optimal. Leaf size=7

$$-\log(\cos(e^x))$$

```
[Out] -Log[Cos[E^x]]
```

Rubi [A] time = 0.0101972, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 3475}

$$-\log(\cos(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Tan[E^x],x]
```

```
[Out] -Log[Cos[E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \tan(e^x)\, dx &= \text{Subst}\left(\int \tan(x)\, dx, x, e^x\right)\\ &= -\log(\cos(e^x))\end{aligned}$$

Mathematica [A] time = 0.0081558, size = 7, normalized size = 1.

$$-\log(\cos(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Tan[E^x],x]
```

```
[Out] -Log[Cos[E^x]]
```

Maple [A] time = 0.002, size = 7, normalized size = 1.

$$-\ln(\cos(\mathrm{e}^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*tan(exp(x)),x)
```

```
[Out] -ln(cos(exp(x)))
```

Maxima [A] time = 0.994762, size = 5, normalized size = 0.71

$$\log(\sec(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="maxima")
```

```
[Out] log(sec(e^x))
```

Fricas [A] time = 0.474122, size = 41, normalized size = 5.86

$$-\frac{1}{2}\log\left(\frac{1}{\tan(e^x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(e^x)^2 + 1))
```

Sympy [A] time = 0.643082, size = 10, normalized size = 1.43

$$\frac{\log\left(\tan^2\left(e^x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x)
```

```
[Out] log(tan(exp(x))**2 + 1)/2
```

Giac [A] time = 1.20248, size = 9, normalized size = 1.29

$$-\log\left(|\cos\left(e^x\right)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*tan(exp(x)),x, algorithm="giac")
```

```
[Out] -log(abs(cos(e^x)))
```

3.70 $\int e^x \sec(e^x)\, dx$

Optimal. Leaf size=5

$$\tanh^{-1}(\sin(e^x))$$

```
[Out] ArcTanh[Sin[E^x]]
```

Rubi [A] time = 0.0097277, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 3770}

$$\tanh^{-1}(\sin(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sec[E^x],x]
```

```
[Out] ArcTanh[Sin[E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int e^x \sec(e^x)\, dx &= \text{Subst}\left(\int \sec(x)\, dx, x, e^x\right) \\ &= \tanh^{-1}(\sin(e^x)) \end{aligned}$$

Mathematica [A] time = 0.005083, size = 5, normalized size = 1.

$$\tanh^{-1}(\sin(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sec[E^x],x]
```

```
[Out] ArcTanh[Sin[E^x]]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.8

$$\ln(\sec(e^x) + \tan(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sec(exp(x)),x)
```

```
[Out] ln(sec(exp(x))+tan(exp(x)))
```

Maxima [B] time = 1.0065, size = 11, normalized size = 2.2

$$\log(\sec(e^x) + \tan(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="maxima")
```

```
[Out] log(sec(e^x) + tan(e^x))
```

Fricas [B] time = 0.482003, size = 65, normalized size = 13.

$$\frac{1}{2}\log(\sin(e^x) + 1) - \frac{1}{2}\log(-\sin(e^x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(e^x) + 1) - 1/2*log(-sin(e^x) + 1)
```

Sympy [A] time = 3.5652, size = 10, normalized size = 2.

$$\log\left(\tan\left(e^{x}\right)+\sec\left(e^{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x)
```

```
[Out] log(tan(exp(x)) + sec(exp(x)))
```

Giac [B] time = 1.18458, size = 39, normalized size = 7.8

$$\frac{1}{4}\,\log\left(\left|\frac{1}{\sin\left(e^{x}\right)}+\sin\left(e^{x}\right)+2\right|\right)-\frac{1}{4}\,\log\left(\left|\frac{1}{\sin\left(e^{x}\right)}+\sin\left(e^{x}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(e^x) + sin(e^x) + 2)) - 1/4*log(abs(1/sin(e^x) + sin(e^x)
 - 2))
```

3.71 $\int e^x \sec(e^x) \tan(e^x)\, dx$

Optimal. Leaf size=4

$$\sec(e^x)$$

[Out] Sec[E^x]

Rubi [A] time = 0.0200626, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 2606, 8}

$$\sec(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Sec[E^x]*Tan[E^x],x]

[Out] Sec[E^x]

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int e^x \sec(e^x)\tan(e^x)\ dx = \text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int 1\,dx, x, \sec(e^x)\right)$$
$$= \sec(e^x)$$

Mathematica [A] time = 0.0102617, size = 4, normalized size = 1.

$$\sec(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sec[E^x]*Tan[E^x],x]
```

```
[Out] Sec[E^x]
```

Maple [A] time = 0.008, size = 4, normalized size = 1.

$$\sec(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sec(exp(x))*tan(exp(x)),x)
```

```
[Out] sec(exp(x))
```

Maxima [A] time = 1.01238, size = 7, normalized size = 1.75

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/cos(e^x)
```

Fricas [A] time = 0.464941, size = 16, normalized size = 4.

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/cos(e^x)
```

Sympy [A] time = 1.05962, size = 3, normalized size = 0.75

$$\sec(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x)
```

```
[Out] sec(exp(x))
```

Giac [A] time = 1.18569, size = 7, normalized size = 1.75

$$\frac{1}{\cos(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sec(exp(x))*tan(exp(x)),x, algorithm="giac")
```

```
[Out] 1/cos(e^x)
```

3.72 $\int e^x \csc^2(e^x)\,dx$

Optimal. Leaf size=6

$$-\cot(e^x)$$

[Out] -Cot[E^x]

Rubi [A] time = 0.0174201, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 3767, 8}

$$-\cot(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Csc[E^x]^2,x]

[Out] -Cot[E^x]

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int e^x \csc^2(e^x)\,dx &= \text{Subst}\left(\int \csc^2(x)\,dx, x, e^x\right)\\ &= -\text{Subst}\left(\int 1\,dx, x, \cot(e^x)\right)\\ &= -\cot(e^x)\end{aligned}$$

Mathematica [A] time = 0.0167705, size = 6, normalized size = 1.

$$-\cot(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Csc[E^x]^2,x]
```

```
[Out] -Cot[E^x]
```

Maple [A] time = 0.054, size = 6, normalized size = 1.

$$-\cot(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*csc(exp(x))^2,x)
```

```
[Out] -cot(exp(x))
```

Maxima [A] time = 1.03101, size = 9, normalized size = 1.5

$$-\frac{1}{\tan(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="maxima")
```

```
[Out] -1/tan(e^x)
```

Fricas [A] time = 0.439292, size = 27, normalized size = 4.5

$$-\frac{\cos(e^x)}{\sin(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="fricas")
```

```
[Out] -cos(e^x)/sin(e^x)
```

Sympy [A] time = 3.2511, size = 5, normalized size = 0.83

$$-\cot(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))**2,x)
```

```
[Out] -cot(exp(x))
```

Giac [A] time = 1.27695, size = 9, normalized size = 1.5

$$-\frac{1}{\tan(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*csc(exp(x))^2,x, algorithm="giac")
```

```
[Out] -1/tan(e^x)
```

3.73 $\int e^x \sin(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{e^x\sin(a+bx)}{b^2+1}-\frac{be^x\cos(a+bx)}{b^2+1}$$

```
[Out] -((b*E^x*Cos[a + b*x])/(1 + b^2)) + (E^x*Sin[a + b*x])/(1 + b^2)
```

Rubi [A] time = 0.0129877, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4432}

$$\frac{e^x\sin(a+bx)}{b^2+1}-\frac{be^x\cos(a+bx)}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + b*x],x]
```

```
[Out] -((b*E^x*Cos[a + b*x])/(1 + b^2)) + (E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(a+bx)\,dx=-\frac{be^x\cos(a+bx)}{1+b^2}+\frac{e^x\sin(a+bx)}{1+b^2}$$

Mathematica [A] time = 0.0579994, size = 27, normalized size = 0.73

$$\frac{e^x(\sin(a+bx)-b\cos(a+bx))}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + b*x],x]
```

```
[Out] (E^x*(-(b*Cos[a + b*x]) + Sin[a + b*x]))/(1 + b^2)
```

Maple [A] time = 0.003, size = 36, normalized size = 1.

$$-\frac{e^{x} b \cos (b x+a)}{b^{2}+1}+\frac{e^{x} \sin (b x+a)}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(b*x+a),x)
```

```
[Out] -b*exp(x)*cos(b*x+a)/(b^2+1)+exp(x)*sin(b*x+a)/(b^2+1)
```

Maxima [A] time = 0.987557, size = 38, normalized size = 1.03

$$-\frac{(b \cos (b x+a)-\sin (b x+a)) e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(b*cos(b*x + a) - sin(b*x + a))*e^x/(b^2 + 1)
```

Fricas [A] time = 0.462817, size = 70, normalized size = 1.89

$$-\frac{b \cos (b x+a) e^{x}-e^{x} \sin (b x+a)}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x + a)*e^x - e^x*sin(b*x + a))/(b^2 + 1)
```

Sympy [A] time = 1.49205, size = 114, normalized size = 3.08

$$\begin{cases} \frac{xe^x \sin(a-ix)}{2} + \frac{ixe^x \cos(a-ix)}{2} - \frac{ie^x \cos(a-ix)}{2} & \text{for } b=-i \\ \frac{xe^x \sin(a+ix)}{2} - \frac{ixe^x \cos(a+ix)}{2} + \frac{e^x \sin(a+ix)}{2} & \text{for } b=i \\ -\frac{be^x \cos(a+bx)}{b^2+1} + \frac{e^x \sin(a+bx)}{b^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sin(a - I*x)/2 + I*x*exp(x)*cos(a - I*x)/2 - I*exp(x)*c
os(a - I*x)/2, Eq(b, -I)), (x*exp(x)*sin(a + I*x)/2 - I*x*exp(x)*cos(a + I*
x)/2 + exp(x)*sin(a + I*x)/2, Eq(b, I)), (-b*exp(x)*cos(a + b*x)/(b**2 + 1)
 + exp(x)*sin(a + b*x)/(b**2 + 1), True))
```

Giac [A] time = 1.28129, size = 47, normalized size = 1.27

$$-\left(\frac{b\cos(bx+a)}{b^2+1} - \frac{\sin(bx+a)}{b^2+1}\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x + a)/(b^2 + 1) - sin(b*x + a)/(b^2 + 1))*e^x
```

3.74 $\int e^x \sin\left(a+cx^2\right)\,dx$

Optimal. Leaf size=115

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c*
x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
(2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rubi [A] time = 0.123032, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4472, 2234, 2204, 2205}

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c*
x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
(2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \sin\left(a+cx^2\right)dx &= \int \left(\frac{1}{2}ie^{-ia+x-icx^2}-\frac{1}{2}ie^{ia+x+icx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-ia+x-icx^2}\,dx-\frac{1}{2}i\int e^{ia+x+icx^2}\,dx\\ &= \frac{1}{2}\left(ie^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\right)\int e^{\frac{i(1-2icx)^2}{4c}}\,dx-\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\right)\int e^{-\frac{i(1+2icx)^2}{4c}}\,dx\\ &= \frac{(-1)^{3/4}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.156724, size = 108, normalized size = 0.94

$$-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{i}{4}/c}\left(e^{\frac{i}{2}/c}(\cos(a)+i\sin(a))\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(2cx-i)}{2\sqrt{c}}\right)+(\sin(a)+i\cos(a))\operatorname{Erfi}\left(\frac{(-1)^{3/4}(2cx+i)}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + c*x^2],x]
```

```
[Out] -((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sqrt[c
])]*(Cos[a] + I*Sin[a]) + Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(I*Cos
[a] + Sin[a])))/(4*Sqrt[c]*E^((I/4)/c))
```

Maple [A] time = 0.124, size = 88, normalized size = 0.8

$$-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{4}(4ac+1)}{c}}\operatorname{Erf}\left(\sqrt{-ic}x-\frac{1}{2}\frac{1}{\sqrt{-ic}}\right)\frac{1}{\sqrt{-ic}}+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{-\frac{i}{4}(4ac+1)}{c}}\operatorname{Erf}\left(\sqrt{ic}x-\frac{1}{2}\frac{1}{\sqrt{ic}}\right)\frac{1}{\sqrt{ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(c*x^2+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*x-1/2/
(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^
(1/2)*x-1/2/(I*c)^(1/2))
```

Maxima [B] time = 1.92927, size = 377, normalized size = 3.28

$$-\frac{\sqrt{\pi}\left(\left(\left(-i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)-i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)-\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2
(0, c)))*cos(1/4*(4*a*c + 1)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1
/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4
*pi + 1/2*arctan2(0, c)))*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqr
t(I*c)) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*arctan
2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arctan2(0, c
)))*cos(1/4*(4*a*c + 1)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi
 + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi +
 1/2*arctan2(0, c)))*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*
c)))/sqrt(abs(c))
```

Fricas [B] time = 0.492526, size = 549, normalized size = 4.77

$$\frac{i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\operatorname{C}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\operatorname{C}\left(-\frac{\sqrt{2}(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\operatorname{S}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_cos(1/2*sqrt(
2)*(2*c*x + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)
/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/
pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)
/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*fresnel_sin(-1/2*sqrt(2
)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \sin\left(a + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sin(a + c*x**2), x)
```

Giac [A] time = 1.14007, size = 171, normalized size = 1.49

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4iac+i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4iac-i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + I/c)*(I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(4*I*a*c + I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) + 1/4*I*s
qrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - I/c)*(-I*c/abs(c) + 1)*sqrt(abs(c))
)*e^(-1/4*(-4*I*a*c - I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.75 $\int e^x \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=144

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + I
*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*E^((-I)*a + ((I/4)
*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rubi [A] time = 0.216196, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4472, 2234, 2204, 2205}

$$\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + I
*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(3/4)*E^((-I)*a + ((I/4)
*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c])
])/(4*Sqrt[c])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^x \sin\left(a+bx+cx^2\right)\,dx &= \int \left(\frac{1}{2} i e^{-ia+(1-ib)x-icx^2} - \frac{1}{2} i e^{ia+(1+ib)x+icx^2}\right) dx \\
&= \frac{1}{2} i \int e^{-ia+(1-ib)x-icx^2}\,dx - \frac{1}{2} i \int e^{ia+(1+ib)x+icx^2}\,dx \\
&= -\left(\frac{1}{2}\left(i e^{\frac{1}{4} i \left(4a+\frac{(1+ib)^2}{c}\right)}\right) \int e^{-\frac{i(1+ib+2icx)^2}{4c}}\,dx\right) + \frac{1}{2}\left(i e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\right) \int e^{\frac{i(1-ib-2icx)^2}{4c}}\,dx \\
&= \frac{(-1)^{3/4} e^{\frac{1}{4} i \left(4a+\frac{(1+ib)^2}{c}\right)} \sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{(-1)^{3/4} e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}} \sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] time = 0.255904, size = 134, normalized size = 0.93

$$
-\frac{\sqrt[4]{-1}\sqrt{\pi} e^{-\frac{i\left(b^2-2ib+1\right)}{4c}} \left(e^{\frac{ib^2}{2c}} (\sin(a)+i\cos(a))\,\mathrm{Erfi}\left(\frac{(-1)^{3/4}(b+2cx+i)}{2\sqrt{c}}\right) + e^{\frac{i}{2}/c} (\cos(a)+i\sin(a))\,\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(b+2cx-i)}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[a + b*x + c*x^2],x]
```

```
[Out] -((-1)^(1/4)*Sqrt[Pi]*(E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b + 2*c*x))/(2*Sq
rt[c])]*(Cos[a] + I*Sin[a]) + E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b + 2
*c*x))/(2*Sqrt[c])]*(I*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^(((I/4)*(1 - (2*I)*b
 + b^2))/c))
```

Maple [A] time = 0.109, size = 117, normalized size = 0.8

$$\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{4}\left(-b^{2}+2ib+4ac+1\right)}{c}}\mathrm{Erf}\left(-\sqrt{-ic}x+\frac{1+ib}{2}\frac{1}{\sqrt{-ic}}\right)\frac{1}{\sqrt{-ic}}+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{4}\left(2ib-4ac+b^{2}-1\right)}{c}}\mathrm{Erf}\left(\sqrt{ic}x-\frac{-ib+1}{2}\frac{1}{\sqrt{ic}}\right)\frac{1}{\sqrt{ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(c*x^2+b*x+a),x)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/(-I*c)^(1/2)*erf(-(-I*c)^(
1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))+1/4*I*Pi^(1/2)*exp(1/4*I*(2*I*b-4*a*c+b^2-
1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2*(-I*b+1)/(I*c)^(1/2))
```

Maxima [B] time = 2.08344, size = 417, normalized size = 2.9

$$-\frac{\sqrt{\pi}\left(\left(\left(i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)+i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)+\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan\left(0,c\right)\right)-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\mathrm{ar}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((I*cos(1/4*pi + 1/2*arctan2(0, c)) + I*cos(-1/4*pi + 1/2*ar
ctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arctan2(
0, c)))*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) +
cos(-1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2
*I*c*x + I*b - 1)*sqrt(I*c)/c) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*c
os(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/
4*pi + 1/2*arctan2(0, c)))*cos(-1/4*(b^2 - 4*a*c - 1)/c) + (cos(1/4*pi + 1/
2*arctan2(0, c)) + cos(-1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*ar
ctan2(0, c)) - I*sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c -
1)/c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(abs(c
))
```

Fricas [B] time = 0.500768, size = 647, normalized size = 4.49

$$\frac{i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^{2}-4iac-2b-i}{4c}\right)}C\left(\frac{\sqrt{2}(2cx+b+i)\sqrt{\frac{c}{\pi}}}{2c}\right)+i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-ib^{2}+4iac-2b+i}{4c}\right)}C\left(-\frac{\sqrt{2}(2cx+b-i)\sqrt{\frac{c}{\pi}}}{2c}\right)+\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^{2}-4iac-2b-i}{4c}\right)}S\left(\frac{\sqrt{2}(\cdots}{}\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_
cos(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) + I*sqrt(2)*pi*sqrt(c/pi)*e^(
1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I
)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/
c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \sin\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sin(a + b*x + c*x**2), x)
```

Giac [A] time = 1.16922, size = 198, normalized size = 1.38

$$\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x + \frac{b-i}{c}\right)\left(-\frac{ic}{|c|} + 1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac+2b-i}{4c}\right)}}{4\left(-\frac{ic}{|c|} + 1\right)\sqrt{|c|}} - \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x + \frac{b+i}{c}\right)\left(\frac{ic}{|c|} + 1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac+2b+i}{4c}\right)}}{4\left(\frac{ic}{|c|} + 1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b - I)/c)*(-I*c/abs(c) + 1)
*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((-I*c/abs(c) + 1)*sq
rt(abs(c))) - 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b + I)/c)*(I*
c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((I*c/a
bs(c) + 1)*sqrt(abs(c)))
```

3.76 $\int e^{x^2} \sin(a+bx)\,dx$

Optimal. Leaf size=81

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rubi [A] time = 0.0549439, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4472, 2234, 2204}

$$\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)-\frac{1}{4}i\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x],x]
```

```
[Out] (I/4)*E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2] - (I/4)*E^(I*a + b
^2/4)*Sqrt[Pi]*Erfi[(I*b + 2*x)/2]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sin(a+bx)\,dx &= \int\left(\frac{1}{2}ie^{-ia-ibx+x^2}-\frac{1}{2}ie^{ia+ibx+x^2}\right)dx\\
&= \frac{1}{2}i\int e^{-ia-ibx+x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+x^2}\,dx\\
&= \frac{1}{2}\left(ie^{-ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(-ib+2x)^2}\,dx-\frac{1}{2}\left(ie^{ia+\frac{b^2}{4}}\right)\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\
&= \frac{1}{4}ie^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)-\frac{1}{4}ie^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0226392, size = 81, normalized size = 1.

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(\cos(a)\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\cos(a)\mathrm{Erf}\left(\frac{b}{2}+ix\right)+\sin(a)\left(\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erf[b/2 - I*x] + Cos[a]*Erf[b/2 + I*x] + (Erfi[
((-I)*b + 2*x)/2] + Erfi[(I*b + 2*x)/2])*Sin[a]))/4
```

Maple [A] time = 0., size = 52, normalized size = 0.6

$$\frac{\sqrt{\pi}\mathrm{e}^{ia}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(-ix+\frac{b}{2}\right)+\frac{\sqrt{\pi}\mathrm{e}^{-ia}}{4}\mathrm{e}^{\frac{b^2}{4}}\mathrm{Erf}\left(ix+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*b^2)*exp(I*a)*erf(-I*x+1/2*b)+1/4*Pi^(1/2)*exp(1/4*b^2
)*exp(-I*a)*erf(I*x+1/2*b)
```

Maxima [A] time = 1.0317, size = 69, normalized size = 0.85

$$\frac{1}{4}\sqrt{\pi}\left((\cos(a) - i\sin(a))\,\mathrm{erf}\left(\frac{1}{2}\,b + i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)} - (\cos(a) + i\sin(a))\,\mathrm{erf}\left(-\frac{1}{2}\,b + i\,x\right)e^{\left(\frac{1}{4}\,b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*((cos(a) - I*sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) - (cos(a) +
I*sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A] time = 0.470909, size = 122, normalized size = 1.51

$$-\frac{1}{4}\sqrt{\pi}\left(\mathrm{erf}\left(-\frac{1}{2}\,b + i\,x\right)e^{\left(\frac{1}{4}\,b^2 + i\,a\right)} - \mathrm{erf}\left(\frac{1}{2}\,b + i\,x\right)e^{\left(\frac{1}{4}\,b^2 - i\,a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*(erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - erf(1/2*b + I*x)*e^(1/
4*b^2 - I*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sin(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(x^2\right)}\sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(b*x + a), x)
```

3.77 $\int e^{x^2} \sin\left(a + cx^2\right) dx$

Optimal. Leaf size=87

$$\frac{i\sqrt{\pi}e^{-ia}\text{Erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} - \frac{i\sqrt{\pi}e^{ia}\text{Erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[Out] ((I/4)*Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(Sqrt[1 - I*c]*E^(I*a)) - ((I/4)*E^(
I*a)*Sqrt[Pi]*Erfi[Sqrt[1 + I*c]*x])/Sqrt[1 + I*c]
```

Rubi [A] time = 0.0984525, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4472, 2204}

$$\frac{i\sqrt{\pi}e^{-ia}\text{Erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}} - \frac{i\sqrt{\pi}e^{ia}\text{Erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(Sqrt[1 - I*c]*E^(I*a)) - ((I/4)*E^(
I*a)*Sqrt[Pi]*Erfi[Sqrt[1 + I*c]*x])/Sqrt[1 + I*c]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\sin\left(a+cx^2\right)dx=\int\left(\frac{1}{2}ie^{-ia+(1-ic)x^2}-\frac{1}{2}ie^{ia+(1+ic)x^2}\right)dx$$
$$=\frac{1}{2}i\int e^{-ia+(1-ic)x^2}\,dx-\frac{1}{2}i\int e^{ia+(1+ic)x^2}\,dx$$
$$=\frac{ie^{-ia}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-ic}\,x\right)}{4\sqrt{1-ic}}-\frac{ie^{ia}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+ic}\,x\right)}{4\sqrt{1+ic}}$$

Mathematica [A] time = 0.218311, size = 129, normalized size = 1.48

$$-\frac{\sqrt[4]{-1}\sqrt{\pi}\left(\sqrt{c+i}\left(\sin(a)\mathrm{Erf}\left(\frac{(1+i)\sqrt{c+i}x}{\sqrt{2}}\right)+\mathrm{Erfi}\left((-1)^{3/4}\sqrt{c+i}x\right)(c\sin(a)+ic\cos(a)+\cos(a))\right)+\sqrt{c-i}(c+i)(\cos(a)+i}{4\left(c^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + c*x^2],x]
```

```
[Out] -((-1)^(1/4)*Sqrt[Pi]*(Sqrt[-I + c]*(I + c)*Erfi[(-1)^(1/4)*Sqrt[-I + c]*x]
*(Cos[a] + I*Sin[a]) + Sqrt[I + c]*(Erf[((1 + I)*Sqrt[I + c]*x)/Sqrt[2]]*Si
n[a] + Erfi[(-1)^(3/4)*Sqrt[I + c]*x]*(Cos[a] + I*c*Cos[a] + c*Sin[a]))))/(
4*(1 + c^2))
```

Maple [A] time = 0.063, size = 62, normalized size = 0.7

$$-\frac{i}{4}\sqrt{\pi}e^{ia}\mathrm{Erf}\left(\sqrt{-ic-1}x\right)\frac{1}{\sqrt{-ic-1}}+\frac{i}{4}\sqrt{\pi}e^{-ia}\mathrm{Erf}\left(\sqrt{-1+ic}x\right)\frac{1}{\sqrt{-1+ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(c*x^2+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)+1/4*I*Pi^(1/2
)*exp(-I*a)/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.482512, size = 192, normalized size = 2.21

$$\frac{\sqrt{\pi}(c+i)\sqrt{-i\,c-1}\,\operatorname{erf}\left(\sqrt{-i\,c-1}x\right)e^{(i\,a)}+\sqrt{\pi}(c-i)\sqrt{i\,c-1}\,\operatorname{erf}\left(\sqrt{i\,c-1}x\right)e^{(-i\,a)}}{4\left(c^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(sqrt(-I*c - 1)*x)*e^(I*a) + sqrt(p
i)*(c - I)*sqrt(I*c - 1)*erf(sqrt(I*c - 1)*x)*e^(-I*a))/(c^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sin\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(x^2\right)}\sin\left(cx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(c*x^2 + a), x)
```

3.78 $\int e^{x^2} \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=155

$$-\frac{i\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{i\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

```
[Out] ((-I/4)*Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 - I
*c]*E^(I*(a - b^2/(4*I + 4*c)))) - ((I/4)*E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[
Pi]*Erfi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/Sqrt[1 + I*c]
```

Rubi [A] time = 0.201997, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4472, 2234, 2204}

$$-\frac{i\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{i\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] ((-I/4)*Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(Sqrt[1 - I
*c]*E^(I*(a - b^2/(4*I + 4*c)))) - ((I/4)*E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[
Pi]*Erfi[(I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/Sqrt[1 + I*c]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\sin\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}ie^{-ia-ibx+(1-ic)x^2}-\frac{1}{2}ie^{ia+ibx+(1+ic)x^2}\right)dx\\
&= \frac{1}{2}i\int e^{-ia-ibx+(1-ic)x^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+(1+ic)x^2}\,dx\\
&= -\left(\frac{1}{2}\left(ie^{ia+\frac{b^2}{4(1+ic)}}\right)\int\exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right)dx\right)+\frac{1}{2}\left(ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\right)\int\exp\left(\frac{(-ib+2(1-ic
\end{aligned}
$$

$$
= -\frac{ie^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}-\frac{ie^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}
$$

Mathematica [A] time = 0.562098, size = 165, normalized size = 1.06

$$
-\frac{(-1)^{3/4}\sqrt{\pi}e^{\frac{ib^2}{-4c+4i}}\left((c-i)\sqrt{c+i}e^{\frac{ib^2c}{2c^2+2}}(\cos(a)-i\sin(a))\mathrm{Erfi}\left(\frac{(-1)^{3/4}(b+2(c+i)x)}{2\sqrt{c+i}}\right)+\sqrt{c-i}(c+i)(\sin(a)-i\cos(a))\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(b+}{2\sqrt{}}\right.\right.}{4\left(c^2+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] -((-1)^(3/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*((-I + c)*Sqrt[I + c]*E^((I*b
^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]*(Co
s[a] - I*Sin[a]) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c)*x)
)/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(4*(1 + c^2))
```

Maple [A] time = 0.208, size = 127, normalized size = 0.8

$$
\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{-4\,ac+4\,ia+b^2}{4\,ic+4}}\mathrm{Erf}\left(-\sqrt{-ic-1}x+\frac{i}{2}b\frac{1}{\sqrt{-ic-1}}\right)\frac{1}{\sqrt{-ic-1}}+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{4\,ia+4\,ac-b^2}{4\,ic-4}}\mathrm{Erf}\left(\sqrt{-1+ic}x+\frac{i}{2}b\frac{1}{\sqrt{-1+ic}}\right)\frac{1}{\sqrt{-1+ic}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*sin(c*x^2+b*x+a),x)
```

```
[Out] 1/4*I*Pi^(1/2)*exp(1/4*(-4*a*c+4*I*a+b^2)/(1+I*c))/(-I*c-1)^(1/2)*erf(-(-I*
c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))+1/4*I*Pi^(1/2)*exp(1/4*(4*I*a+4*a*c-b^
2)/(-1+I*c))/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x+1/2*I*b/(-1+I*c)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.499874, size = 427, normalized size = 2.75

$$-\frac{\sqrt{\pi}(c-i)\sqrt{i\,c-1}\,\text{erf}\left(-\frac{\left(bc+2\left(c^2+1\right)x-i\,b\right)\sqrt{i\,c-1}}{2\left(c^2+1\right)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\left(c^2+1\right)}\right)}-\sqrt{\pi}(c+i)\sqrt{-i\,c-1}\,\text{erf}\left(\frac{\left(bc+2\left(c^2+1\right)x+i\,b\right)\sqrt{-i\,c-1}}{2\left(c^2+1\right)}\right)e^{\left(\frac{-i\,b^2c+4}{4}\right.}}{4\left(c^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c - I)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*s
qrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 + 1
)) - sqrt(pi)*(c + I)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)*sq
rt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2 +
1)))/(c^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\sin\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sin(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(x^2\right)} \sin\left(cx^2 + bx + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(e^(x^2)*sin(c*x^2 + b*x + a), x)
```

3.79 $\int f^{a+bx}\sin\left(d+fx^2\right)\,dx$

Optimal. Leaf size=142

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2}{2\sqrt{f}}\right.$$

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[((
-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*f^(-1/2 + a
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I/
4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rubi [A] time = 0.210487, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4472, 2287, 2234, 2204, 2205}

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2}{2\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[((
-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*f^(-1/2 + a
)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I/
4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sin\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+bx}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+bx}\,dx\\
&= \frac{1}{2}i\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx-\frac{1}{2}i\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(ie^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\,dx\\
&= \frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+
\end{aligned}
$$

Mathematica [A] time = 0.2288, size = 132, normalized size = 0.93

$$
-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{ib^2\log^2(f)}{4f}}\left(e^{\frac{ib^2\log^2(f)}{2f}}(\cos(d)+i\sin(d))\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)+(\sin(d)+i\cos(d))\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2fx}{2}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(
1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Erfi[((-1)^(3
/4)*(2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(I*Cos[d] + Sin[d])))/(4*E^(((I/4)*b
^2*Log[f]^2)/f))
```

Maple [A] time = 0.267, size = 116, normalized size = 0.8

$$\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{4}\left(\left(\ln(f)\right)^{2}b^{2}+4df\right)}{f}}\mathrm{Erf}\left(-\sqrt{-if}x+\frac{b\ln\left(f\right)}{2}\frac{1}{\sqrt{-if}}\right)\frac{1}{\sqrt{-if}}-\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{\frac{-\frac{i}{4}\left(\left(\ln(f)\right)^{2}b^{2}+4df\right)}{f}}\mathrm{Erf}\left(-\sqrt{if}x+\frac{b\ln\left(f\right)}{2}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f
)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*
b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.504809, size = 749, normalized size = 5.27

$$\frac{i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^{2}\log(f)^{2}+4af\log(f)-4idf}{4f}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)+i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^{2}\log(f)^{2}+4af\log(f)+4idf}{4f}\right)}\mathrm{C}\left(-\frac{\sqrt{2}(2fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)+\cdots}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d
*f)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + I*sqrt(
2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresne
l_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/p
i)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqr
t(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sin\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2), x)
```

Giac [B] time = 1.28821, size = 405, normalized size = 2.85

$$\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\,\mathrm{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi a\mathrm{s}\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
+ 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f
))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(a
bs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sqrt(pi)*erf
(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f
```

```
) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*s
gn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f
))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((I*f/abs(f)
 + 1)*sqrt(abs(f)))
```

3.80 $\int f^{a+bx}\sin^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=157

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\mathrm{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\right.$$

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] + ((1/16 + I/16)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(1
6*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rubi [A] time = 0.203078, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4472, 2194, 2287, 2234, 2204, 2205}

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\mathrm{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\mathrm{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-b\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] + ((1/16 + I/16)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(1
6*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\sin^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx-\frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx-\frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-4ifx+b\log}{8f}}\\
&= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log}{}\right.}
\end{aligned}
$$

Mathematica [A] time = 1.07239, size = 156, normalized size = 0.99

$$\frac{1}{16}f^a\left(-\frac{(1-i)\sqrt{\pi}e^{-\frac{ib^2\log^2(f)}{8f}}(\cos(d)-i\sin(d))^2\operatorname{Erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)}{\sqrt{f}}-\frac{(1-i)\sqrt{\pi}e^{\frac{ib^2\log^2(f)}{8f}}(\cos(d)+i\sin(d))^2\operatorname{Erfi}\left(\frac{(1-i}{}\right.}{\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) - ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
)*Sqrt[f]) - ((1 - I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] + I*Sin[d])^2)/Sqrt[f]))/16
```

Maple [A] time = 0.374, size = 139, normalized size = 0.9

$$\frac{\sqrt{2}f^a\sqrt{\pi}}{16}e^{\frac{-\frac{i}{8}\left(\left(\ln(f)\right)^2b^2+16df\right)}{f}}\operatorname{Erf}\left(-\sqrt{2}\sqrt{if}x+\frac{\ln(f)b\sqrt{2}}{4}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}+\frac{f^a\sqrt{\pi}}{8}e^{\frac{\frac{i}{8}\left(\left(\ln(f)\right)^2b^2+16df\right)}{f}}\operatorname{Erf}\left(-\sqrt{-2\,if}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^2,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*er
f(-2^(1/2)*(I*f)^(1/2)*x+1/4*ln(f)*b*2^(1/2)/(I*f)^(1/2))+1/8*Pi^(1/2)*f^a*
exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*
ln(f)*b/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.516861, size = 770, normalized size = 4.9

$$-\frac{2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log\left(f\right)^2+8\,af\log\left(f\right)-16i\,df}{8\,f}\right)}\mathrm{C}\left(\frac{\left(4\,fx+i\,b\log\left(f\right)\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log\left(f\right)-2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log\left(f\right)^2+8\,af\log\left(f\right)+16i\,df}{8\,f}\right)}\mathrm{C}\left(-\frac{\left(4\,fx-i\,b\log\left(f\right)\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)
/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - 2*pi*b*sqrt
(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/
2*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(-
I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*l
og(f))*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 +
8*a*f*log(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi
)/f)*log(f) - 8*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sin^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**2, x)
```

Giac [B] time = 1.38776, size = 703, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f)
- 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) + 2*I*e^(-1/2*
I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b
*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f)))
+ 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(
f)))/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs
(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*lo
g(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sq
rt(f)*(-I*f/abs(f) + 1)) + 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(f
) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(
f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log
(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a
*log(abs(f)) - 2*I*d)/(sqrt(f)*(I*f/abs(f) + 1))
```

3.81 $\int f^{a+bx}\sin^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=298

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)}{\sqrt{6}\sqrt{f}}\right.$$

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[
((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/16)*E^((3*
I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*((
6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(3/4)*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4*
d + (b^2*Log[f]^2)/f))) - ((1/16 - I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/2
 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2*
Log[f]^2)/f))
```

Rubi [A] time = 0.366547, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4472, 2287, 2234, 2204, 2205}

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)}{\sqrt{6}\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```


```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[
((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/16)*E^((3*
I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*((
6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(3/4)*f^(-1/2 + a)*Sqrt[
Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4*
d + (b^2*Log[f]^2)/f))) - ((1/16 - I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/2
 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2*
Log[f]^2)/f))
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx}\sin^3\left(d+fx^2\right)dx = \int\left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+bx}\right)dx$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+bx}\,dx-\frac{3}{8}i\int e^{id+i}$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}i\int e^{-id-ifx^2+}$$

$$= \frac{1}{8}\left(ie^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(3ie^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx-\frac{1}{8}$$

$$= \frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-\frac{1}{2}}$$

Mathematica [A] time = 0.884968, size = 268, normalized size = 0.9

$$\frac{1}{48}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{ib^2\log^2(f)}{4f}}\left(9ie^{\frac{ib^2\log^2(f)}{2f}}(\cos(d)+i\sin(d))\text{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)+\sqrt{3}e^{\frac{ib^2\log^2(f)}{6f}}\left(e^{\frac{ib^2\log^2(f)}{6f}}(\sin(3d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f])
)/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + (9*I)*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[
((-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d]) + Sqrt[3
]*E^(((I/6)*b^2*Log[f]^2)/f)*(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sqr
t[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d]) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[((
6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[
3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

Maple [A] time = 0.481, size = 239, normalized size = 0.8

$$-\frac{i}{16}f^a\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{12}\left((\ln(f))^2b^2+36df\right)}{f}}\text{Erf}\left(-\sqrt{-3if}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-3if}}\right)\frac{1}{\sqrt{-3if}}+\frac{i}{48}\sqrt{3}f^a\sqrt{\pi}\mathrm{e}^{\frac{-\frac{i}{12}\left((\ln(f))^2b^2+36df\right)}{f}}\text{Erf}\left(-\sqrt{3}\sqrt{if}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(
-(-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-3*I*f)^(1/2))+1/48*I*Pi^(1/2)*f^a*exp(-1/12
*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1
/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2
+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))+3/16*I*P
i^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)
*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.545741, size = 1519, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 3
6*I*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*
sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f
)*fresnel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)
*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel
_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f
/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*s
qrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(
-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f
*x + I*b*log(f))*sqrt(f/pi)/f) + sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f
)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*lo
g(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a
*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f
/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4
*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx} \sin^3\left(d + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + f*x**2)**3, x)
```

Giac [B] time = 1.46531, size = 803, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
 + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(
f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(
abs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*I*sqrt(6)*sqrt(pi)*e
rf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f
)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))*
sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt(
f)*(-I*f/abs(f) + 1)) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(
12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/
24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2
/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sg
n(f) + 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(I*f/abs(f) + 1)) - 3/1
6*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*lo
g(abs(f)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f -
1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))
/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs
(f)) - I*d)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.82 $\int f^{a+bx}\sin\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=162

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)\cdots}{2\sqrt{f}}\right.$$

```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*E
^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^(
1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.333609, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4472, 2287, 2234, 2204, 2205}

$$\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)\cdots}{2\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```


```
[Out] ((-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*Er
f[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(3/4)*E
^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^(
1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\sin\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+bx}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+bx}\right)dx\\ &=\frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+bx}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+bx}\,dx\\ &=\frac{1}{2}i\int\exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+ifx^2+a\log(\right.\\ &=\frac{1}{2}\left(ie^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}dx-\frac{1}{2}\left(ie^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log}{4f}}\\ &=\frac{1}{4}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}(-1)^{3/4}e^{-id+\frac{i(e+}{}}\end{aligned}$$

Mathematica [A] time = 0.385259, size = 162, normalized size = 1.

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{i\left(b^2\log^2(f)+e^2\right)}{4f}}\left(e^{\frac{ib^2\log^2(f)}{2f}}(\cos(d)+i\sin(d))\text{Erfi}\left(\frac{\sqrt[4]{-1}(-ib\log(f)+e+2fx)}{2\sqrt{f}}\right)+e^{\frac{ie^2}{2f}}(\sin(d)+i\cos(d))\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^(((I/2)*b^2*Log[f]^2)/f)*E
rfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] + I*Sin[d])
+ E^(((I/2)*e^2)/f)*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[f])]
*(I*Cos[d] + Sin[d])))/(4*E^(((I/4)*(e^2 + b^2*Log[f]^2))/f))
```

Maple [A] time = 0.195, size = 152, normalized size = 0.9

$$\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{4}\left(\left(\ln\left(f\right)\right)^{2}b^{2}+2\,i\ln\left(f\right)be-e^{2}+4\,df\right)}{f}}\mathrm{Erf}\left(-\sqrt{-if}x+\frac{ie+b\ln\left(f\right)}{2}\frac{1}{\sqrt{-if}}\right)\frac{1}{\sqrt{-if}}-\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{\frac{-\frac{i}{4}\left(\left(\ln\left(f\right)\right)^{2}b^{2}-2\,i\ln\left(f\right)be-e^{2}+4\,df\right)}{f}}\mathrm{Erf}\left(-\sqrt{if}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f
)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/4*I*Pi^(1/2)*
f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I
*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.520653, size = 868, normalized size = 5.36

$$\frac{i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^{2}\log\left(f\right)^{2}+ie^{2}-4idf-2\left(be-2af\right)\log\left(f\right)}{4f}\right)}\mathrm{C}\left(\frac{\sqrt{2}\left(2fx+ib\log\left(f\right)+e\right)\sqrt{\frac{f}{\pi}}}{2f}\right)+i\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^{2}\log\left(f\right)^{2}-ie^{2}+4idf-2\left(be-2af\right)\log\left(f\right)}{4f}\right)}\mathrm{C}\left(-\frac{\sqrt{2}\left(2fx\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*
(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*s
qrt(f/pi)/f) + I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I
*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log
(f) + e)*sqrt(f/pi)/f) + sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*
e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x +
 I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx} \sin \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2), x)
```

Giac [B] time = 1.31703, size = 518, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn
(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log
(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*s
gn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*I*sqrt(2)*sq
rt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*
e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b
^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4
```

```
*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*
pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I
*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.83 $\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=179

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\text{Erfi}\left(\left(\frac{1}{4}\cdots\right.\right.$$

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*S
qrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] + (1/16
 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*x
)/(2*b*Log[f])
```

Rubi [A] time = 0.352211, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4472, 2194, 2287, 2234, 2204, 2205}

$$\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\text{Erfi}\left(\left(\frac{1}{4}\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (1/16 + I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*S
qrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] + (1/16
 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*x
)/(2*b*Log[f])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx}\,dx\right)-\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\int\exp\left(-2id-2ifx^2+a\log(f)-x(2ie-b\log(f))\right)dx-\frac{1}{4}\int\exp\left(2id\right.\\
&= \frac{f^{a+bx}}{2b\log(f)}-\frac{1}{4}\exp\left(-2id+a\log(f)-\frac{i(-2ie+b\log(f))^2}{8f}\right)\int e^{\frac{i(-2ie-4ifx+b\log(f))^2}{8f}}dx-\frac{1}{4}\left(\right.\\
&= \left(\frac{1}{16}+\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)+\left(\frac{1}{16}+\frac{i}{16}\right.
\end{aligned}$$

Mathematica [A] time = 1.12311, size = 244, normalized size = 1.36

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{i\left(b^2\log^2(f)+4e^2\right)}{8f}}\left(\sqrt[4]{-1}\sqrt{2\pi}b\log(f)e^{\frac{ib^2\log^2(f)}{4f}}(\cos(2d)+i\sin(2d))\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2i(e+2fx))}{\sqrt{f}}\right)+8f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{i\left(b^2\log^2 \right.}{8}}\right.}{16b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b
*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erf[((
1/4 + I/4)*((2*I)*(e + 2*f*x) + b*Log[f]))/Sqrt[f]]*Log[f]*(Cos[2*d] + I*Si
n[2*d]) + (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f
*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)
*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

Maple [A] time = 0.34, size = 175, normalized size = 1.

$$\frac{\sqrt{2}f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{-\frac{i}{8}\left(\left(\ln(f)\right)^2b^2-4\,i\ln(f)be-4\,e^2+16\,df\right)}{f}}\mathrm{Erf}\left(-\sqrt{2}\sqrt{if}x+\frac{\left(b\ln\left(f\right)-2\,ie\right)\sqrt{2}}{4}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}+\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{\frac{\frac{i}{8}\left(\left(\ln(f)\right)^2b^2+4\,i\ln(f)be-4\,e^2+16\,df\right.}{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2-4*I*ln(f)*b*e-4*e^2+16*d*f)/f)*2^
(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/(I
*f)^(1/2))+1/8*Pi^(1/2)*f^a*exp(1/8*I*(ln(f)^2*b^2+4*I*ln(f)*b*e-4*e^2+16*d
*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)^(1
/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.518888, size = 910, normalized size = 5.08

$$-\frac{2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log\left(f\right)^2+4i\,e^2-16i\,d\,f-4\,\left(be-2\,af\right)\log\left(f\right)}{8\,f}\right)}\mathrm{C}\left(\frac{\left(4\,fx+i\,b\log\left(f\right)+2\,e\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log\left(f\right)-2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log\left(f\right)^2-4i\,e^2+16i\,d\,f-4\,\left(be-2\,af\right)\log\left(f\right)}{8\,f}\right)}\mathrm{C}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(2*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(
b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/p
i)/f)*log(f) - 2*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*
f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*
sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^
2 - 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(
f) + 2*e)*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2
 - 4*I*e^2 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x
- I*b*log(f) + 2*e)*sqrt(f/pi)/f)*log(f) - 8*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\sin^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sin(d + e*x + f*x**2)**2, x)
```

Giac [B] time = 1.35416, size = 817, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f)
- 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) + 2*I*e^(-1/2*
I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b
*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f)))
+ 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(
f)) - 4*e)/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*l
og(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*
b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*
a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*
e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) + 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (
pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(I*f/abs(f) + 1))*e^(-1/16
*I*pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f
+ 1/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f)
+ 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2
*b*e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1))
```

3.84 $\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=340

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\cdots}{\cdots}\right.$$

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/
16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]
*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3
*(-1)^(3/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]
*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 -
I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rubi [A] time = 0.601827, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4472, 2287, 2234, 2204, 2205}

$$\frac{3}{16}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*(-1)^(3/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*
Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 + (1/16 - I/
16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]
*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3
*(-1)^(3/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]
*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 -
I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx}\sin^3\left(d+ex+fx^2\right)dx=\int\left(-\frac{1}{8}ie^{-3i\left(d+ex+fx^2\right)}f^{a+bx}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}-\right.$$

$$=-\left(\frac{1}{8}i\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx}\,dx\right)+\frac{1}{8}i\int\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)$$

$$=-\left(\frac{1}{8}i\int\exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)dx\right)+\frac{1}{8}i\int\exp\left(3id+3i\right.$$

$$=-\left(\frac{1}{8}\left(i\exp\left(-3id+a\log(f)-\frac{i(-3ie+b\log(f))^2}{12f}\right)\right)\int e^{\frac{i(-3ie-6ifx+b\log(f))^2}{12f}}dx\right)+\frac{1}{8}\left(3ie\right.$$

$$=\frac{3}{16}(-1)^{3/4}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)+\left(\frac{1}{16}-\frac{i}{16}\right)e^{3i}$$

Mathematica [A] time = 1.5456, size = 323, normalized size = 0.95

$$\frac{1}{48}(-1)^{3/4}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{i(b^2\log^2(f)+3e^2)}{4f}}\left(9i(\cos(d)+i\sin(d))e^{\frac{i(b^2\log^2(f)+e^2)}{2f}}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(-ib\log(f)+e+2fx)}{2\sqrt{f}}\right)+e^{\frac{ie^2}{f}}\left(\sqrt{3}(\cos(3d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(3/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*((9*I)*E^(((I/2)*(e^2 + b^2*Lo
g[f]^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d]
 + I*Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2
))/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Co
s[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2
)*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d])
))/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A] time = 0.486, size = 311, normalized size = 0.9

$$-\frac{i}{16}f^a\sqrt{\pi}\mathrm{e}^{\frac{\frac{i}{12}\left((\ln(f))^2b^2+6i\ln(f)be-9e^2+36df\right)}{f}}\operatorname{Erf}\left(-\sqrt{-3if}x+\frac{3ie+b\ln(f)}{2}\frac{1}{\sqrt{-3if}}\right)\frac{1}{\sqrt{-3if}}+\frac{i}{48}\sqrt{3}f^a\sqrt{\pi}\mathrm{e}^{\frac{-\frac{i}{12}\left((\ln(f))^2b^2-6i\ln(f)be\right.}{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+6*I*ln(f)*b*e-9*e^2+36*d*f)/f)
/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))+1
/48*I*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2-6*I*ln(f)*b*e-9*e^2+36*d*f)/f)*
3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^(1/2)/
(I*f)^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+
4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))+3/1
6*I*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)^
(1/2)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.555856, size = 1773, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(-I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*
f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f)
+ 3*e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*
I*e^2 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*(6*f
*x - I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I
*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) + 9*I*sqrt(2)*pi*sqrt(f/pi
)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fre
snel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - sqrt(6)*pi*s
qrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e - 2*a*f)*l
og(f))/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f)
+ sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*I*d*f - 6*(b
*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*
sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4
*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*lo
g(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 -
I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*
x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.55873, size = 1030, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b
*log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sg
n(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*lo
g(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*
sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f)
)/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*I*sqrt(6)*
sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(
abs(f)) - 6*e)/f)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*
b^2*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f +
1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/
2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d -
 3/4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) + 1/48*I*sqrt(6)*sqrt(pi)*erf(-1/
24*sqrt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/f
)*(I*f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*
sgn(f)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(a
bs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I
*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/(s
qrt(f)*(I*f/abs(f) + 1)) - 3/16*I*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x +
(pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs
(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I
*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*
pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(a
bs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(
abs(f)))
```

3.85 $\int f^{a+cx^2} \sin(d+ex)\,dx$

Optimal. Leaf size=151

$$-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\operatorname{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] ((-I/4)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f]
)/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - ((I/4)*E^(I*d + e^2/(
4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.207281, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4472, 2287, 2234, 2204}

$$-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\operatorname{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x],x]
```


```
[Out] ((-I/4)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f]
)/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - ((I/4)*E^(I*d + e^2/(
4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]])
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sin(d+ex)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex}f^{a+cx^2}-\frac{1}{2}ie^{id+iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}i\int e^{-id-iex}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex}f^{a+cx^2}\,dx\\
&= \frac{1}{2}i\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx-\frac{1}{2}i\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{2}\left(ie^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{2}\left(ie^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{ie^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 0.153063, size = 119, normalized size = 0.79

$$
\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}}\left(i(\cos(d)+i\sin(d))\text{Erfi}\left(\frac{-2cx\log(f)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\sin(d)+i\cos(d))\text{Erfi}\left(\frac{2cx\log(f)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(I*Erfi[((-I)*e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqr
t[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.203, size = 123, normalized size = 0.8

$$\frac{i}{4} f^{a} \sqrt{\pi}\,\mathrm{e}^{\frac{4 i d \ln(f) c+e^{2}}{4 c \ln(f)}} \operatorname{Erf}\left(-\sqrt{-c \ln(f)}\, x+\frac{i}{2} e \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}}+\frac{i}{4} f^{a} \sqrt{\pi}\,\mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 c \ln(f)}} \operatorname{Erf}\left(\sqrt{-c \ln(f)}\, x+\frac{i}{2} e \frac{1}{\sqrt{-c \ln(f)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))+1/4*I*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.496638, size = 413, normalized size = 2.74

$$\frac{i \sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2 c x \log(f)+i e) \sqrt{-c \log(f)}}{2 c \log(f)}\right) e^{\left(\frac{4 a c \log(f)^{2}+4 i c d \log(f)+e^{2}}{4 c \log(f)}\right)}-i \sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{(2 c x \log(f)-i e) \sqrt{-c \log(f)}}{2 c \log(f)}\right) e^{\left(\frac{4 a c \log \ldots}{\ldots}\right)}}{4 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) -
```

```
I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log
(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d), x)
```

3.86 $\int f^{a+cx^2} \sin^2(d+ex)\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)}-2id}\operatorname{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)}+2id}\operatorname{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rubi [A] time = 0.217943, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4472, 2204, 2287, 2234}

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)}-2id}\operatorname{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)}+2id}\operatorname{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin^2(d+ex)\,dx &= \int \left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+cx^2}\right)dx\\ &= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\,dx-\frac{1}{4}\int e^{2id+2iex+a\log(f)+cx^2\log(f)}\,d\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{4}\left(e^{2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(2ie+2}{4c}}\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+cx\log(}{\sqrt{c}\sqrt{\log(}}\right.}{8\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.250202, size = 132, normalized size = 0.77

$$\frac{\sqrt{\pi}f^a\left(2\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)-e^{\frac{e^2}{c\log(f)}}\left((\cos(2d)-i\sin(2d))\text{Erfi}\left(\frac{cx\log(f)-ie}{\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(2d)+i\sin(2d))\text{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)\right)\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] - E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
```

```
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.2, size = 145, normalized size = 0.9

$$-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2id\ln(f)c-e^{2}}{c\ln(f)}}\,\mathrm{Erf}\left(\sqrt{-c\ln(f)}\,x+ie\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}+\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{\frac{2id\ln(f)c+e^{2}}{c\ln(f)}}\,\mathrm{Erf}\left(-\sqrt{-c\ln(f)}\,x+ie\frac{1}{\sqrt{-c\ln(f}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-
c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+
e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2)
)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.497576, size = 452, normalized size = 2.64

$$-\frac{2\sqrt{\pi}\sqrt{-c\log(f)}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log(f)}\,x\right)-\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(cx\log(f)+ie)\sqrt{-c\log(f)}}{c\log(f)}\right)e^{\left(\frac{ac\log(f)^{2}+2icd\log(f)+e^{2}}{c\log(f)}\right)}-\sqrt{\pi}\sqrt{-c\,\mathrm{lo}}}{8\,c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) - sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) - sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^2, x)
```

3.87 $\int f^{a+cx^2} \sin^3(d+ex)\,dx$

Optimal. Leaf size=301

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+$$

```
[Out] (((-3*I)/16)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((-3*I
)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + e^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*
Sqrt[Log[f]])
```

Rubi [A] time = 0.345859, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4472, 2287, 2234, 2204}

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```


```
[Out] (((-3*I)/16)*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((-3*I
)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + e^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*
Sqrt[Log[f]])
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\sin^3(d+ex)\,dx &= \int \left(\frac{3}{8}ie^{-id-iex}f^{a+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}ie^{3id+3iex}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex}f^{a+cx^2}\,dx+\frac{3}{8}i\int e^{-id-iex}f^{a+cx^2}\,dx-\frac{3}{8}i\int e^{id+iex}f^{a
\end{aligned}$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}i\int e^{-id-iex+a\log(}$$

$$= \frac{1}{8}\left(3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{8}\left(3ie^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{8}\left(ie^{-3id+\frac{9}{4c\,\mathrm{lo}}}\right.$$

$$= -\frac{3ie^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{ie^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3ie^{id+\frac{e^2}{4c\log(f)}}f}{16}$$

Mathematica [A] time = 0.430038, size = 224, normalized size = 0.74

$$\frac{\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}}\left(-ie^{\frac{2e^2}{c\log(f)}}\left((\cos(3d)-i\sin(3d))\mathrm{Erfi}\left(\frac{2cx\log(f)-3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)-(\cos(3d)+i\sin(3d))\mathrm{Erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)+3i(\cos(d)+\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*((3*I)*Erfi[((-I)*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + 3*Erfi[((-I)*e + 2*c*x*Log[f])/
(2*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - I*E^((2*e^2)/(c*Log[f]))*(E
rfi[((-3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3
*d]) - Erfi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] +
I*Sin[3*d]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.335, size = 246, normalized size = 0.8

$$-\frac{i}{16}f^a\sqrt{\pi}\mathrm{e}^{\frac{12\,id\ln(f)c+9\,e^2}{4\,c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln\left(f\right)}x+\frac{3\,i}{2}e\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}-\frac{i}{16}\sqrt{\pi}f^a\mathrm{e}^{-\frac{12\,id\ln(f)c-9\,e^2}{4\,c\ln(f)}}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)}x+\frac{3}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(e*x+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2
)*erf(-(-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))-1/16*I*Pi^(1/2)*f^a*exp
(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*
x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2)
)+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)
*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.516084, size = 826, normalized size = 2.74

$$-i\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+3i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+12i\,cd\log\left(f\right)+9\,e^{2}}{4\,c\log\left(f\right)}\right)}+3i\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(2\,cx\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(
f))) + 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*lo
g(f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))
) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f
))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) +
 I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/
(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/
(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(ex+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(e*x + d)^3, x)
```

3.88 $\int f^{a+cx^2}\sin\left(d+fx^2\right)dx$

Optimal. Leaf size=107

$$\frac{i\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] ((I/4)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c*Log[
f]]) - ((I/4)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/Sqrt[I*f +
 c*Log[f]]
```

Rubi [A] time = 0.198138, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4472, 2287, 2205, 2204}

$$\frac{i\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((I/4)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c*Log[
f]]) - ((I/4)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/Sqrt[I*f +
 c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sin\left(d+fx^2\right)\,dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+cx^2}\,dx\\
&= \frac{1}{2}i\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx-\frac{1}{2}i\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\
&= \frac{ie^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}
\end{aligned}
$$

Mathematica [A] time = 0.462602, size = 170, normalized size = 1.59

$$
-\frac{\sqrt[4]{-1}\sqrt{\pi}f^a\left(\sqrt{f+ic\log(f)}\left(c\sin(d)\log(f)\mathrm{Erf}\left(\frac{(1+i)x\sqrt{f+ic\log(f)}}{\sqrt{2}}\right)+\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{f+ic\log(f)}\right)(f\sin(d)+\cos(d)(c\,\mathrm{l}\ldots}{4\left(c^2\log^2(f)+f^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f -
 I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]]*(c
*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Log[f]*Sin[d] + Erfi[(-1)^(3
/4)*x*Sqrt[f + I*c*Log[f]]]*(Cos[d]*(I*f + c*Log[f]) + f*Sin[d]))))/(4*(f^2
 + c^2*Log[f]^2))
```

Maple [A] time = 0.129, size = 84, normalized size = 0.8

$$-\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{id}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-if}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-if}}+\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{-id}\mathrm{Erf}\left(x\sqrt{if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{if-c\ln\left(f\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d),x)
```

```
[Out] -1/4*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*
x)+1/4*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.499155, size = 301, normalized size = 2.81

$$\frac{\sqrt{\pi}\left(i\,c\log\left(f\right)+f\right)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}x\right)e^{\left(a\log\left(f\right)+i\,d\right)}+\sqrt{\pi}\left(-i\,c\log\left(f\right)+f\right)\sqrt{-c\log\left(f\right)+i\,f}\,\mathrm{erf}\left(\sqrt{\right.}\right.}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I
*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(fx^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d), x)
```

3.89 $\int f^{a+cx^2} \sin^2\left(d+fx^2\right) dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{\pi}e^{-2id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}e^{2id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) - (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.227606, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4472, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}e^{-2id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}e^{2id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) - (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx=\int\left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}\right)dx$$
$$=-\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$
$$=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\int\exp\left(-2id+a\log(f)-x^2(2if-c\log(f))\right)dx-\frac{1}{4}\int\exp$$
$$=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{2if+c\right.}{8\sqrt{2if+c\log(f)}}$$

Mathematica [A] time = 0.7892, size = 188, normalized size = 1.34

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2\operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\sqrt{2f+ic\log(f)}(c\log(f)+2if)(\cos(2d)-i\sin(2d))\operatorname{Erf}\left(\sqrt[4]{-1}x\sqrt{2f+ic\log(f)}\right)\right.}{c^2\log^2(f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(Erf[(-1)^(1/4)*x*Sqrt[2*f + I*c*Log[f]]]*Sqrt[2*f + I*c*Log[f]]*
((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]) + Erf[(-1)^(3/4)*x*Sqrt[2*f -
I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*
d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] time = 0.15, size = 107, normalized size = 0.8

$$-\frac{f^{a}\sqrt{\pi}\,\mathrm{e}^{-2\,id}}{8}\mathrm{Erf}\left(x\sqrt{2\,if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{2\,if-c\ln\left(f\right)}}-\frac{f^{a}\sqrt{\pi}\,\mathrm{e}^{2\,id}}{8}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-2\,if}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-2\,if}}+\frac{f^{a}\sqrt{\pi}}{4}\mathrm{Er}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(
1/2))-1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*
f)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.505152, size = 479, normalized size = 3.42

$$-\frac{2\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+4\,f^{2}\right)\sqrt{-c\log\left(f\right)}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}-2ic\,f\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\mathrm{erf}\left(\sqrt{-\right.}{8\left(c^{3}\log\left(f\right)^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) - sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
```

```
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + f*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(fx^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^2, x)
```

3.90 $\int f^{a+cx^2} \sin^3\left(d+fx^2\right) dx$

Optimal. Leaf size=213

$$\frac{3i\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{-3id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}e^{3id}}{}$$

```
[Out] (((3*I)/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c
*Log[f]]) - ((I/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(E^((3*I)
*d)*Sqrt[(3*I)*f - c*Log[f]]) - (((3*I)/16)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqr
t[I*f + c*Log[f]]])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d)*f^a*Sqrt[Pi]
*Erfi[x*Sqrt[(3*I)*f + c*Log[f]]])/Sqrt[(3*I)*f + c*Log[f]]
```

Rubi [A] time = 0.332318, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4472, 2287, 2205, 2204}

$$\frac{3i\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}e^{-3id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}-\frac{3i\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{i\sqrt{\pi}e^{3id}}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(E^(I*d)*Sqrt[I*f - c
*Log[f]]) - ((I/16)*f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(E^((3*I)
*d)*Sqrt[(3*I)*f - c*Log[f]]) - (((3*I)/16)*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqr
t[I*f + c*Log[f]]])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d)*f^a*Sqrt[Pi]
*Erfi[x*Sqrt[(3*I)*f + c*Log[f]]])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sin^3\left(d+fx^2\right)dx=\int\left(\frac{3}{8}ie^{-id-ifx^2}f^{a+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+cx^2}\right)dx$$

$$=-\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+cx^2}\,dx-\frac{3}{8}i\int$$

$$=-\left(\frac{1}{8}i\int\exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\int\exp\left(3id+a\log(f)+x^2(\right.$$

$$=\frac{3ie^{-id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}-\frac{3ie^{id}f^a\sqrt{\pi}\operatorname{erfi}\left(\right.}{16\sqrt{if+}}$$

Mathematica [A] time = 2.28787, size = 386, normalized size = 1.81

$$\frac{\sqrt[4]{-1}\sqrt{\pi}f^a\left((f-ic\log(f))\left(\sqrt{3f-ic\log(f)}\left(-c^2\log^2(f)+4icf\log(f)+3f^2\right)(\cos(3d)+i\sin(3d))\operatorname{Erfi}\left(\sqrt[4]{-1}x\sqrt{3f-ic}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
 - I*c*Log[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(Erfi[(-1)^(1/4)*x*Sqrt[3*f - I*c
*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*
(Cos[3*d] + I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*Erfi[(-1)^(3/4)*x*Sqrt[f +
```

```
I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(c*Cos[d]*Log[f] - 3*f*Sin[d]) + 3*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*(3*f*Cos[d] +
c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f + I*c*Log[f
])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f
^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] time = 0.271, size = 166, normalized size = 0.8

$$\frac{i}{16}f^{a}\sqrt{\pi}e^{3\,id}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-3\,if}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-3\,if}}-\frac{i}{16}f^{a}\sqrt{\pi}e^{-3\,id}\mathrm{Erf}\left(x\sqrt{3\,if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}+\frac{3\,i}{16}f^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+d)^3,x)
```

```
[Out] 1/16*I*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*I*f)^
(1/2)*x)-1/16*I*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I
*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-
c*ln(f)-I*f)^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.549795, size = 871, normalized size = 4.09

$$\sqrt{\pi}\left(-i\,c^{3}\log\left(f\right)^{3}-3\,c^{2}f\log\left(f\right)^{2}-i\,cf^{2}\log\left(f\right)-3\,f^{3}\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}x\right)e^{\left(a\log\left(f\right)+3i\,d\right)}+\sqrt{\pi}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) + sqrt(pi)*(3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 + 27*I*c*f^2*log(f) + 27
*f^3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d)
 + sqrt(pi)*(-3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 - 27*I*c*f^2*log(f) + 27*
f^3)*sqrt(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d)
+ sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqr
t(-c*log(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(
c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a} \sin\left(fx^2+d\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + d)^3, x)
```

3.91 $\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=187

$$\frac{i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] ((I/4)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*
(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((I/4)*
E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*
Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rubi [A] time = 0.366748, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4472, 2287, 2234, 2205, 2204}

$$\frac{i\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```


```
[Out] ((I/4)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*
(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((I/4)*
E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*
Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+cx^2}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}i\int\exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+iex+a\log(f\right.\\ &= \frac{1}{2}\left(ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{2}\left(ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\\ &= \frac{ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.967429, size = 216, normalized size = 1.16

$$-\frac{(-1)^{3/4}\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)+4if}}\left((f-ic\log(f))\sqrt{f+ic\log(f)}(\cos(d)-i\sin(d))e^{\frac{ie^2f}{2\left(c^2\log^2(f)+f^2\right)}}\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2icx\log(f)+e+2fx)}{2\sqrt{f+ic\log(f)}}\right)+\sqrt{f}\right.}{4\left(c^2\log^2(f)+f^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -((-1)^(3/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(E^(((I/2)*e^2*f)/
(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*S
qrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Sin
[d]) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log
[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(4*(f
^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.379, size = 169, normalized size = 0.9

$$\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{\frac{4id\ln(f)c-4df+e^{2}}{4if+4c\ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-if}\,x+\frac{i}{2}e\frac{1}{\sqrt{-c\ln(f)-if}}\right)\frac{1}{\sqrt{-c\ln(f)-if}}+\frac{i}{4}f^{a}\sqrt{\pi}\mathrm{e}^{-\frac{4id\ln(f)c+4df-e^{2}}{4c\ln(f)-4if}}\operatorname{Erf}\left(x\sqrt{if}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(
f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))+1/4
*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*l
n(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.548508, size = 765, normalized size = 4.09

$$\frac{\sqrt{\pi}\left(i\,c\log\left(f\right)+f\right)\sqrt{-c\log\left(f\right)-i\,f}\operatorname{erf}\left(\frac{\left(2\,c^2x\log\left(f\right)^2+2\,f^2x+i\,ce\log\left(f\right)+ef\right)\sqrt{-c\log\left(f\right)-i\,f}}{2\left(c^2\log\left(f\right)^2+f^2\right)}\right)e^{\left(\frac{4\,ac^2\log\left(f\right)^3+4i\,c^2d\log\left(f\right)^2-i\,e^2f+4i\,df^2+\left(ce^2+4\,a\right.}{4\left(c^2\log\left(f\right)^2+f^2\right)}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f
)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f
^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 +
(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(fx^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d), x)
```

3.92 $\int f^{a+cx^2} \sin^2\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=211

$$-\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id} \text{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)+2if}+2id} \text{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi} f^a \text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.418573, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4472, 2204, 2287, 2234, 2205}

$$-\frac{\sqrt{\pi} f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id} \text{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi} f^a e^{\frac{e^2}{c\log(f)+2if}+2id} \text{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi} f^a \text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+cx^2}-\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2}-\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\int\exp\left(-2id-2iex+a\log(f)-x^2(2if-c\log(f))\right)dx\\
&= \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2ie+2x(-2if+c\log(f)))}{4(-2if+c\log(f))}\right.\\
&= \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id+\frac{e^2}{2if+c\log(f)}}f}{8\sqrt{}}
\end{aligned}
$$

Mathematica [A] time = 2.25287, size = 251, normalized size = 1.19

$$
\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2\operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\cos(2d)+i\sin(2d))e^{\frac{e^2}{c\log(f)+2if}}\operatorname{Erf}\left(\frac{(-1)^{3/4}(-icx\log(f)+}{\sqrt{2f-ic\log(f}}\right.\right.}{c^2\,l}\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(E^(e^2/((2*I)*f + c*Log[f]))*Erf[((-1)^(3/4)*(e + 2*f*x - I*c*x*
Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*(2*f + I*c*Log[f])*
(Cos[2*d] + I*Sin[2*d]) + E^(e^2/((-2*I)*f + c*Log[f]))*Erf[((-1)^(1/4)*(e
+ 2*f*x + I*c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*
f + I*c*Log[f]]*(I*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] time = 0.39, size = 191, normalized size = 0.9

$$-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2 i d \ln\left(f\right) c+4 d f-e^{2}}{-2 i f+c \ln\left(f\right)}}\mathrm{Erf}\left(x\sqrt{2 i f-c\ln\left(f\right)}+i e\frac{1}{\sqrt{2 i f-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{2 i f-c\ln\left(f\right)}}+\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{\frac{2 i d \ln\left(f\right) c-4 d f+e^{2}}{2 i f+c \ln\left(f\right)}}\mathrm{Erf}\left(-\sqrt{-c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c
*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))+1/8*Pi
^(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^
(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^
(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.556525, size = 927, normalized size = 4.39

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)-\sqrt{\pi}\left(c^2\log\left(f\right)^2-2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2if}\operatorname{erf}\left(\frac{\left(c^2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) - sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2*I*
f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*I*e^
2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - sqrt(
pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8*I*d*f^2
 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\sin^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sin(d + e*x + f*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(fx^2+ex+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^2, x)
```

3.93 $\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=377

$$-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3i\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{3i\sqrt{\pi}f^ae^{\cdots}}{\cdots}$$

```
[Out] (((3*I)/16)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e +
 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((
I/16)*E^((-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*
I)*e + 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*
f - c*Log[f]] - (((3*I)/16)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
c*Log[f]] + ((I/16)*E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])
])/Sqrt[(3*I)*f + c*Log[f]]
```

Rubi [A] time = 0.655414, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4472, 2287, 2234, 2205, 2204}

$$-\frac{i\sqrt{\pi}f^a\exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3i\sqrt{\pi}f^ae^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{3i\sqrt{\pi}f^ae^{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (((3*I)/16)*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e +
 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*Log[f]] - ((
I/16)*E^((-3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*
I)*e + 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*
f - c*Log[f]] - (((3*I)/16)*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
c*Log[f]] + ((I/16)*E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])
])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\sin^3\left(d+ex+fx^2\right)dx = \int \Big(-\frac{1}{8}ie^{-3i\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+cx^2}$$

$$= -\left(\frac{1}{8}i\int e^{-3i\left(d+ex+fx^2\right)}f^{a+cx^2}\,dx\right)+\frac{1}{8}i\int\exp\Big(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)$$

$$= -\left(\frac{1}{8}i\int\exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\int\exp\Big(3id+3i$$

$$= \frac{1}{8}\left(3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{8}\Big(ie^{-3id-\frac{9e^2}{4(3if-c\log(f}}$$

$$= \frac{3ie^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{ie^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\Big(\frac{3ie+2x(3if-c\log}{2\sqrt{3if-c\log}}}{16\sqrt{3if-c\log(f)}}$$

Mathematica [A] time = 6.6193, size = 490, normalized size = 1.3

$$\frac{\sqrt[4]{-1}\sqrt{\pi}f^a\left((f-ic\log(f))\left(\sqrt{3f-ic\log(f)}\left(-c^2\log^2(f)+4icf\log(f)+3f^2\right)(\cos(3d)+i\sin(3d))e^{\frac{9e^2}{4(c\log(f)+3if)}}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(\cdots}{\cdots}\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(-3*E^(e^2/((4*I)*f + 4*c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Lo
g[f]]*(9*f^3 + (9*I)*c*f^2*Log[f] + c^2*f*Log[f]^2 + I*c^3*Log[f]^3)*(Cos[d
] + I*Sin[d]) + (f - I*c*Log[f])*(E^((9*e^2)/(4*((3*I)*f + c*Log[f])))*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*
Sqrt[3*f - I*c*Log[f]]*(3*f^2 + (4*I)*c*f*Log[f] - c^2*Log[f]^2)*(Cos[3*d]
+ I*Sin[3*d]) + (3*f - I*c*Log[f])*(3*E^(e^2/((-4*I)*f + 4*c*Log[f]))*Erfi[
((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[
f + I*c*Log[f]]*((-3*I)*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^((9*e^2)/(4*(
(-3*I)*f + c*Log[f])))*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[3*f + I*c*Log[f]])]*(f + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(I*Cos[3
*d] + Sin[3*d])))))/(16*(9*f^4 + 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] time = 0.613, size = 338, normalized size = 0.9

$$-\frac{i}{16}f^a\sqrt{\pi}\mathrm{e}^{\frac{12id\ln(f)c-36df+9e^2}{4c\ln(f)+12if}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)-3if}\,x+\frac{3i}{2}e\frac{1}{\sqrt{-c\ln(f)-3if}}\right)\frac{1}{\sqrt{-c\ln(f)-3if}}-\frac{i}{16}\sqrt{\pi}f^a\mathrm{e}^{-\frac{12id\ln(f)c+36\cdots}{4c\ln(f)-12\cdots}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/
(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*
f)^(1/2))-1/16*I*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(-3*I*f
+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f
-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I
*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*l
n(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*l
n(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I
```

```
*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.668306, size = 1840, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
g(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2 + 4
*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 + 27*I
*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
+ sqrt(pi)*(3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 + 27*I*c*f^2*log(f) + 27*f^
3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f)
 + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^
3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c
^2*log(f)^2 + f^2)) + sqrt(pi)*(-3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 - 27*I
*c*f^2*log(f) + 27*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2
*f^2*x - I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^
(1/4*(4*a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2
+ 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)
^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\sin\left(fx^2+ex+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*sin(f*x^2 + e*x + d)^3, x)
```

3.94 $\int f^{a+bx+cx^2}\sin(d+ex)\,dx$

Optimal. Leaf size=176

$$-\frac{i\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] ((-I/4)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e
 - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]
]) - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*
e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f
]])
```

Rubi [A] time = 0.334657, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4472, 2287, 2234, 2204}

$$-\frac{i\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{i\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] ((-I/4)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e
 - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]
]) - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*
e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f
]])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sin(d+ex)\,dx &= \int\left(\frac{1}{2}ie^{-id-iex}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+iex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}i\int \exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx-\frac{1}{2}i\int \exp\left(id+a\log(f)+cx\right.\\ &= -\left(\frac{1}{2}\left(ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right)+\frac{1}{2}\left(ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right.\\ &= -\frac{ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{ie+b\log(f)+2cx\log(f}{2\sqrt{c}\sqrt{\log(f)}}\right.}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.332312, size = 155, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}\left(i(\cos(d)+i\sin(d))\text{Erfi}\left(\frac{-\log(f)(b+2cx)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+e^{\frac{ibe}{c}}(\sin(d)+i\cos(d))\text{Erfi}\left(\frac{\log(f)(b+2cx)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(I*Er
fi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[
d]) + E^((I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[
```

```
f]])]*(I*Cos[d] + Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.234, size = 170, normalized size = 1.

$$\frac{i}{4} f^{a} \sqrt{\pi} \mathrm{e}^{-\frac{(\ln(f))^{2} b^{2}+2 i \ln(f) b e-4 i d \ln(f) c-e^{2}}{4 c \ln(f)}} \mathrm{Erf}\left(-\sqrt{-c \ln(f)} x+\frac{i e+b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)}}\right) \frac{1}{\sqrt{-c \ln(f)}}-\frac{i}{4} f^{a} \sqrt{\pi} \mathrm{e}^{-\frac{(\ln(f))^{2} b^{2}-2 i \ln(f}{4 c \ln}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/l
n(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f
))^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f
)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e
)/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.497979, size = 487, normalized size = 2.77

$$\frac{i \sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{((2 c x+b) \log(f)+i e) \sqrt{-c \log(f)}}{2 c \log(f)}\right) e^{\left(-\frac{(b^{2}-4 a c) \log(f)^{2}-e^{2}-(4 i c d-2 i b e) \log(f)}{4 c \log(f)}\right)}-i \sqrt{\pi} \sqrt{-c \log(f)} \operatorname{erf}\left(\frac{((2 c x+b) \log(f)-i e)}{2 c \log(f)}\right.}{4 c \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 - (4*I*c*d - 2*I*
b*e)*log(f))/(c*log(f))) - I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*
log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 -
 e^2 - (-4*I*c*d + 2*I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sin(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d), x)
```

3.95 $\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\,\mathrm{l}}{2\sqrt{c}\sqrt{\mathrm{lo}}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.382583, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4472, 2234, 2204, 2287}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\operatorname{Erfi}\left(\frac{b\log(f)+2cx\,\mathrm{l}}{2\sqrt{c}\sqrt{\mathrm{lo}}}\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```


```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) - (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx = \int\left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)dx$$

$$= -\left(\frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)dx\right)-\frac{1}{4}\int \exp\left(2id+a\log\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\left(\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-2ie+b\log}{4}\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}-$$

Mathematica [A] time = 0.716334, size = 204, normalized size = 0.88

$$-\frac{\sqrt{\pi}e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\left((\cos(2d)+i\sin(2d))e^{\frac{e^2}{c\log(f)}}\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(2d)-i\sin(2d))e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)-2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^2,x]
```

```
[Out] -(f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[Log[f
]])/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)*e +
 (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + E^
(e^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]
])]*(Cos[2*d] + I*Sin[2*d])))/(8*Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

Maple [A] time = 0.414, size = 217, normalized size = 0.9

$$\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4i\ln(f)be+8id\ln(f)c-4e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{b\ln(f)-2ie}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}+\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2+4i\ln(f}{4c\,l}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c-4*e^2)/l
n(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*ln
(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f
)*c-4*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*l
n(f))/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.505868, size = 617, normalized size = 2.67

$$\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{\left((2cx+b)\log(f)+2ie\right)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2-4e^2-(8icd-4ibe)\log(f)}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{\left((2cx+b)\log(f)-2ie}{2c\log(f}\right.}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 2*I*e)*sqrt(-c*
log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 - (8*I*c*d - 4*
I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*
log(f) - 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*e^2 - (-8*I*c*d + 4*I*b*e)*log(f))/(c*log(f))) - 2*sqrt(pi)*sqrt(-c*lo
g(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*lo
g(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sin^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^2, x)
```

3.96 $\int f^{a+bx+cx^2} \sin^3(d+ex)\,dx$

Optimal. Leaf size=354

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))}{4c\log(f)}}}{1}$$

```
[Out] (((-3*I)/16)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[L
og[f]]) + ((I/16)*E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(S
qrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f])
)*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.489745, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4472, 2287, 2234, 2204}

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{3i\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))}{4c\log(f)}}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```


```
[Out] (((-3*I)/16)*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[L
og[f]]) + ((I/16)*E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt
[Pi]*Erfi[((3*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(S
qrt[c]*Sqrt[Log[f]]) - (((3*I)/16)*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f])
)*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]]
)])/(Sqrt[c]*Sqrt[Log[f]]) + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4
*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c
]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]])
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^3(d+ex)\,dx = \int\left(\frac{3}{8}ie^{-id-iex}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+iex}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}ie^{3id+3iex}f^{a+bx+cx^2}\right)d$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}i\int e^{-id-iex}f^{a+bx+cx^2}\,dx-\frac{3}{8}$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)+cx^2\log(f)-x(3ie-b\log(f))\right)dx\right)+\frac{1}{8}i\int \exp\left(3id+a\,\mathrm{lo}\right.$$

$$= -\left(\frac{1}{8}\left(3ie^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\right)+\frac{1}{8}\left(3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}\right.$$

$$= -\frac{3ie^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{i\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3}{\cdots}\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 1.00392, size = 391, normalized size = 1.1

$$\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}\left(-\sin(3d)e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{\log(f)(b+2cx)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+i\cos(3d)e^{\frac{2e^2}{c\log(f)}}\mathrm{Erfi}\left(\frac{\log(f)(b+2cx)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)-\sin(3d)e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\mathrm{E}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*((-I)
*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2
*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + I*E^((2*e^2)/(c*Log[f]))*Cos[3*d]
*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + (3*I)*E^((
I*b*e)/c)*Erfi[((-I)*e - (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] + I*Sin[d]) + 3*E^(((2*I)*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])]*(I*Cos[d] + Sin[d]) - E^((e*(2*e + (3*I)*b*Log[f]))
/(c*Log[f]))*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*Sin[3*d] - E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.453, size = 338, normalized size = 1.

$$-\frac{i}{16}f^{a}\sqrt{\pi}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+6\,i\ln(f)be-12\,id\ln(f)c-9\,e^{2}}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln\left(f\right)}x+\frac{3\,ie+b\ln\left(f\right)}{2}\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}+\frac{i}{16}f^{a}\sqrt{\pi}\mathrm{e}^{-\frac{(\ln(f))^{2}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c-9*e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*I*e+b*ln(f))/(
-c*ln(f))^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12
*I*d*ln(f)*c-9*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(
b*ln(f)-3*I*e)/(-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-
2*I*ln(f)*b*e+4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^
(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-1/4*(l
n(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.526348, size = 973, normalized size = 2.75

$$\frac{3i\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2-e^2-\left(4i\,cd-2i\,be\right)\log\left(f\right)}{4\,c\log\left(f\right)}\right)}-3i\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)-i\,e}{2\,c\log(f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(
-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 - (4*I*c*d - 2
*I*b*e)*log(f))/(c*log(f))) - 3*I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x
+ b)*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f
)^2 - e^2 - (-4*I*c*d + 2*I*b*e)*log(f))/(c*log(f))) - I*sqrt(pi)*sqrt(-c*l
og(f))*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(
-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 - (12*I*c*d - 6*I*b*e)*log(f))/(c*log(
f))) + I*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt
(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 - (-12*I*c*
d + 6*I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x+d)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \sin(ex+d)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x + d)^3, x)
```

3.97 $\int f^{a+bx+cx^2} \sin\left(d+fx^2\right) dx$

Optimal. Leaf size=193

$$-\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] ((-I/4)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf
[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*
Log[f]] - ((I/4)*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I
*f + c*Log[f]]
```

Rubi [A] time = 0.384992, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4472, 2287, 2234, 2205, 2204}

$$-\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] ((-I/4)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf
[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f - c*
Log[f]] - ((I/4)*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[P
i]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I
*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}i\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}i\int\exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id+a\log(f)\right.\\
&= \frac{1}{2}\left(ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{2}\left(ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}\right.\\
&= -\frac{ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{ie^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(if+}{2\sqrt{if+c\log}}\right.}{4\sqrt{if+c\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.967958, size = 230, normalized size = 1.19

$$-\frac{\sqrt[4]{-1}\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}}\left(\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))e^{\frac{ib^2f\log^2(f)}{2\left(c^2\log^2(f)+f^2\right)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-i\log(f)(b+2cx))}{2\sqrt{f-ic\log(f)}}\right)+\sqrt{f}\right.}{4\left(c^2\log^2(f)+f^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[(
(-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f
 + I*c*Log[f]]*(I*f + c*Log[f])*(Cos[d] - I*Sin[d]) + E^(((I/2)*b^2*f*Log[f
]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*Log[f]))
/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] +
I*Sin[d])))/(4*(f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.378, size = 180, normalized size = 0.9

$$\frac{i}{4} f^{a}\sqrt{\pi}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-4\,i d\ln(f)c+4\,d f}{4\,i f+4\,c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln\left(f\right)-i f}\,x+\frac{b\ln\left(f\right)}{2}\frac{1}{\sqrt{-c\ln\left(f\right)-i f}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)-i f}}-\frac{i}{4} f^{a}\sqrt{\pi}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+4\,i d\ln}{4\,c\ln(f)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f))
)/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*
f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I
*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*
f-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.544457, size = 778, normalized size = 4.03

$$\sqrt{\pi}\left(ic\log\left(f\right)+f\right)\sqrt{-c\log\left(f\right)-if}\,\operatorname{erf}\left(\frac{\left(2f^2x-ibf\log(f)+\left(2c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-if}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(\frac{4af^2\log(f)-\left(b^2c-4ac^2\right)\log(f)^3+4idf^2+\left(4ic^2d\cdots\right.}{4\left(c^2\log(f)^2+f^2\right)}\right.}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b
*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
 f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4
*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f)
 + f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sin\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin\left(fx^2+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d), x)
```

3.98 $\int f^{a+bx+cx^2} \sin^2\left(d+fx^2\right) dx$

Optimal. Leaf size=245

$$\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} - \frac{\sqrt{\pi} f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log}}{}\right.}{4\sqrt{c}\sqrt{\log(}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.462039, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4472, 2234, 2204, 2287, 2205}

$$\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}} - \frac{\sqrt{\pi} f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}} + \frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log}}{}\right.}{4\sqrt{c}\sqrt{\log(}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```


```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```


Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^2\left(d+fx^2\right)dx = \int\left(\frac{1}{2}f^{a+bx+cx^2}-\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}-\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)dx$$

$$= -\left(\frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx\right)-\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)dx\right)-\frac{1}{4}\int \exp\left(2id+a\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(b\log(f)+2x(-2if+c\log}{4(-2if+c\log(f))}\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\,\text{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}-\frac{e^{2id-\frac{b^2}{8if}}}{}$$

Mathematica [A] time = 3.08969, size = 299, normalized size = 1.22

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2f^{-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}}\left(\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\cos(2d)+i\sin(2d))e^{\frac{ib^2f\log^2(f)}{c^2\log^2(f)+4f^2}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*(Erf[((-1)^(1/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*c*L
og[f]])]*Sqrt[2*f + I*c*Log[f]]*((2*I)*f + c*Log[f])*(Cos[2*d] - I*Sin[2*d]
) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erf[((-1)^(3/4)*(4*f*x -
I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]]*(
2*f + I*c*Log[f])*(Cos[2*d] + I*Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] time = 0.424, size = 227, normalized size = 0.9

$$\frac{f^a\sqrt{\pi}}{8}e^{-\frac{(\ln(f))^2b^2+8id\ln(f)c+16df}{4c\ln(f)-8if}}\text{Erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{2if-c\ln(f)}}\right)\frac{1}{\sqrt{2if-c\ln(f)}}+\frac{f^a\sqrt{\pi}}{8}e^{-\frac{(\ln(f))^2b^2-8id}{8if+4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(-2*I*f+c*ln(f
)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-c
*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f)
/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*
ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^
(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.559602, size = 1025, normalized size = 4.18

$$\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2 - 2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right) - 2if}\,\mathrm{erf}\left(\frac{\left(8f^2x-2ibf\log(f)+\left(2c^2x+bc\right)\log(f)^2\right)\sqrt{-c\log(f)-2if}}{2\left(c^2\log(f)^2+4f^2\right)}\right)e^{\left(\frac{16af^2\log(f)-\left(b^2c-4ac^2\right)\log}{4\left(c^2\right.}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1
/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - 2
*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*l
og(f)^3 + 32*I*d*f^2 + (8*I*c^2*d + 2*I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4*
f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*er
f(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f)
+ 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2
)*log(f)^3 - 32*I*d*f^2 + (-8*I*c^2*d - 2*I*b^2*f)*log(f)^2)/(c^2*log(f)^2
+ 4*f^2)) - 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*
x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*
log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \sin\left(fx^2+d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^2, x)
```

3.99 $\int f^{a+bx+cx^2} \sin^3\left(d+fx^2\right) dx$

Optimal. Leaf size=386

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{i\sqrt{\pi}f^a\exp\left(3i\cdots\right.}{\cdots}$$

```
[Out] (((-3*I)/16)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi
]*Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f
 - c*Log[f]] + ((I/16)*E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f -
c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d - (b^2*Log[f]^2)
/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f])
)/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d - (b
^2*Log[f]^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((
3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rubi [A] time = 0.572557, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4472, 2287, 2234, 2205, 2204}

$$-\frac{3i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{i\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{i\sqrt{\pi}f^a\exp\left(3i\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (((-3*I)/16)*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi
]*Erf[(b*Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt[I*f
 - c*Log[f]] + ((I/16)*E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f])
)*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f -
c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/16)*E^(I*d - (b^2*Log[f]^2)
/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f])
)/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f + c*Log[f]] + ((I/16)*E^((3*I)*d - (b
^2*Log[f]^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((
3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^3\left(d+fx^2\right)dx = \int\left(\frac{3}{8}ie^{-id-ifx^2}f^{a+bx+cx^2}-\frac{3}{8}ie^{id+ifx^2}f^{a+bx+cx^2}-\frac{1}{8}ie^{-3id-3ifx^2}f^{a+bx+cx^2}+\frac{1}{8}ie^{3id+3ifx^2}f^{a+$$

$$= -\left(\frac{1}{8}i\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}i\int e^{-id-ifx^2}f^{a+bx+cx^2}$$

$$= -\left(\frac{1}{8}i\int \exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\int \exp\left(3id+\right.$$

$$= \frac{1}{8}\left(3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx-\frac{1}{8}\left(ie^{-3id+\frac{b^2\log}{12if-4c}}\right.$$

$$= -\frac{3ie^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{ie^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-}{2\sqrt{3}}\right.}{16\sqrt{3if-c\log(f)}}$$

Mathematica [B] time = 7.01153, size = 3291, normalized size = 8.53

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
```

```
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) - (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) - 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c^3*E^(((I/4)*b^2*Log[f
]^2)/(3*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d
] + (3*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)
*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(6*
f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sq
rt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/
E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^(3
/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f]))))/(16*(I*f - c*Log[f])*(f - I*c*Log[f])*(3*f - I*c*Log[f])*(3*f +
I*c*Log[f]))
```

Maple [A] time = 0.629, size = 358, normalized size = 0.9

$$-\frac{i}{16} f^{a} \sqrt{\pi}\, \mathrm{e}^{-\frac{(\ln(f))^{2} b^{2}-12 i d \ln(f) c+36 d f}{12 i f+4 c \ln(f)}} \mathrm{Erf}\left(-\sqrt{-c \ln(f)-3 i f}\, x+\frac{b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)-3 i f}}\right) \frac{1}{\sqrt{-c \ln(f)-3 i f}}+\frac{i}{16} f^{a} \sqrt{\pi}\, \mathrm{e}^{-\frac{(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-12*I*d*ln(f)*c+36*d*f)/(3*I*f+c*
ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-
c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f
)*c+36*d*f)/(-3*I*f+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^
(1/2)+1/2*ln(f)*b/(3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f
)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*
f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*exp(-
1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*e
rf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.672334, size = 1854, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 + (12*I*c^2*d + 3*I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(I*c^3*log(f)^3 - 3*c^2*
f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 + (-12*I*c^2*d - 3*I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2))
 + sqrt(pi)*(3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 + 27*I*c*f^2*log(f) + 27*f
^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)
*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(
f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^
2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-3*I*c^3*log(f)^3 + 3*c^2*f*log(f)^2 -
 27*I*c*f^2*log(f) + 27*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f
*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f
^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*
I*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*
f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \sin\left(fx^2+d\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + d)^3, x)
```

3.100 $\int f^{a+bx+cx^2} \sin\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=212

$$\frac{i\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}} - \frac{i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] ((I/4)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*
Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt
[I*f - c*Log[f]] - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rubi [A] time = 0.566377, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4472, 2287, 2234, 2205, 2204}

$$\frac{i\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}} - \frac{i\sqrt{\pi}f^a\exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] ((I/4)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*
Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/Sqrt
[I*f - c*Log[f]] - ((I/4)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
c*Log[f]])])/Sqrt[I*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}ie^{-id-iex-ifx^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{id+iex+ifx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}i\int e^{-id-iex-ifx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{id+iex+ifx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}i\int\exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)dx-\frac{1}{2}i\int\exp\left(id\right.\\ &= \frac{1}{2}\left(i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right.\\ &= \frac{i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}-\frac{i\exp\left(id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right.}{4}\end{aligned}$$

Mathematica [A] time = 2.18063, size = 347, normalized size = 1.64

$$-\frac{\sqrt[4]{-1}\sqrt{\pi}f^{\frac{f(af-be)+ac^2\log^2(f)}{c^2\log^2(f)+f^2}}\exp\left(-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f+ic\log(f)}+\frac{e^2}{f-ic\log(f)}\right)\right)\left(\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))e^{\frac{ib^2f\log^2(f)}{2\left(c^2\log^2(f)+f^2\right)}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2))*S
qrt[Pi]*(E^(((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f + (2
*I)*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt
[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]
) + E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Log[f]))
*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(I*Cos[d] + Sin[d])))/(4*E^((I/4
)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2*Log[
f]^2))
```

Maple [A] time = 0.289, size = 216, normalized size = 1.

$$\frac{i}{4} f^a \sqrt{\pi} \mathrm{e}^{-\frac{(\ln(f))^2 b^2 + 2i\ln(f) be - 4id\ln(f)c + 4df - e^2}{4if + 4c\ln(f)}} \mathrm{Erf}\left(-\sqrt{-c\ln(f) - if}\,x + \frac{ie + b\ln(f)}{2}\frac{1}{\sqrt{-c\ln(f) - if}}\right)\frac{1}{\sqrt{-c\ln(f) - if}} - \frac{i}{4} f^a \sqrt{\vphantom{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-
e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I
*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-
2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*
erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.565179, size = 938, normalized size = 4.42

$$\sqrt{\pi}\left(ic\log\left(f\right)+f\right)\sqrt{-c\log\left(f\right)-if}\,\operatorname{erf}\left(\frac{\left(2f^2x+\left(2c^2x+bc\right)\log(f)^2+ef+\left(ice-ibf\right)\log(f)\right)\sqrt{-c\log(f)-if}}{2\left(c^2\log(f)^2+f^2\right)}\right)e^{\left(-\frac{\left(b^2c-4ac^2\right)\log(f)^3+ie^2f-4idf^2-(4i\cdots}{4(\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2*
c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f)
/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I*
d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a
*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-I*c*log(f) + f)*sqrt(-c*lo
g(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I
*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c -
 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*
f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sin\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin\left(fx^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d), x)
```

3.101 $\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx$

Optimal. Leaf size=268

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f}{4c\log(f)+8}\right.}{}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.645554, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4472, 2234, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}-\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f}{4c\log(f)+8}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) - (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\sin^2\left(d+ex+fx^2\right)dx = \int \left(\frac{1}{2}f^{a+bx+cx^2} - \frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2} - \frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\right)dx$$

$$= -\left(\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx\right) - \frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,dx + \frac{1}{2}\int f^{a+bx}$$

$$= -\left(\frac{1}{4}\int \exp\left(-2id+a\log(f)-x(2ie-b\log(f))-x^2(2if-c\log(f))\right)dx\right) - \frac{1}{4}\int$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{1}{4}\left(\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-2}{}\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2ie-b\log(f)+2x(}{2\sqrt{2if-c\log}}\right.}{8\sqrt{2if-c\log(f)}}$$

Mathematica [B] time = 6.72516, size = 1120, normalized size = 4.18

$$f^a\sqrt{\pi}\left(8\sqrt{c}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}f^{2-\frac{b^2}{4c}}+2c^{5/2}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)f^{-\frac{b^2}{4c}}+2(-1)^{3/4}ce^{\frac{i\left(-4e^2+4ib\log(f)e+b^2\log^2(f)\right)}{4(2f-ic\log(f))}}\text{Er}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erf[((-1)^(
3/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f
]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4*
I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erf[((-1)^(3/4)
*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f - I*c*Log[f]] + (2*(-1)^(3/4)*c*f*Cos[2*d]*Erf[((-1)^(1/
4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]]
)]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Cos[2*d]*Erf[((-1)^(1/4)
*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]
*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^
2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (4*I)
*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erf[((-1)^(3/4)*(2*e + 4
*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]*S
qrt[2*f - I*c*Log[f]]*Sin[2*d] - (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4*I)*b
*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erf[((-1)^(3/4)*(2*e + 4*f*x
 - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(1/4)*c*f*Erf[((-1)^(1/4)*(2*e + 4*f
*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f])) - ((-1)^(3/4)*c^2*Erf[((-1)^(1/4)*(2*e + 4*f*x
 + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[f]^2*Sqr
t[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] + b^2*Lo
g[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f + I*c*Lo
g[f]))
```

Maple [A] time = 0.362, size = 263, normalized size = 1.

$$\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4i\ln(f)be+8id\ln(f)c+16df-4e^2}{4c\ln(f)-8if}}\mathrm{Erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{b\ln(f)-2ie}{2}\frac{1}{\sqrt{2if-c\ln(f)}}\right)\frac{1}{\sqrt{2if-c\ln(f)}}+\frac{f^a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-4
*e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+
1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))+1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)
^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)
-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-2
*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f
))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.574545, size = 1204, normalized size = 4.49

$$\frac{\sqrt{\pi}\left(c^2\log(f)^2-2icf\log(f)\right)\sqrt{-c\log(f)-2if}\,\mathrm{erf}\left(\frac{\left(8f^2x+\left(2c^2x+bc\right)\log(f)^2+4ef+(2ice-2ibf)\log(f)\right)\sqrt{-c\log(f)-2if}}{2\left(c^2\log(f)^2+4f^2\right)}\right)e^{\left(-\frac{(b^2c-4ac}{}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(1
/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f + (2*I*c*e - 2*I*b*f)*log(f)
)*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2
)*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 - (8*I*c^2*d - 4*I*b*c*e + 2*I*b^2*f)*l
og(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + s
qrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1/2*(8*
f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f + (-2*I*c*e + 2*I*b*f)*log(f))*sqr
t(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log
(f)^3 - 8*I*e^2*f + 32*I*d*f^2 - (-8*I*c^2*d + 4*I*b*c*e - 2*I*b^2*f)*log(f
)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) - 2*sqr
t(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*lo
g(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \sin\left(fx^2+ex+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^2, x)
```

3.102 $\int f^{a+bx+cx^2} \sin^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=430

$$\frac{3i\sqrt{\pi}f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a \exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\ldots}{2\sqrt{-c\log\ldots}}\right.}{16\sqrt{-c\log(f)+3if}}$$

```
[Out] (((3*I)/16)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])
/Sqrt[I*f - c*Log[f]] - ((I/16)*E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I
)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*L
og[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/1
6)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
 c*Log[f]] + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(
2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rubi [A] time = 0.910781, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4472, 2287, 2234, 2205, 2204}

$$\frac{3i\sqrt{\pi}f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{i\sqrt{\pi}f^a \exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\mathrm{Erf}\left(\frac{-b\log(f)+2x(-c\ldots}{2\sqrt{-c\log\ldots}}\right.}{16\sqrt{-c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```


```
[Out] (((3*I)/16)*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt
[Pi]*Erf[(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])
/Sqrt[I*f - c*Log[f]] - ((I/16)*E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I
)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*L
og[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/Sqrt[(3*I)*f - c*Log[f]] - (((3*I)/1
6)*E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I
*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/Sqrt[I*f +
 c*Log[f]] + ((I/16)*E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Lo
g[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(
2*Sqrt[(3*I)*f + c*Log[f]])])/Sqrt[(3*I)*f + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\sin^3\left(d+ex+fx^2\right)dx &= \int\left(-\frac{1}{8}ie^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}i\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a}\right.\\ &= -\left(\frac{1}{8}i\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}i\int\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+\right.\right.\\ &= -\left(\frac{1}{8}i\int\exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f))\right)dx\right)+\frac{1}{8}i\int\\ &= \frac{1}{8}\left(3i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f))}{4(-if+c\log(f))}\right.\\ &= \frac{3i\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{i\exp\left(-3id-\frac{(3e+ib\,}{4(3if-c}\right.}{}\end{aligned}$$

Mathematica [B] time = 7.24826, size = 3835, normalized size = 8.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] + 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] + 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] - (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) - (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) + (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 - (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
```

```
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] - (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) - (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqr
t[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*(-e^2 -
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - 3*(-1)^(1/4)*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I
*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c*E^(((I/4)*(-9*e
^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - 3*(-1)^(1/4)*c^2*E^(((I/4)*(-
9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Erfi[((-1)^
(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[
f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] - (-1)^(3/4)*c^3*E^(((I/4)*(
-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
```

```
]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (3*(-1)^(3/4)*f^3*Erfi[((-1
)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Lo
g[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log[
f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4
)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log
[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(
3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[
f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b
*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f]))))/(16*(I*f - c*Log[f])*(f -
I*c*Log[f])*(3*f - I*c*Log[f])*(3*f + I*c*Log[f]))
```

Maple [A] time = 0.611, size = 430, normalized size = 1.

$$-\frac{i}{16} f^{a} \sqrt{\pi} \mathrm{e}^{-\frac{(\ln(f))^{2} b^{2}+6 i \ln(f) b e-12 i d \ln(f) c+36 d f-9 e^{2}}{12 i f+4 c \ln(f)}} \mathrm{Erf}\left(-\sqrt{-c \ln(f)-3 i f}\, x+\frac{3 i e+b \ln(f)}{2} \frac{1}{\sqrt{-c \ln(f)-3 i f}}\right) \frac{1}{\sqrt{-c \ln(f)-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*
d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1
/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^(1/2))+1/16*I*Pi^(1/2)*f^a*exp(-
1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*f-9*e^2)/(-3*I*f+c*ln(f)
))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*I*e)/(
3*I*f-c*ln(f))^(1/2))-3/16*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b
*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f
-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2))+3/16*I*Pi^(1/2)*f^a*
exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))
/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(
f)-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.698121, size = 2196, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(pi)*(-I*c^3*log(f)^3 - 3*c^2*f*log(f)^2 - I*c*f^2*log(f) - 3*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*
e*f + (3*I*c*e - 3*I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9
*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 - (12
*I*c^2*d - 6*I*b*c*e + 3*I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*
log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(3*I*c^3*log(f)^3 + 3*c^2*f*log(
f)^2 + 27*I*c*f^2*log(f) + 27*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x +
 (2*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) -
I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f -
4*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f +
 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(-3*I*c^3*log(f)^3 + 3*c
^2*f*log(f)^2 - 27*I*c*f^2*log(f) + 27*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(
2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqrt(-c
*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 -
I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (c*e^2
- 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(I*c^3*log(f)
^3 - 3*c^2*f*log(f)^2 + I*c*f^2*log(f) - 3*f^3)*sqrt(-c*log(f) + 3*I*f)*erf
(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f + (-3*I*c*e + 3*I*b*f)*lo
g(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a
*c^2)*log(f)^3 - 27*I*e^2*f + 108*I*d*f^2 - (-12*I*c^2*d + 6*I*b*c*e - 3*I*
b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f
^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin\left(fx^2+ex+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(f*x^2 + e*x + d)^3, x)
```

3.103 $\int f^{a+bx+cx^2} \sin\left(a+bx+ex^2\right) dx$

Optimal. Leaf size=213

$$\frac{i\sqrt{\pi}\exp\left(-(-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\mathrm{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}-\frac{i\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\mathrm{Er}}{4\sqrt{c\log(f)+ie}}$$

```
[Out] ((I/4)*Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c
*Log[f]])])/(E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])
))*Sqrt[I*e - c*Log[f]]) - ((I/4)*E^((I + Log[f])*(a - (b^2*(I + Log[f]))/(
(4*I)*e + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]
))/(2*Sqrt[I*e + c*Log[f]])])/Sqrt[I*e + c*Log[f]]
```

Rubi [A] time = 0.795216, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4472, 2287, 2234, 2205, 2204}

$$\frac{i\sqrt{\pi}\exp\left(-(-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\mathrm{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}-\frac{i\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\mathrm{Er}}{4\sqrt{c\log(f)+ie}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] ((I/4)*Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c
*Log[f]])])/(E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])
))*Sqrt[I*e - c*Log[f]]) - ((I/4)*E^((I + Log[f])*(a - (b^2*(I + Log[f]))/(
(4*I)*e + 4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]
))/(2*Sqrt[I*e + c*Log[f]])])/Sqrt[I*e + c*Log[f]]
```

Rule 4472

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)dx &= \int\left(\frac{1}{2}ie^{-ia-ibx-iex^2}f^{a+bx+cx^2}-\frac{1}{2}ie^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}i\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}i\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}i\int\exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)dx-\frac{1}{2}i\int\exp\left(a(i-\right.\\
&= \frac{1}{2}\left(i\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\right)\int\exp\left(\frac{(-b(i-\log(f))+2x(-ie+}{4(-ie+c\log(f))}\right.\\
&= \frac{i\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}-\frac{i\exp\left((i+\right.}{}
\end{aligned}$$

Mathematica [A] time = 1.8534, size = 324, normalized size = 1.52

$$\frac{\sqrt{\pi}e^{-\frac{b^2c\log^3(f)}{2\left(c^2\log^2(f)+e^2\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\left((\cos(a)+i\sin(a))(e+ic\log(f))\sqrt{c\log(f)+ie}\exp\left(\frac{1}{4}b^2\left(\frac{\log^2(f)}{c\log(f)-ie}+\frac{1}{c\log(f)+ie}\right)\right)\mathrm{Erfi}\left(\frac{-}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Sin[a + b*x + e*x^2],x]
```

```
[Out] (f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c*Log[f])^
(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c^2*Log[f]
^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e + c*Log[f]
])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a])) + E^((b^2
*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[((-I)*(b +
 2*e*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[f])*Sq
rt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(4*E^((b^2*c*Log[f]^3)/(2*(e^2 + c
^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.454, size = 218, normalized size = 1.

$$\frac{i}{4} f^{a} \sqrt{\pi} \mathrm{e}^{-\frac{(\ln(f))^{2} b^{2}-4 i \ln(f) a c+2 i \ln(f) b^{2}+4 a e-b^{2}}{4 i e+4 c \ln(f)}} \operatorname{Erf}\left(-\sqrt{-c \ln\left(f\right)-i e}x+\frac{b \ln\left(f\right)+i b}{2} \frac{1}{\sqrt{-c \ln\left(f\right)-i e}}\right) \frac{1}{\sqrt{-c \ln\left(f\right)-i e}}-\frac{i}{4} f^{a} \sqrt{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x)
```

```
[Out] 1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*a*c+2*I*ln(f)*b^2+4*a*e-
b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-(-c*ln(f)-I*e)^(1/2)*x+1/2*(b
*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))-1/4*I*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+
4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b^2)/(-I*e+c*ln(f)))/(I*e-c*ln(f))^(1/2)*
erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*b)/(I*e-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.558113, size = 938, normalized size = 4.4

$$\sqrt{\pi}\left(i\,c\log\left(f\right)+e\right)\sqrt{-c\log\left(f\right)-i\,e}\,\operatorname{erf}\left(\frac{\left(2\,e^2x+\left(2\,c^2x+bc\right)\log\left(f\right)^2+be+\left(i\,bc-i\,be\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-i\,e}}{2\left(c^2\log\left(f\right)^2+e^2\right)}\right)e^{\left(-\frac{\left(b^2c-4\,ac^2\right)\log\left(f\right)^3+i\,b^2e-4i\,ae^2-\left(-2i\right.}{4\left(\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c*log(f) + e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (2*
c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e)
/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - 4*I*
a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*
a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(-I*c*log(f) + e)*sqrt(-c*l
og(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*b*c +
I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*
e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c
^2*log(f)^2 + e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\sin\left(a+bx+ex^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*sin(e*x**2+b*x+a),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sin(a + b*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\sin\left(ex^2+bx+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*sin(e*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*sin(e*x^2 + b*x + a), x)
```

3.104 $\int e^x \cos(a+bx)\,dx$

Optimal. Leaf size=36

$$\frac{be^x\sin(a+bx)}{b^2+1}+\frac{e^x\cos(a+bx)}{b^2+1}$$

```
[Out] (E^x*Cos[a + b*x])/(1 + b^2) + (b*E^x*Sin[a + b*x])/(1 + b^2)
```

Rubi [A] time = 0.0123241, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{be^x\sin(a+bx)}{b^2+1}+\frac{e^x\cos(a+bx)}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + b*x],x]
```

```
[Out] (E^x*Cos[a + b*x])/(1 + b^2) + (b*E^x*Sin[a + b*x])/(1 + b^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(a+bx)\,dx=\frac{e^x\cos(a+bx)}{1+b^2}+\frac{be^x\sin(a+bx)}{1+b^2}$$

Mathematica [A] time = 0.0504032, size = 26, normalized size = 0.72

$$\frac{e^x(b\sin(a+bx)+\cos(a+bx))}{b^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + b*x],x]
```

```
[Out] (E^x*(Cos[a + b*x] + b*Sin[a + b*x]))/(1 + b^2)
```

Maple [A] time = 0.008, size = 35, normalized size = 1.

$$\frac{e^{x} \cos \left(b x+a\right)}{b^{2}+1}+\frac{e^{x} b \sin \left(b x+a\right)}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(b*x+a),x)
```

```
[Out] exp(x)*cos(b*x+a)/(b^2+1)+b*exp(x)*sin(b*x+a)/(b^2+1)
```

Maxima [A] time = 1.09355, size = 34, normalized size = 0.94

$$\frac{\left(b \sin \left(b x+a\right)+\cos \left(b x+a\right)\right) e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] (b*sin(b*x + a) + cos(b*x + a))*e^x/(b^2 + 1)
```

Fricas [A] time = 0.461251, size = 69, normalized size = 1.92

$$\frac{b e^{x} \sin \left(b x+a\right)+\cos \left(b x+a\right) e^{x}}{b^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*e^x*sin(b*x + a) + cos(b*x + a)*e^x)/(b^2 + 1)
```

Sympy [A] time = 2.47465, size = 114, normalized size = 3.17

$$\begin{cases} -\frac{ixe^x\sin(a-ix)}{2} + \frac{xe^x\cos(a-ix)}{2} + \frac{e^x\cos(a-ix)}{2} & \text{for } b=-i \\ \frac{ixe^x\sin(a+ix)}{2} + \frac{xe^x\cos(a+ix)}{2} - \frac{ie^x\sin(a+ix)}{2} & \text{for } b=i \\ \frac{be^x\sin(a+bx)}{b^2+1} + \frac{e^x\cos(a+bx)}{b^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x)
```

```
[Out] Piecewise((-I*x*exp(x)*sin(a - I*x)/2 + x*exp(x)*cos(a - I*x)/2 + exp(x)*co
s(a - I*x)/2, Eq(b, -I)), (I*x*exp(x)*sin(a + I*x)/2 + x*exp(x)*cos(a + I*x
)/2 - I*exp(x)*sin(a + I*x)/2, Eq(b, I)), (b*exp(x)*sin(a + b*x)/(b**2 + 1)
 + exp(x)*cos(a + b*x)/(b**2 + 1), True))
```

Giac [A] time = 1.13738, size = 45, normalized size = 1.25

$$\left(\frac{b\sin(bx+a)}{b^2+1} + \frac{\cos(bx+a)}{b^2+1}\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] (b*sin(b*x + a)/(b^2 + 1) + cos(b*x + a)/(b^2 + 1))*e^x
```

3.105 $\int e^x \cos\left(a + cx^2\right) dx$

Optimal. Leaf size=115

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c
*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(1/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
 (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rubi [A] time = 0.09922, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4473, 2234, 2204, 2205}

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + c*x^2],x]
```

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*a + c^(-1)))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 + (2*I)*c
*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(1/4)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 -
 (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]*E^((I/4)*(4*a + c^(-1))))
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \cos\left(a+cx^2\right)dx &= \int \left(\frac{1}{2}e^{-ia+x-icx^2}+\frac{1}{2}e^{ia+x+icx^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia+x-icx^2}\,dx+\frac{1}{2}\int e^{ia+x+icx^2}\,dx\\ &= \frac{1}{2}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{\frac{i(1-2icx)^2}{4c}}\,dx+\frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\int e^{-\frac{i(1+2icx)^2}{4c}}\,dx\\ &= -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(1+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4a+\frac{1}{c}\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt[4]{-1}(1-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.159783, size = 109, normalized size = 0.95

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{i}{4}/c}\left(e^{\frac{i}{2}/c}(\sin(a)-i\cos(a))\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(2cx-i)}{2\sqrt{c}}\right)-(\cos(a)-i\sin(a))\mathrm{Erfi}\left(\frac{(-1)^{3/4}(2cx+i)}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(I + 2*c*x))/(2*Sqrt[c])]*(Cos[a]
- I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + 2*c*x))/(2*Sqrt[c])]*((-I
)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^((I/4)/c))
```

Maple [A] time = 0.054, size = 86, normalized size = 0.8

$$\frac{\sqrt{\pi}}{4}e^{\frac{-\frac{i}{4}(4ac+1)}{c}}\mathrm{Erf}\left(\sqrt{ic}x-\frac{1}{2}\frac{1}{\sqrt{ic}}\right)\frac{1}{\sqrt{ic}}+\frac{\sqrt{\pi}}{4}e^{\frac{\frac{i}{4}(4ac+1)}{c}}\mathrm{Erf}\left(\sqrt{-ic}x-\frac{1}{2}\frac{1}{\sqrt{-ic}}\right)\frac{1}{\sqrt{-ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-1/4*I*(4*a*c+1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x-1/2/(I*c
)^(1/2))+1/4*Pi^(1/2)*exp(1/4*I*(4*a*c+1)/c)/(-I*c)^(1/2)*erf((-I*c)^(1/2)*
x-1/2/(-I*c)^(1/2))
```

Maxima [B] time = 2.10717, size = 378, normalized size = 3.29

$$\frac{\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(0, c\right)\right) + \cos\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\left(0, c\right)\right) - i\sin\left(\frac{1}{4}\pi + \frac{1}{2}\arctan\left(0, c\right)\right) + i\sin\left(-\frac{1}{4}\pi + \frac{1}{2}\arctan\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi + 1/2*arctan2
(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/2*arctan2(0
, c)))*cos(1/4*(4*a*c + 1)/c) + (-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos
(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*
pi + 1/2*arctan2(0, c)))*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x - 1)/sqrt
(I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi + 1/2*arctan2(0, c
)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/2*arctan2(0, c))
)*cos(1/4*(4*a*c + 1)/c) - (-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4
*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi +
1/2*arctan2(0, c)))*sin(1/4*(4*a*c + 1)/c))*erf(1/2*(2*I*c*x + 1)/sqrt(-I*c
)))/sqrt(abs(c))
```

Fricas [B] time = 0.491066, size = 549, normalized size = 4.77

$$\frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right) - \sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{4i\,ac+i}{4\,c}\right)}\,\mathrm{C}\left(-\frac{\sqrt{2}(2\,cx-i)\sqrt{\frac{c}{\pi}}}{2\,c}\right) - i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-4i\,ac-i}{4\,c}\right)}\,\mathrm{S}\left(\frac{\sqrt{2}(2\,cx+i)\sqrt{\frac{c}{\pi}}}{2\,c}\right) - i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\right.}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_cos(1/2*sqrt(2)
*(2*c*x + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*
fresnel_cos(-1/2*sqrt(2)*(2*c*x - I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi
)*e^(1/4*(-4*I*a*c - I)/c)*fresnel_sin(1/2*sqrt(2)*(2*c*x + I)*sqrt(c/pi)/c
) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(4*I*a*c + I)/c)*fresnel_sin(-1/2*sqrt(2
)*(2*c*x - I)*sqrt(c/pi)/c))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\cos\left(a+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*cos(a + c*x**2), x)
```

Giac [A] time = 1.14323, size = 171, normalized size = 1.49

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{4i\,ac+i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-4i\,ac-i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + I/c)*(I*c/abs(c) + 1)*sqrt(ab
s(c)))*e^(-1/4*(4*I*a*c + I)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) - 1/4*sqrt(
2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - I/c)*(-I*c/abs(c) + 1)*sqrt(abs(c)))*e^
(-1/4*(-4*I*a*c - I)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.106 $\int e^x \cos \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=144

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\operatorname{Erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 +
I*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(1/4)*E^((-I)*a + ((I/4
)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c]
)])/(4*Sqrt[c])
```

Rubi [A] time = 0.168718, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4473, 2234, 2204, 2205}

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{i(b+i)^2}{4c}-ia}\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(-ib-2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\operatorname{Erf}\left(\frac{\sqrt[4]{-1}(ib+2icx+1)}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[a + b*x + c*x^2],x]
```


```
[Out] -((-1)^(1/4)*E^((I/4)*(4*a + (1 + I*b)^2/c))*Sqrt[Pi]*Erf[((-1)^(1/4)*(1 +
I*b + (2*I)*c*x))/(2*Sqrt[c])])/(4*Sqrt[c]) + ((-1)^(1/4)*E^((-I)*a + ((I/4
)*(I + b)^2)/c)*Sqrt[Pi]*Erfi[((-1)^(1/4)*(1 - I*b - (2*I)*c*x))/(2*Sqrt[c]
)])/(4*Sqrt[c])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^x \cos\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-ia+(1-ib)x-icx^2}+\frac{1}{2}e^{ia+(1+ib)x+icx^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia+(1-ib)x-icx^2}\,dx+\frac{1}{2}\int e^{ia+(1+ib)x+icx^2}\,dx\\ &= \frac{1}{2}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\int e^{-\frac{i(1+ib+2icx)^2}{4c}}\,dx+\frac{1}{2}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\int e^{\frac{i(1-ib-2icx)^2}{4c}}\,dx\\ &= -\frac{\sqrt[4]{-1}e^{\frac{1}{4}i\left(4a+\frac{(1+ib)^2}{c}\right)}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(1+ib+2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt[4]{-1}e^{-\frac{i\left(1-2ib-b^2+4ac\right)}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}(1-ib-2icx)}{2\sqrt{c}}\right)}{4\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.254125, size = 135, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\sqrt{\pi}e^{-\frac{i\left(b^2-2ib+1\right)}{4c}}\left(e^{\frac{i}{2}/c}(\sin(a)-i\cos(a))\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(b+2cx-i)}{2\sqrt{c}}\right)-e^{\frac{ib^2}{2c}}(\cos(a)-i\sin(a))\operatorname{Erfi}\left(\frac{(-1)^{3/4}(b+2cx+i)}{2\sqrt{c}}\right)\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-(E^(((I/2)*b^2)/c)*Erfi[((-1)^(3/4)*(I + b + 2*c*x))
/(2*Sqrt[c])]*(Cos[a] - I*Sin[a])) + E^((I/2)/c)*Erfi[((-1)^(1/4)*(-I + b +
 2*c*x))/(2*Sqrt[c])]*((-I)*Cos[a] + Sin[a])))/(4*Sqrt[c]*E^(((I/4)*(1 - (2
*I)*b + b^2))/c))
```

Maple [A] time = 0.05, size = 115, normalized size = 0.8

$$\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{\frac{i}{4}(2ib-4ac+b^2-1)}{c}}\mathrm{Erf}\left(\sqrt{ic}\,x-\frac{-ib+1}{2}\frac{1}{\sqrt{ic}}\right)\frac{1}{\sqrt{ic}}-\frac{\sqrt{\pi}}{4}\mathrm{e}^{\frac{\frac{i}{4}(-b^2+2ib+4ac+1)}{c}}\mathrm{Erf}\left(-\sqrt{-ic}\,x+\frac{1+ib}{2}\frac{1}{\sqrt{-ic}}\right)\frac{1}{\sqrt{-ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*I*(2*I*b-4*a*c+b^2-1)/c)/(I*c)^(1/2)*erf((I*c)^(1/2)*x
-1/2*(-I*b+1)/(I*c)^(1/2))-1/4*Pi^(1/2)*exp(1/4*I*(-b^2+2*I*b+4*a*c+1)/c)/(
-I*c)^(1/2)*erf(-(-I*c)^(1/2)*x+1/2*(1+I*b)/(-I*c)^(1/2))
```

Maxima [B] time = 2.36101, size = 420, normalized size = 2.92

$$-\frac{\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-i\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+i\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\mathrm{arc}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi + 1/2*arctan
2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/2*arctan2(
0, c)))*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (I*cos(1/4*pi + 1/2*arctan2(0, c))
+ I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c - 1)/c))*erf(1/2*I*(2
*I*c*x + I*b - 1)*sqrt(I*c)/c) + ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1
/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4
*pi + 1/2*arctan2(0, c)))*cos(-1/4*(b^2 - 4*a*c - 1)/c) - (-I*cos(1/4*pi +
1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*
arctan2(0, c)) - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c -
1)/c))*erf(1/2*I*(2*I*c*x + I*b + 1)*sqrt(-I*c)/c))*e^(-1/2*b/c)/sqrt(abs(c
))
```

Fricas [B] time = 0.500248, size = 647, normalized size = 4.49

$$\frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^2-4iac-2b-i}{4c}\right)}\mathrm{C}\left(\frac{\sqrt{2}(2cx+b+i)\sqrt{\frac{c}{\pi}}}{2c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{-ib^2+4iac-2b+i}{4c}\right)}\mathrm{C}\left(-\frac{\sqrt{2}(2cx+b-i)\sqrt{\frac{c}{\pi}}}{2c}\right)-i\sqrt{2}\pi\sqrt{\frac{c}{\pi}}e^{\left(\frac{ib^2-4iac-2b-i}{4c}\right)}\mathrm{S}\left(\frac{\sqrt{2}(2cx\cdots}{2\cdots}\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)*fresnel_co
s(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*e^(1/4*
(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_cos(-1/2*sqrt(2)*(2*c*x + b - I)*sq
rt(c/pi)/c) - I*sqrt(2)*pi*sqrt(c/pi)*e^(1/4*(I*b^2 - 4*I*a*c - 2*b - I)/c)
*fresnel_sin(1/2*sqrt(2)*(2*c*x + b + I)*sqrt(c/pi)/c) - I*sqrt(2)*pi*sqrt(
c/pi)*e^(1/4*(-I*b^2 + 4*I*a*c - 2*b + I)/c)*fresnel_sin(-1/2*sqrt(2)*(2*c*
x + b - I)*sqrt(c/pi)/c))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x}\cos\left(a+bx+cx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*cos(a + b*x + c*x**2), x)
```

Giac [A] time = 1.15175, size = 198, normalized size = 1.38

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b-i}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^{2}-4iac+2b-i}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b+i}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^{2}+4iac+2b+i}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b - I)/c)*(-I*c/abs(c) + 1)*
sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c + 2*b - I)/c)/((-I*c/abs(c) + 1)*sqr
t(abs(c))) - 1/4*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + (b + I)/c)*(I*c/a
bs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c + 2*b + I)/c)/((I*c/abs(
c) + 1)*sqrt(abs(c)))
```

3.107 $\int e^{x^2} \cos(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rubi [A] time = 0.0567589, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4473, 2234, 2204}

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}-ia}\text{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}+ia}\text{Erfi}\left(\frac{1}{2}(2x+ib)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^((-I)*a + b^2/4)*Sqrt[Pi]*Erfi[((-I)*b + 2*x)/2])/4 + (E^(I*a + b^2/4)*S
qrt[Pi]*Erfi[(I*b + 2*x)/2])/4
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\cos(a+bx)\,dx &= \int\left(\frac{1}{2}e^{-ia-ibx+x^2}+\frac{1}{2}e^{ia+ibx+x^2}\right)dx\\ &= \frac{1}{2}\int e^{-ia-ibx+x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+x^2}\,dx\\ &= \frac{1}{2}e^{-ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(-ib+2x)^2}\,dx+\frac{1}{2}e^{ia+\frac{b^2}{4}}\int e^{\frac{1}{4}(ib+2x)^2}\,dx\\ &= \frac{1}{4}e^{-ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(-ib+2x)\right)+\frac{1}{4}e^{ia+\frac{b^2}{4}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1}{2}(ib+2x)\right)\end{aligned}$$

Mathematica [A] time = 0.0826598, size = 82, normalized size = 1.06

$$\frac{1}{4}\sqrt{\pi}e^{\frac{b^2}{4}}\left(-\sin(a)\left(\mathrm{Erf}\left(\frac{b}{2}-ix\right)+\mathrm{Erf}\left(\frac{b}{2}+ix\right)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(2x-ib)\right)+\cos(a)\mathrm{Erfi}\left(\frac{1}{2}(2x+ib)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x],x]
```

```
[Out] (E^(b^2/4)*Sqrt[Pi]*(Cos[a]*Erfi[((-I)*b + 2*x)/2] + Cos[a]*Erfi[(I*b + 2*x
)/2] - (Erf[b/2 - I*x] + Erf[b/2 + I*x])*Sin[a]))/4
```

Maple [A] time = 0.002, size = 54, normalized size = 0.7

$$-\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{-ia}\mathrm{Erf}\left(ix+\frac{b}{2}\right)+\frac{i}{4}\sqrt{\pi}\mathrm{e}^{\frac{b^2}{4}}\mathrm{e}^{ia}\mathrm{Erf}\left(-ix+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(b*x+a),x)
```

```
[Out] -1/4*I*Pi^(1/2)*exp(1/4*b^2)*exp(-I*a)*erf(I*x+1/2*b)+1/4*I*Pi^(1/2)*exp(1/
4*b^2)*exp(I*a)*erf(-I*x+1/2*b)
```

Maxima [A] time = 1.16329, size = 70, normalized size = 0.91

$$-\frac{1}{4}\sqrt{\pi}\left((i\cos(a)+\sin(a))\,\mathrm{erf}\left(\frac{1}{2}b+i\,x\right)e^{\left(\frac{1}{4}b^2\right)}+(i\cos(a)-\sin(a))\,\mathrm{erf}\left(-\frac{1}{2}b+i\,x\right)e^{\left(\frac{1}{4}b^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*((I*cos(a) + sin(a))*erf(1/2*b + I*x)*e^(1/4*b^2) + (I*cos(a)
 - sin(a))*erf(-1/2*b + I*x)*e^(1/4*b^2))
```

Fricas [A] time = 0.475381, size = 127, normalized size = 1.65

$$\frac{1}{4}\sqrt{\pi}\left(-i\,\mathrm{erf}\left(-\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2+i\,a\right)}-i\,\mathrm{erf}\left(\frac{1}{2}\,b+i\,x\right)e^{\left(\frac{1}{4}\,b^2-i\,a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*(-I*erf(-1/2*b + I*x)*e^(1/4*b^2 + I*a) - I*erf(1/2*b + I*x)*e
^(1/4*b^2 - I*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(bx+a\right)e^{\left(x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*e^(x^2), x)
```

3.108 $\int e^{x^2}\cos\left(a+cx^2\right)\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{\pi}e^{-ia}\operatorname{Erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}}+\frac{\sqrt{\pi}e^{ia}\operatorname{Erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(4*Sqrt[1 - I*c]*E^(I*a)) + (E^(I*a)*Sqrt[
Pi]*Erfi[Sqrt[1 + I*c]*x])/(4*Sqrt[1 + I*c])
```

Rubi [A] time = 0.078642, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4473, 2204}

$$\frac{\sqrt{\pi}e^{-ia}\operatorname{Erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}}+\frac{\sqrt{\pi}e^{ia}\operatorname{Erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erfi[Sqrt[1 - I*c]*x])/(4*Sqrt[1 - I*c]*E^(I*a)) + (E^(I*a)*Sqrt[
Pi]*Erfi[Sqrt[1 + I*c]*x])/(4*Sqrt[1 + I*c])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\cos\left(a+cx^2\right)dx=\int\left(\frac{1}{2}e^{-ia+(1-ic)x^2}+\frac{1}{2}e^{ia+(1+ic)x^2}\right)dx$$
$$=\frac{1}{2}\int e^{-ia+(1-ic)x^2}dx+\frac{1}{2}\int e^{ia+(1+ic)x^2}dx$$
$$=\frac{e^{-ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1-ic}x\right)}{4\sqrt{1-ic}}+\frac{e^{ia}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1+ic}x\right)}{4\sqrt{1+ic}}$$

Mathematica [A] time = 0.193847, size = 107, normalized size = 1.29

$$\frac{\sqrt[4]{-1}\sqrt{\pi}\left((1-ic)\sqrt{c-i}(\cos(a)+i\sin(a))\operatorname{Erfi}\left(\sqrt[4]{-1}\sqrt{c-i}x\right)-(c-i)\sqrt{c+i}(\cos(a)-i\sin(a))\operatorname{Erfi}\left((-1)^{3/4}\sqrt{c+i}x\right)\right)}{4\left(c^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*Erfi[(-1)^(3/4)*Sqrt[I + c]*x]
*(Cos[a] - I*Sin[a])) + (1 - I*c)*Sqrt[-I + c]*Erfi[(-1)^(1/4)*Sqrt[-I + c]
*x]*(Cos[a] + I*Sin[a])))/(4*(1 + c^2))
```

Maple [A] time = 0.043, size = 60, normalized size = 0.7

$$\frac{\sqrt{\pi}e^{-ia}}{4}\operatorname{Erf}\left(\sqrt{-1+ic}x\right)\frac{1}{\sqrt{-1+ic}}+\frac{\sqrt{\pi}e^{ia}}{4}\operatorname{Erf}\left(\sqrt{-ic-1}x\right)\frac{1}{\sqrt{-ic-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(c*x^2+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(-I*a)/(-1+I*c)^(1/2)*erf((-1+I*c)^(1/2)*x)+1/4*Pi^(1/2)*ex
p(I*a)/(-I*c-1)^(1/2)*erf((-I*c-1)^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.479143, size = 198, normalized size = 2.39

$$\frac{\sqrt{\pi}(i\,c-1)\sqrt{-i\,c-1}\,\mathrm{erf}\left(\sqrt{-i\,c-1}x\right)e^{(i\,a)}+\sqrt{\pi}\sqrt{i\,c-1}(-i\,c-1)\,\mathrm{erf}\left(\sqrt{i\,c-1}x\right)e^{(-i\,a)}}{4\left(c^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(sqrt(-I*c - 1)*x)*e^(I*a) + sqrt
(pi)*sqrt(I*c - 1)*(-I*c - 1)*erf(sqrt(I*c - 1)*x)*e^(-I*a))/(c^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cos\left(a+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\cos\left(cx^2+a\right)e^{\left(x^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cos(c*x^2 + a)*e^(x^2), x)
```

3.109 $\int e^{x^2} \cos \left(a+bx+cx^2\right) dx$

Optimal. Leaf size=151

$$\frac{\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}-\frac{\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}$$

```
[Out] -(Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(4*Sqrt[1 - I*c]*
E^(I*(a - b^2/(4*I + 4*c)))) + (E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[Pi]*Erfi[(
I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/(4*Sqrt[1 + I*c])
```

Rubi [A] time = 0.172238, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4473, 2234, 2204}

$$\frac{\sqrt{\pi}e^{ia+\frac{b^2}{4(1+ic)}}\text{Erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}-\frac{\sqrt{\pi}e^{-i\left(a-\frac{b^2}{4c+4i}\right)}\text{Erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*Cos[a + b*x + c*x^2],x]
```


```
[Out] -(Sqrt[Pi]*Erfi[(I*b - 2*(1 - I*c)*x)/(2*Sqrt[1 - I*c])])/(4*Sqrt[1 - I*c]*
E^(I*(a - b^2/(4*I + 4*c)))) + (E^(I*a + b^2/(4*(1 + I*c)))*Sqrt[Pi]*Erfi[(
I*b + 2*(1 + I*c)*x)/(2*Sqrt[1 + I*c])])/(4*Sqrt[1 + I*c])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\cos\left(a+bx+cx^2\right)dx &= \int\left(\frac{1}{2}e^{-ia-ibx+(1-ic)x^2}+\frac{1}{2}e^{ia+ibx+(1+ic)x^2}\right)dx \\
&= \frac{1}{2}\int e^{-ia-ibx+(1-ic)x^2}\,dx+\frac{1}{2}\int e^{ia+ibx+(1+ic)x^2}\,dx \\
&= \frac{1}{2}e^{ia+\frac{b^2}{4(1+ic)}}\int\exp\left(\frac{(ib+2(1+ic)x)^2}{4(1+ic)}\right)dx+\frac{1}{2}e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\int\exp\left(\frac{(-ib+2(1-ic)x)^2}{4(1-ic)}\right)dx \\
&= -\frac{e^{-i\left(a-\frac{b^2}{4i+4c}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib-2(1-ic)x}{2\sqrt{1-ic}}\right)}{4\sqrt{1-ic}}+\frac{e^{ia+\frac{b^2}{4(1+ic)}}\sqrt{\pi}\operatorname{erfi}\left(\frac{ib+2(1+ic)x}{2\sqrt{1+ic}}\right)}{4\sqrt{1+ic}}
\end{aligned}
$$

Mathematica [A] time = 0.576526, size = 166, normalized size = 1.1

$$
\frac{\sqrt[4]{-1}\sqrt{\pi}e^{\frac{ib^2}{-4c+4i}}\left(\sqrt{c-i}(c+i)(\sin(a)-i\cos(a))\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(b+2(c-i)x)}{2\sqrt{c-i}}\right)-(c-i)\sqrt{c+i}e^{\frac{ib^2c}{2c^2+2}}(\cos(a)-i\sin(a))\operatorname{Erfi}\left(\frac{(-1)^{3/4}(b+2(\cdots}{2\sqrt{c+i}}\right.\right.}{4\left(c^2+1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*Cos[a + b*x + c*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^((I*b^2)/(4*I - 4*c))*Sqrt[Pi]*(-((-I + c)*Sqrt[I + c]*E^((I*
b^2*c)/(2 + 2*c^2))*Erfi[((-1)^(3/4)*(b + 2*(I + c)*x))/(2*Sqrt[I + c])]*(C
os[a] - I*Sin[a])) + Sqrt[-I + c]*(I + c)*Erfi[((-1)^(1/4)*(b + 2*(-I + c)*
x))/(2*Sqrt[-I + c])]*((-I)*Cos[a] + Sin[a])))/(4*(1 + c^2))
```

Maple [A] time = 0.066, size = 125, normalized size = 0.8

$$
\frac{\sqrt{\pi}}{4}e^{\frac{4\,ia+4\,ac-b^2}{4\,ic-4}}\operatorname{Erf}\left(\sqrt{-1+ic}x+\frac{i}{2}b\frac{1}{\sqrt{-1+ic}}\right)\frac{1}{\sqrt{-1+ic}}-\frac{\sqrt{\pi}}{4}e^{\frac{-4\,ac+4\,ia+b^2}{4\,ic+4}}\operatorname{Erf}\left(-\sqrt{-ic-1}x+\frac{i}{2}b\frac{1}{\sqrt{-ic-1}}\right)\frac{1}{\sqrt{-ic-1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*cos(c*x^2+b*x+a),x)
```

```
[Out] 1/4*Pi^(1/2)*exp(1/4*(4*I*a+4*a*c-b^2)/(-1+I*c))/(-1+I*c)^(1/2)*erf((-1+I*c
)^(1/2)*x+1/2*I*b/(-1+I*c)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(-4*a*c+4*I*a+b^2)/(
1+I*c))/(-I*c-1)^(1/2)*erf(-(-I*c-1)^(1/2)*x+1/2*I*b/(-I*c-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.503148, size = 431, normalized size = 2.85

$$\frac{\sqrt{\pi}(i\,c+1)\sqrt{i\,c-1}\,\mathrm{erf}\left(-\frac{\left(bc+2\left(c^2+1\right)x-i\,b\right)\sqrt{i\,c-1}}{2\left(c^2+1\right)}\right)e^{\left(\frac{i\,b^2c-4i\,ac^2+b^2-4i\,a}{4\left(c^2+1\right)}\right)}+\sqrt{\pi}(i\,c-1)\sqrt{-i\,c-1}\,\mathrm{erf}\left(\frac{\left(bc+2\left(c^2+1\right)x+i\,b\right)\sqrt{-i\,c-1}}{2\left(c^2+1\right)}\right)e^{\left(\frac{-i\,b^2c+4}{4(}\right.}}{4\left(c^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(I*c + 1)*sqrt(I*c - 1)*erf(-1/2*(b*c + 2*(c^2 + 1)*x - I*b)*
sqrt(I*c - 1)/(c^2 + 1))*e^(1/4*(I*b^2*c - 4*I*a*c^2 + b^2 - 4*I*a)/(c^2 +
1)) + sqrt(pi)*(I*c - 1)*sqrt(-I*c - 1)*erf(1/2*(b*c + 2*(c^2 + 1)*x + I*b)
*sqrt(-I*c - 1)/(c^2 + 1))*e^(1/4*(-I*b^2*c + 4*I*a*c^2 + b^2 + 4*I*a)/(c^2
 + 1)))/(c^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{x^2}\cos\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*cos(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*cos(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos \left(cx^2 + bx + a\right) e^{\left(x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*cos(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(c*x^2 + b*x + a)*e^(x^2), x)
```

3.110 $\int f^{a+bx}\cos\left(d+fx^2\right)\,dx$

Optimal. Leaf size=142

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(
(-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(1/4)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I
/4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rubi [A] time = 0.169169, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4473, 2287, 2234, 2204, 2205}

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+2ifx)}{2\sqrt{f}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(
(-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(1/4)*f^(-1/2 +
a)*Sqrt[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(4*E^((I
/4)*(4*d + (b^2*Log[f]^2)/f)))
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\cos\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+ifx^2}f^{a+bx}\right)dx \\
&= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx}\,dx \\
&= \frac{1}{2}\int e^{-id-ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{2}\int e^{id+ifx^2+a\log(f)+bx\log(f)}\,dx \\
&= \frac{1}{2}\left(e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx+\frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{-\frac{i(2ifx+b\log(f))^2}{4f}}\,dx \\
&= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a
\end{aligned}
$$

Mathematica [A] time = 0.238734, size = 133, normalized size = 0.94

$$
\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{ib^2\log^2(f)}{4f}}\left(e^{\frac{ib^2\log^2(f)}{2f}}(\sin(d)-i\cos(d))\text{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)-(\cos(d)-i\sin(d))\text{Erfi}\left(\frac{(-1)^{3/4}(2fx}{2
$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-(Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]))
/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^
(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I
/4)*b^2*Log[f]^2)/f))
```

Maple [A] time = 0.066, size = 114, normalized size = 0.8

$$-\frac{f^a\sqrt{\pi}}{4}e^{\frac{-\frac{i}{4}\left((\ln(f))^2b^2+4df\right)}{f}}\operatorname{Erf}\left(-\sqrt{if}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{f^a\sqrt{\pi}}{4}e^{\frac{\frac{i}{4}\left((\ln(f))^2b^2+4df\right)}{f}}\operatorname{Erf}\left(-\sqrt{-if}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{-if}}\right)\frac{1}{\sqrt{-if}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^
(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*
d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.511301, size = 749, normalized size = 5.27

$$\frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^2\log(f)^2+4af\log(f)-4idf}{4f}\right)}C\left(\frac{\sqrt{2}(2fx+ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^2\log(f)^2+4af\log(f)+4idf}{4f}\right)}C\left(-\frac{\sqrt{2}(2fx-ib\log(f))\sqrt{\frac{f}{\pi}}}{2f}\right)-i\sqrt{}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f
)/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(2)*p
i*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_co
s(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)
*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(
2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b
^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x -
I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cos\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2), x)
```

Giac [B] time = 1.30176, size = 405, normalized size = 2.85

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{8}\sqrt{2}\left(4x-\frac{\pi b\,\mathrm{sgn}(f)-\pi b+2ib\log(|f|)}{f}\right)\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}\right)e^{\left(\frac{i\pi^2b^2\mathrm{sgn}(f)}{8f}+\frac{\pi b^2\log(|f|)\mathrm{sgn}(f)}{4f}-\frac{i\pi^2b^2}{8f}-\frac{\pi b^2\log(|f|)}{4f}+\frac{ib^2\log(|f|)^2}{4f}-\frac{1}{2}i\pi\right.}}{4\left(-\frac{if}{|f|}+1\right)\sqrt{|f|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*l
og(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f +
 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f)
)/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(ab
s(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*sqrt(2)*sqrt(pi)*erf(-1
/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) +
```

```
 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(
f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^
2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) - I*d)/((I*f/abs(f) +
1)*sqrt(abs(f)))
```

3.111 $\int f^{a+bx}\cos^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=157

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\text{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(\cdots}{\cdots}\right)$$

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] - ((1/16 + I/16)*f^(-1/2 +
 a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(
16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```

Rubi [A] time = 0.173463, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4473, 2194, 2287, 2234, 2204, 2205}

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{1}{8}i\left(\frac{b^2\log^2(f)}{f}+16d\right)}\text{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(\cdots}{\cdots}\right)$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```


```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((1/4 + I/4)*((4*I)*f*x + b*Log[f]))/Sqrt[f]] - ((1/16 + I/16)*f^(-1/2 +
 a)*Sqrt[Pi]*Erfi[((1/4 + I/4)*((4*I)*f*x - b*Log[f]))/Sqrt[f]])/E^((I/8)*(
16*d + (b^2*Log[f]^2)/f)) + f^(a + b*x)/(2*b*Log[f])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}\cos^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx}\right)dx \\
&= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2id-2ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2id+2ifx^2+a\log(f)+bx\log(f)}\,dx \\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2id+\frac{ib^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{i(4ifx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\left(e^{-\frac{1}{8}i\left(16d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-4ifx+b\log}{8f}} \\
&= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{ib^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(4ifx+b\log(f))}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{-\frac{1}{8}i\left(16d+\frac{b^2\log}{}\right.}
\end{aligned}
$$

Mathematica [A] time = 1.09174, size = 158, normalized size = 1.01

$$\frac{1}{16}f^a\left(\frac{(1-i)\sqrt{\pi}e^{-\frac{ib^2\log^2(f)}{8f}}(\cos(d)-i\sin(d))^2\mathrm{Erf}\left(\frac{(4+4i)fx-(1-i)b\log(f)}{4\sqrt{f}}\right)}{\sqrt{f}}+\frac{(1+i)\sqrt{\pi}e^{\frac{ib^2\log^2(f)}{8f}}(\sin(2d)-i\cos(2d))\mathrm{Erfi}\left(\frac{(1}{}\right.}{\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*((8*f^(b*x))/(b*Log[f]) + ((1 - I)*Sqrt[Pi]*Erf[((4 + 4*I)*f*x - (1 -
I)*b*Log[f])/(4*Sqrt[f])]*(Cos[d] - I*Sin[d])^2)/(E^(((I/8)*b^2*Log[f]^2)/f
)*Sqrt[f]) + ((1 + I)*E^(((I/8)*b^2*Log[f]^2)/f)*Sqrt[Pi]*Erfi[((4 + 4*I)*f
*x + (1 - I)*b*Log[f])/(4*Sqrt[f])]*((-I)*Cos[2*d] + Sin[2*d]))/Sqrt[f]))/1
6
```

Maple [A] time = 0.109, size = 139, normalized size = 0.9

$$-\frac{\sqrt{2}f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{-\frac{i}{8}\left((\ln(f))^2b^2+16df\right)}{f}}\mathrm{Erf}\left(-\sqrt{2}\sqrt{if}x+\frac{\ln\left(f\right)b\sqrt{2}}{4}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{\frac{\frac{i}{8}\left((\ln(f))^2b^2+16df\right)}{f}}\mathrm{Erf}\left(-\sqrt{-2\,if}x+\frac{b\ln\left(f\right)}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2+16*d*f)/f)*2^(1/2)/(I*f)^(1/2)*e
rf(-2^(1/2)*(I*f)^(1/2)*x+1/4*ln(f)*b*2^(1/2)/(I*f)^(1/2))-1/8*Pi^(1/2)*f^a
*exp(1/8*I*(ln(f)^2*b^2+16*d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2
*ln(f)*b/(-2*I*f)^(1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.516288, size = 768, normalized size = 4.89

$$\frac{2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log\left(f\right)^2+8\,af\log\left(f\right)-16i\,df}{8\,f}\right)}\mathrm{C}\left(\frac{\left(4\,fx+i\,b\log\left(f\right)\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log\left(f\right)-2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log\left(f\right)^2+8\,af\log\left(f\right)+16i\,df}{8\,f}\right)}\mathrm{C}\left(-\frac{\left(4\,fx-i\,b\log\left(f\right)\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/
f)*fresnel_cos(1/2*(4*f*x + I*b*log(f))*sqrt(f/pi)/f)*log(f) - 2*pi*b*sqrt(
f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8*a*f*log(f) + 16*I*d*f)/f)*fresnel_cos(-1/2
*(4*f*x - I*b*log(f))*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(-I
*b^2*log(f)^2 + 8*a*f*log(f) - 16*I*d*f)/f)*fresnel_sin(1/2*(4*f*x + I*b*lo
g(f))*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 + 8
*a*f*log(f) + 16*I*d*f)/f)*fresnel_sin(-1/2*(4*f*x - I*b*log(f))*sqrt(f/pi)
/f)*log(f) + 8*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cos^2\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**2, x)
```

Giac [B] time = 1.34673, size = 703, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f)
- 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) + 2*I*e^(-1/2*
I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b
*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f)))
- 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(
f)))/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*log(abs
(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*b^2*lo
g(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 2*I*d)/(sq
rt(f)*(-I*f/abs(f) + 1)) - 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (pi*b*sgn(f
) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/16*I*pi^2*b^2*sgn(
f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f + 1/8*pi*b^2*log
(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a
*log(abs(f)) - 2*I*d)/(sqrt(f)*(I*f/abs(f) + 1))
```

3.112 $\int f^{a+bx} \cos^3\left(d+fx^2\right)\,dx$

Optimal. Leaf size=298

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+}{\sqrt{6}\sqrt{f}}\right.$$

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I/16)*E^((3
*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*(
(6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(1/4)*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4
*d + (b^2*Log[f]^2)/f))) - ((1/16 + I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/
2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2
*Log[f]^2)/f))
```

Rubi [A] time = 0.330467, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4473, 2287, 2234, 2204, 2205}

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f}+4d\right)}\mathrm{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{ib^2\log^2(f)}{12f}+3id}\mathrm{Erf}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(b\log(f)+}{\sqrt{6}\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```



```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (b^2*Log[f]^2)/f))*f^(-1/2 + a)*Sqrt[Pi]*Erf
[((-1)^(1/4)*((2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/16 - (1/16 + I/16)*E^((3
*I)*d + ((I/12)*b^2*Log[f]^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6]*Erf[((1/2 + I/2)*(
(6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (3*(-1)^(1/4)*f^(-1/2 + a)*Sqrt
[Pi]*Erfi[((-1)^(1/4)*((2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/(16*E^((I/4)*(4
*d + (b^2*Log[f]^2)/f))) - ((1/16 + I/16)*f^(-1/2 + a)*Sqrt[Pi/6]*Erfi[((1/
2 + I/2)*((6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])])/E^((I/12)*(36*d + (b^2
*Log[f]^2)/f))
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)dx = \int\left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx}+\frac{3}{8}e^{id+ifx^2}f^{a+bx}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx}+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx}\right)dx$$

$$= \frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a+bx}\,dx+\frac{3}{8}\int e^{id+ifx^2}f^{a}$$

$$= \frac{1}{8}\int e^{-3id-3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{8}\int e^{3id+3ifx^2+a\log(f)+bx\log(f)}\,dx+\frac{3}{8}\int e^{-id-ifx^2+a\log(}$$

$$= \frac{1}{8}\left(e^{3id+\frac{ib^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{i(6ifx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(3e^{-\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^a\right)\int e^{\frac{i(-2ifx+b\log(f))^2}{4f}}\,dx+\frac{1}{8}$$

$$= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{b^2\log^2(f)}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt[4]{-1}(2ifx+b\log(f))}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+\frac{ib^2\log^2(f)}{12f}}f^{-}$$

Mathematica [A] time = 0.898799, size = 267, normalized size = 0.9

$$\frac{1}{48}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{-\frac{ib^2\log^2(f)}{4f}}\left(9e^{\frac{ib^2\log^2(f)}{2f}}(\sin(d)-i\cos(d))\text{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-ib\log(f))}{2\sqrt{f}}\right)+\sqrt{3}e^{\frac{ib^2\log^2(f)}{6f}}\left(e^{\frac{ib^2\log^2(f)}{6f}}(\sin(3d)-i\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(-1/2 + a)*Sqrt[Pi]*(-9*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) + 9*E^(((I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d] + Sin[d]) + Sqrt[3]*E^(((I/6)*b^2*Log[f]^2)/f)*(-(Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Cos[3*d] - I*Sin[3*d])) + E^(((I/6)*b^2*Log[f]^2)/f)*Erfi[((6 + 6*I)*f*x + (1 - I)*b*Log[f])/(2*Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d]))))/(48*E^(((I/4)*b^2*Log[f]^2)/f))
```

Maple [A] time = 0.217, size = 235, normalized size = 0.8

$$-\frac{\sqrt{3}f^a\sqrt{\pi}}{48}e^{\frac{-\frac{i}{12}\left((\ln(f))^2b^2+36df\right)}{f}}\text{Erf}\left(-\sqrt{3}\sqrt{if}x+\frac{\ln(f)b\sqrt{3}}{6}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{3f^a\sqrt{\pi}}{16}e^{\frac{-\frac{i}{4}\left((\ln(f))^2b^2+4df\right)}{f}}\text{Erf}\left(-\sqrt{if}x+\frac{b\ln(f)}{2}\frac{1}{\sqrt{i}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2+36*d*f)/f)*3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*ln(f)*b*3^(1/2)/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*ln(f)*b/(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^(1/2)*x+1/2*ln(f)*b/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(1/12*I*(ln(f)^2*b^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-3*I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.5599, size = 1517, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 12*a*f*log(f) - 36*I
*d*f)/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f))*sqrt(f/pi)/f) - sqrt(
6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 + 12*a*f*log(f) + 36*I*d*f)/f)*fre
snel_cos(-1/6*sqrt(6)*(6*f*x - I*b*log(f))*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqr
t(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*log(f) - 4*I*d*f)/f)*fresnel_cos(1/
2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1
/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*I*d*f)/f)*fresnel_cos(-1/2*sqrt(2)*(2
*f*x - I*b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*
log(f)^2 + 12*a*f*log(f) - 36*I*d*f)/f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*
b*log(f))*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 +
 12*a*f*log(f) + 36*I*d*f)/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f))
*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + 4*a*f*
log(f) - 4*I*d*f)/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f))*sqrt(f/pi
)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 + 4*a*f*log(f) + 4*
I*d*f)/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x - I*b*log(f))*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cos^3\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + f*x**2)**3, x)
```

Giac [B] time = 1.4692, size = 803, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)))/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(f)/f
+ 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(abs(f
))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(a
bs(f)) + I*d)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqrt(pi)*erf(
-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(
-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2*log(abs(f))*sgn
(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/12*I*b^2*log(abs(
f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) + 3*I*d)/(sqrt(f)*
(-I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x
+ (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)))/f)*(I*f/abs(f) + 1))*e^(-1/24*I*
pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f)/f + 1/24*I*pi^2*b^2/f +
1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f)
+ 1/2*I*pi*a + a*log(abs(f)) - 3*I*d)/(sqrt(f)*(I*f/abs(f) + 1)) - 3/16*sqr
t(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f
)))/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*
b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/
4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a + a*log(abs(f)) -
I*d)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.113 $\int f^{a+bx}\cos\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=162

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+}{2\sqrt{f}}\right.$$

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(1/4)*
E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^
(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rubi [A] time = 0.254745, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {4473, 2287, 2234, 2204, 2205}

$$-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(e+ib\log(f))^2}{4f}-id}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-b\log(f)+}{2\sqrt{f}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] -((-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]*E
rf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])])/4 - ((-1)^(1/4)*
E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi]*Erfi[((-1)^
(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])])/4
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx}\cos\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx}\right)dx\\ &= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+bx}\,dx\\ &= \frac{1}{2}\int\exp\left(-id-ifx^2+a\log(f)-x(ie-b\log(f))\right)dx+\frac{1}{2}\int\exp\left(id+ifx^2+a\log(f)\right.\\ &= \frac{1}{2}\left(e^{-id+\frac{i(e+ib\log(f))^2}{4f}}f^a\right)\int e^{\frac{i(-ie-2ifx+b\log(f))^2}{4f}}dx+\frac{1}{2}\left(e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^a\right)\int e^{-\frac{i(ie+2ifx+b\log(f)}{4f}}\\ &= -\frac{1}{4}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt[4]{-1}e^{-id+\frac{i(e+ib\log(}{4f}}\end{aligned}$$

Mathematica [A] time = 0.390433, size = 163, normalized size = 1.01

$$\frac{1}{4}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{i\left(b^2\log^2(f)+e^2\right)}{4f}}\left(e^{\frac{ib^2\log^2(f)}{2f}}(\sin(d)-i\cos(d))\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(-ib\log(f)+e+2fx)}{2\sqrt{f}}\right)-e^{\frac{ie^2}{2f}}(\cos(d)-i\sin(d))\mathrm{Erfi}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^(((I/2)*e^2)/f)*Erfi[((-1
)^(3/4)*(e + 2*f*x + I*b*Log[f]))/(2*Sqrt[f])]*(Cos[d] - I*Sin[d])) + E^(((
I/2)*b^2*Log[f]^2)/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f]
)]*((-I)*Cos[d] + Sin[d])))/(4*E^(((I/4)*(e^2 + b^2*Log[f]^2))/f))
```

Maple [A] time = 0.071, size = 150, normalized size = 0.9

$$-\frac{f^a\sqrt{\pi}}{4}e^{\frac{-\frac{i}{4}\left((\ln(f))^2b^2-2i\ln(f)be-e^2+4df\right)}{f}}\mathrm{Erf}\left(-\sqrt{if}x+\frac{b\ln(f)-ie}{2}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{f^a\sqrt{\pi}}{4}e^{\frac{\frac{i}{4}\left((\ln(f))^2b^2+2i\ln(f)be-e^2+4df\right)}{f}}\mathrm{Erf}\left(-\sqrt{-if}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4*d*f)/f)/(I*f)
^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-1/4*Pi^(1/2)*f^a*e
xp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)^(1/2)*erf(-(-I*f)^
(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.511775, size = 868, normalized size = 5.36

$$\frac{\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{-ib^2\log(f)^2+ie^2-4idf-2(be-2af)\log(f)}{4f}\right)}C\left(\frac{\sqrt{2}(2fx+ib\log(f)+e)\sqrt{\frac{f}{\pi}}}{2f}\right)-\sqrt{2}\pi\sqrt{\frac{f}{\pi}}e^{\left(\frac{ib^2\log(f)^2-ie^2+4idf-2(be-2af)\log(f)}{4f}\right)}C\left(-\frac{\sqrt{2}(2fx-\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*d*f - 2*(b
*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(2)*(2*f*x + I*b*log(f) + e)*sqr
t(f/pi)/f) - sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f
 - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*sqrt(2)*(2*f*x - I*b*log(f)
+ e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^
2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I
*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)
^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*
(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cos\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2), x)
```

Giac [B] time = 1.3256, size = 518, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*l
og(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn(
f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log(
abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sg
n(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/
f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/4*sqrt(2)*sqrt(
pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/
f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*
log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*
```

```
b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*
a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^
2/f)/((I*f/abs(f) + 1)*sqrt(abs(f)))
```

3.114 $\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=179

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\text{Erfi}\left(\left(\frac{1}{4}+\right.\right.$$

```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*
Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] - (1/1
6 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[P
i]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*
x)/(2*b*Log[f])
```

Rubi [A] time = 0.283542, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4473, 2194, 2287, 2234, 2204, 2205}

$$\left(-\frac{1}{16}-\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+2ie)^2}{8f}+2id}\text{Erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(b\log(f)+2ie+4ifx)}{\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{i(2e+ib\log(f))^2}{8f}-2id}\text{Erfi}\left(\left(\frac{1}{4}+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```


```
[Out] (-1/16 - I/16)*E^((2*I)*d + ((I/8)*((2*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*
Sqrt[Pi]*Erf[((1/4 + I/4)*((2*I)*e + (4*I)*f*x + b*Log[f]))/Sqrt[f]] - (1/1
6 + I/16)*E^((-2*I)*d + ((I/8)*(2*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[P
i]*Erfi[((1/4 + I/4)*((2*I)*e + (4*I)*f*x - b*Log[f]))/Sqrt[f]] + f^(a + b*
x)/(2*b*Log[f])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx}\,dx+\frac{1}{2}\int f^{a+bx}\,dx\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int\exp\left(-2id-2ifx^2+a\log(f)-x(2ie-b\log(f))\right)dx+\frac{1}{4}\int\exp\left(2\right.\\
&= \frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\exp\left(-2id+a\log(f)-\frac{i(-2ie+b\log(f))^2}{8f}\right)\int e^{\frac{i(-2ie-4ifx+b\log(f))^2}{8f}}\,dx+\frac{1}{4}\\
&= \left(-\frac{1}{16}-\frac{i}{16}\right)e^{2id+\frac{i(2ie+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(2ie+4ifx+b\log(f))}{\sqrt{f}}\right)-\left(\frac{1}{16}+\right.
\end{aligned}$$

Mathematica [A] time = 1.14282, size = 245, normalized size = 1.37

$$\frac{f^{a-\frac{be+f}{2f}}e^{-\frac{i\left(b^2\log^2(f)+4e^2\right)}{8f}}\left(\sqrt[4]{-1}\sqrt{2\pi}b\log(f)e^{\frac{ib^2\log^2(f)}{4f}}(\sin(2d)-i\cos(2d))\operatorname{Erfi}\left(\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(-ib\log(f)+2e+4fx)}{\sqrt{f}}\right)+8f^{b\left(\frac{e}{2f}+x\right)+\frac{1}{2}}e^{\frac{i\left(b^2\log^2(f\right.}{8f}}\right.}{16b\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(8*E^(((I/8)*(4*e^2 + b^2*Log[f]^2))/f)*f^(1/2 + b
*(e/(2*f) + x)) + (-1)^(1/4)*b*E^(((I/4)*b^2*Log[f]^2)/f)*Sqrt[2*Pi]*Erfi[(
(1/4 + I/4)*(2*e + 4*f*x - I*b*Log[f]))/Sqrt[f]]*Log[f]*((-I)*Cos[2*d] + Si
n[2*d]) - (-1)^(1/4)*b*E^((I*e^2)/f)*Sqrt[2*Pi]*Erf[((1/4 + I/4)*(2*e + 4*f
*x + I*b*Log[f]))/Sqrt[f]]*Log[f]*(I*Cos[2*d] + Sin[2*d])))/(16*b*E^(((I/8)
*(4*e^2 + b^2*Log[f]^2))/f)*Log[f])
```

Maple [A] time = 0.14, size = 175, normalized size = 1.

$$-\frac{\sqrt{2}f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{-\frac{i}{8}\left(\left(\ln(f)\right)^2b^2-4i\ln(f)be-4e^2+16df\right)}{f}}\operatorname{Erf}\left(-\sqrt{2}\sqrt{if}x+\frac{\left(b\ln\left(f\right)-2ie\right)\sqrt{2}}{4}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{\frac{\frac{i}{8}\left(\left(\ln(f)\right)^2b^2+4i\ln(f)be-4e^2+16df\right.}{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/8*I*(ln(f)^2*b^2-4*I*ln(f)*b*e-4*e^2+16*d*f)/f)*2
^(1/2)/(I*f)^(1/2)*erf(-2^(1/2)*(I*f)^(1/2)*x+1/4*(b*ln(f)-2*I*e)*2^(1/2)/(
I*f)^(1/2))-1/8*Pi^(1/2)*f^a*exp(1/8*I*(ln(f)^2*b^2+4*I*ln(f)*b*e-4*e^2+16*
d*f)/f)/(-2*I*f)^(1/2)*erf(-(-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-2*I*f)^(
1/2))+1/2*f^(b*x+a)/b/ln(f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.522357, size = 909, normalized size = 5.08

$$2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{-i\,b^2\log\left(f\right)^2+4i\,e^2-16i\,df-4\left(be-2\,af\right)\log\left(f\right)}{8\,f}\right)}\mathrm{C}\left(\frac{\left(4\,fx+i\,b\log\left(f\right)+2\,e\right)\sqrt{\frac{f}{\pi}}}{2\,f}\right)\log\left(f\right)-2\,\pi b\sqrt{\frac{f}{\pi}}e^{\left(\frac{i\,b^2\log\left(f\right)^2-4i\,e^2+16i\,df-4\left(be-2\,af\right)\log\left(f\right)}{8\,f}\right)}\mathrm{C}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2 - 16*I*d*f - 4*(b
*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*(4*f*x + I*b*log(f) + 2*e)*sqrt(f/pi
)/f)*log(f) - 2*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2 - 4*I*e^2 + 16*I*d*f
 - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/2*(4*f*x - I*b*log(f) + 2*e)*s
qrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(-I*b^2*log(f)^2 + 4*I*e^2
 - 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*(4*f*x + I*b*log(f
) + 2*e)*sqrt(f/pi)/f)*log(f) - 2*I*pi*b*sqrt(f/pi)*e^(1/8*(I*b^2*log(f)^2
- 4*I*e^2 + 16*I*d*f - 4*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*(4*f*x -
 I*b*log(f) + 2*e)*sqrt(f/pi)/f)*log(f) + 8*f*f^(b*x + a))/(b*f*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx}\cos^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*cos(d + e*x + f*x**2)**2, x)
```

Giac [B] time = 1.38031, size = 817, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(
abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) - (pi*b*sgn(f) - pi*
b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2
*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(f)) + a*log(abs(f)
)) - 1/2*I*(-2*I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f)
- 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) + 2*I*e^(-1/2*
I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b
*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log(abs(f)))
- 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(
f)) - 4*e)/f)*(-I*f/abs(f) + 1))*e^(1/16*I*pi^2*b^2*sgn(f)/f + 1/8*pi*b^2*l
og(abs(f))*sgn(f)/f - 1/16*I*pi^2*b^2/f - 1/8*pi*b^2*log(abs(f))/f + 1/8*I*
b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*
a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 2*I*d - 1/2*I*
e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) - 1/8*sqrt(pi)*erf(-1/8*sqrt(f)*(8*x + (
pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 4*e)/f)*(I*f/abs(f) + 1))*e^(-1/16
*I*pi^2*b^2*sgn(f)/f - 1/8*pi*b^2*log(abs(f))*sgn(f)/f + 1/16*I*pi^2*b^2/f
+ 1/8*pi*b^2*log(abs(f))/f - 1/8*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f)
+ 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2
*b*e*log(abs(f))/f - 2*I*d + 1/2*I*e^2/f)/(sqrt(f)*(I*f/abs(f) + 1))
```

3.115 $\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=340

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\cdots\right.$$

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]
*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]])/16 - (1/16 + I
/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (
3*(-1)^(1/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])]])/16 - (1/16 +
 I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/
6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rubi [A] time = 0.519674, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4473, 2287, 2234, 2204, 2205}

$$-\frac{3}{16}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{1}{4}i\left(4d+\frac{(b\log(f)+ie)^2}{f}\right)}\text{Erf}\left(\frac{\sqrt[4]{-1}(b\log(f)+ie+2ifx)}{2\sqrt{f}}\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{\frac{\pi}{6}}f^{a-\frac{1}{2}}e^{\frac{i(b\log(f)+3ie)^2}{12f}+3id}\text{Erf}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (-3*(-1)^(1/4)*E^((I/4)*(4*d + (I*e + b*Log[f])^2/f))*f^(-1/2 + a)*Sqrt[Pi]
*Erf[((-1)^(1/4)*(I*e + (2*I)*f*x + b*Log[f]))/(2*Sqrt[f])]])/16 - (1/16 + I
/16)*E^((3*I)*d + ((I/12)*((3*I)*e + b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/6
]*Erf[((1/2 + I/2)*((3*I)*e + (6*I)*f*x + b*Log[f]))/(Sqrt[6]*Sqrt[f])] - (
3*(-1)^(1/4)*E^((-I)*d + ((I/4)*(e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi
]*Erfi[((-1)^(1/4)*(I*e + (2*I)*f*x - b*Log[f]))/(2*Sqrt[f])]])/16 - (1/16 +
 I/16)*E^((-3*I)*d + ((I/12)*(3*e + I*b*Log[f])^2)/f)*f^(-1/2 + a)*Sqrt[Pi/
6]*Erfi[((1/2 + I/2)*((3*I)*e + (6*I)*f*x - b*Log[f]))/(Sqrt[6]*Sqrt[f])]
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
```

```
x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+bx}\cos^3\left(d+ex+fx^2\right)\,dx = \int\left(\frac{1}{8}e^{-3i\left(d+ex+fx^2\right)}f^{a+bx} + \frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx} + \frac{3}{8}\text{ex}\right.$$

$$= \frac{1}{8}\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx}\,dx + \frac{1}{8}\int\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\,d$$

$$= \frac{1}{8}\int\exp\left(-3id-3ifx^2+a\log(f)-x(3ie-b\log(f))\right)\,dx + \frac{1}{8}\int\exp\left(3id+3ifx^2+a\right.$$

$$= \frac{1}{8}\exp\left(-3id+a\log(f)-\frac{i(-3ie+b\log(f))^2}{12f}\right)\int e^{\frac{i(-3ie-6ifx+b\log(f))^2}{12f}}\,dx + \frac{1}{8}\left(3e^{-id+\frac{i(e+ib\log}{4f}}\right.$$

$$= -\frac{3}{16}\sqrt[4]{-1}e^{\frac{1}{4}i\left(4d+\frac{(ie+b\log(f))^2}{f}\right)}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt[4]{-1}(ie+2ifx+b\log(f))}{2\sqrt{f}}\right) - \left(\frac{1}{16}+\frac{i}{16}\right)e^{3id+}$$

Mathematica [A] time = 1.61065, size = 322, normalized size = 0.95

$$\frac{1}{48}\sqrt[4]{-1}\sqrt{\pi}f^{a-\frac{be+f}{2f}}e^{-\frac{i\left(b^2\log^2(f)+3e^2\right)}{4f}}\left(9(\sin(d)-i\cos(d))e^{\frac{i\left(b^2\log^2(f)+e^2\right)}{2f}}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-ib\log(f)+e+2fx)}{2\sqrt{f}}\right)+e^{\frac{ie^2}{f}}\left(-\sqrt{3}(\cos(3d\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(9*E^(((I/2)*(e^2 + b^2*Log[f]
^2))/f)*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f]))/(2*Sqrt[f])]*((-I)*Cos[d
] + Sin[d]) + E^((I*e^2)/f)*(-9*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f]))/
(2*Sqrt[f])]*(Cos[d] - I*Sin[d]) - Sqrt[3]*E^(((I/6)*(3*e^2 + b^2*Log[f]^2)
)/f)*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f]))/(2*Sqrt[3]*Sqrt[f])]*(Cos
[3*d] - I*Sin[3*d])) + Sqrt[3]*E^(((I/3)*b^2*Log[f]^2)/f)*Erfi[((1/2 + I/2)
*(3*e + 6*f*x - I*b*Log[f]))/(Sqrt[6]*Sqrt[f])]*((-I)*Cos[3*d] + Sin[3*d]))
)/(48*E^(((I/4)*(3*e^2 + b^2*Log[f]^2))/f))
```

Maple [A] time = 0.252, size = 307, normalized size = 0.9

$$-\frac{\sqrt{3}f^a\sqrt{\pi}}{48}\mathrm{e}^{\frac{-\frac{i}{12}\left(\left(\ln\left(f\right)\right)^2b^2-6\,i\ln\left(f\right)be-9\,e^2+36\,df\right)}{f}}\mathrm{Erf}\left(-\sqrt{3}\sqrt{if}x+\frac{\left(b\ln\left(f\right)-3\,ie\right)\sqrt{3}}{6}\frac{1}{\sqrt{if}}\right)\frac{1}{\sqrt{if}}-\frac{3\,f^a\sqrt{\pi}}{16}\mathrm{e}^{\frac{-\frac{i}{4}\left(\left(\ln(f)\right)^2b^2-2\,i\ln(f)be-e^2\right.}{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/48*Pi^(1/2)*f^a*exp(-1/12*I*(ln(f)^2*b^2-6*I*ln(f)*b*e-9*e^2+36*d*f)/f)*
3^(1/2)/(I*f)^(1/2)*erf(-3^(1/2)*(I*f)^(1/2)*x+1/6*(b*ln(f)-3*I*e)*3^(1/2)/
(I*f)^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*I*(ln(f)^2*b^2-2*I*ln(f)*b*e-e^2+4*
d*f)/f)/(I*f)^(1/2)*erf(-(I*f)^(1/2)*x+1/2*(b*ln(f)-I*e)/(I*f)^(1/2))-3/16*
Pi^(1/2)*f^a*exp(1/4*I*(ln(f)^2*b^2+2*I*ln(f)*b*e-e^2+4*d*f)/f)/(-I*f)^(1/2
)*erf(-(-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp
(1/12*I*(ln(f)^2*b^2+6*I*ln(f)*b*e-9*e^2+36*d*f)/f)/(-3*I*f)^(1/2)*erf(-(-3
*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-3*I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.559289, size = 1771, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3
*e)*sqrt(f/pi)/f) - sqrt(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2
 + 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(-1/6*sqrt(6)*(6*f*x -
I*b*log(f) + 3*e)*sqrt(f/pi)/f) + 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*lo
g(f)^2 + I*e^2 - 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(1/2*sqrt(
2)*(2*f*x + I*b*log(f) + e)*sqrt(f/pi)/f) - 9*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*
(I*b^2*log(f)^2 - I*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_cos(
-1/2*sqrt(2)*(2*f*x - I*b*log(f) + e)*sqrt(f/pi)/f) - I*sqrt(6)*pi*sqrt(f/p
i)*e^(1/12*(-I*b^2*log(f)^2 + 9*I*e^2 - 36*I*d*f - 6*(b*e - 2*a*f)*log(f))/
f)*fresnel_sin(1/6*sqrt(6)*(6*f*x + I*b*log(f) + 3*e)*sqrt(f/pi)/f) - I*sqr
t(6)*pi*sqrt(f/pi)*e^(1/12*(I*b^2*log(f)^2 - 9*I*e^2 + 36*I*d*f - 6*(b*e -
2*a*f)*log(f))/f)*fresnel_sin(-1/6*sqrt(6)*(6*f*x - I*b*log(f) + 3*e)*sqrt(
f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(-I*b^2*log(f)^2 + I*e^2 - 4*I*
d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(1/2*sqrt(2)*(2*f*x + I*b*log(f
) + e)*sqrt(f/pi)/f) - 9*I*sqrt(2)*pi*sqrt(f/pi)*e^(1/4*(I*b^2*log(f)^2 - I
*e^2 + 4*I*d*f - 2*(b*e - 2*a*f)*log(f))/f)*fresnel_sin(-1/2*sqrt(2)*(2*f*x
 - I*b*log(f) + e)*sqrt(f/pi)/f))/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60652, size = 1030, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x - (pi*b*sgn(f) - pi*b + 2*I*b*
log(abs(f)) - 2*e)/f)*(-I*f/abs(f) + 1)*sqrt(abs(f)))*e^(1/8*I*pi^2*b^2*sgn
(f)/f + 1/4*pi*b^2*log(abs(f))*sgn(f)/f - 1/8*I*pi^2*b^2/f - 1/4*pi*b^2*log
(abs(f))/f + 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*s
gn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))
/f + I*d - 1/4*I*e^2/f)/((-I*f/abs(f) + 1)*sqrt(abs(f))) - 1/48*sqrt(6)*sqr
t(pi)*erf(-1/24*sqrt(6)*sqrt(f)*(12*x - (pi*b*sgn(f) - pi*b + 2*I*b*log(abs
(f)) - 6*e)/f)*(-I*f/abs(f) + 1))*e^(1/24*I*pi^2*b^2*sgn(f)/f + 1/12*pi*b^2
*log(abs(f))*sgn(f)/f - 1/24*I*pi^2*b^2/f - 1/12*pi*b^2*log(abs(f))/f + 1/1
2*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I
*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f + 3*I*d - 3/
4*I*e^2/f)/(sqrt(f)*(-I*f/abs(f) + 1)) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/24*sq
rt(6)*sqrt(f)*(12*x + (pi*b*sgn(f) - pi*b + 2*I*b*log(abs(f)) + 6*e)/f)*(I*
f/abs(f) + 1))*e^(-1/24*I*pi^2*b^2*sgn(f)/f - 1/12*pi*b^2*log(abs(f))*sgn(f
)/f + 1/24*I*pi^2*b^2/f + 1/12*pi*b^2*log(abs(f))/f - 1/12*I*b^2*log(abs(f)
)^2/f - 1/2*I*pi*a*sgn(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b
*e/f + a*log(abs(f)) - 1/2*b*e*log(abs(f))/f - 3*I*d + 3/4*I*e^2/f)/(sqrt(f
)*(I*f/abs(f) + 1)) - 3/16*sqrt(2)*sqrt(pi)*erf(-1/8*sqrt(2)*(4*x + (pi*b*s
gn(f) - pi*b + 2*I*b*log(abs(f)) + 2*e)/f)*(I*f/abs(f) + 1)*sqrt(abs(f)))*e
^(-1/8*I*pi^2*b^2*sgn(f)/f - 1/4*pi*b^2*log(abs(f))*sgn(f)/f + 1/8*I*pi^2*b
^2/f + 1/4*pi*b^2*log(abs(f))/f - 1/4*I*b^2*log(abs(f))^2/f - 1/2*I*pi*a*sg
n(f) + 1/4*I*pi*b*e*sgn(f)/f + 1/2*I*pi*a - 1/4*I*pi*b*e/f + a*log(abs(f))
- 1/2*b*e*log(abs(f))/f - I*d + 1/4*I*e^2/f)/((I*f/abs(f) + 1)*sqrt(abs(f))
)
```

3.116 $\int f^{a+cx^2}\cos(d+ex)\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(I*d + e^2/(4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqr
t[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.16782, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4473, 2287, 2234, 2204}

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x],x]
```


```
[Out] -(E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(I*d + e^2/(4*c*Log[f]
))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqr
t[c]*Sqrt[Log[f]])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+cx^2}\cos(d+ex)\,dx &= \int \left(\frac{1}{2}e^{-id-iex}f^{a+cx^2}+\frac{1}{2}e^{id+iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+cx^2}\,dx\\
&= \frac{1}{2}\int e^{-id-iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{2}\int e^{id+iex+a\log(f)+cx^2\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{2}\left(e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.153937, size = 116, normalized size = 0.79

$$\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}}\left((\cos(d)-i\sin(d))\mathbf{Erfi}\left(\frac{2cx\log(f)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(d)+i\sin(d))\mathbf{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])]*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.066, size = 121, normalized size = 0.8

$$\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4 i d \ln(f) c-e^{2}}{4 c \ln(f)}}\operatorname{Erf}\left(\sqrt{-c\ln(f)}\,x+\frac{i}{2}e\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{\frac{4 i d \ln(f) c+e^{2}}{4 c \ln(f)}}\operatorname{Erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{i}{2}e\frac{1}{\sqrt{-c\ln(f)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf
((-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(1/4*(4*I
*d*ln(f)*c+e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(
-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.491779, size = 409, normalized size = 2.78

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(f)^{2}+4icd\log(f)+e^{2}}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx\log(f)-ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(\frac{4ac\log(\right.}{}\right.}}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f))
/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) + s
```

```
qrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*log
(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))))/(c*log(f)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos(ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d), x)
```

3.117 $\int f^{a+cx^2}\cos^2(d+ex)\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}-2id}\text{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}+2id}\text{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rubi [A] time = 0.199762, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4473, 2204, 2287, 2234}

$$-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}-2id}\text{Erfi}\left(\frac{-cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)}+2id}\text{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(
(-2*I)*d + e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - c*x*Log[f])/(Sqrt[c]*Sq
rt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d + e^2/(c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos^2(d+ex)\,dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2iex}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2iex}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2id-2iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{4}\int e^{2id+2iex+a\log(f)+cx^2\log(f}\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(-2ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{4}\left(e^{2id+\frac{e^2}{c\log(f)}}f^a\right)\int e^{\frac{(2i}{}}\\
&= \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2id+\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+cx\,\mathrm{lo}}{\sqrt{c}\sqrt{\mathrm{lo}}}\right.}{8\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.257265, size = 131, normalized size = 0.77

$$\frac{\sqrt{\pi}f^a\left(2\mathrm{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+e^{\frac{e^2}{c\log(f)}}\left((\cos(2d)-i\sin(2d))\mathrm{Erfi}\left(\frac{cx\log(f)-ie}{\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(2d)+i\sin(2d))\mathrm{Erfi}\left(\frac{cx\log(f)+ie}{\sqrt{c}\sqrt{\log(f)}}\right)\right)\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + E^(e^2/(c*Log[f]))*(Erfi[((
-I)*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + Erfi[
```

```
(I*e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] + I*Sin[2*d]))))/(8*Sq
rt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.092, size = 145, normalized size = 0.9

$$\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2id\ln(f)c-e^{2}}{c\ln(f)}}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)}x+ie\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{\frac{2id\ln(f)c+e^{2}}{c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln\left(f\right)}x+ie\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c
*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp((2*I*d*ln(f)*c+e
^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+I*e/(-c*ln(f))^(1/2))
+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.497368, size = 452, normalized size = 2.64

$$-\frac{2\sqrt{\pi}\sqrt{-c\log\left(f\right)}f^{a}\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(cx\log\left(f\right)+ie\right)\sqrt{-c\log\left(f\right)}}{c\log\left(f\right)}\right)e^{\left(\frac{ac\log\left(f\right)^{2}+2icd\log\left(f\right)+e^{2}}{c\log\left(f\right)}\right)}+\sqrt{\pi}\sqrt{-c\log}}{8c\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log(f))*x) + sqrt(pi)*sqrt
(-c*log(f))*erf((c*x*log(f) + I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(
f)^2 + 2*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf((c*
x*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^((a*c*log(f)^2 - 2*I*c*d*log(
f) + e^2)/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos^2(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^2, x)
```

3.118 $\int f^{a+cx^2}\cos^3(d+ex)\,dx$

Optimal. Leaf size=293

$$-\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\cdots}{}$$

```
[Out] (-3*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^((-3*I)*d + (9*e^2)
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(I*d + e^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt
[Log[f]]) + (E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.330865, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4473, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}-id}\text{Erfi}\left(\frac{-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{4c\log(f)}-3id}\text{Erfi}\left(\frac{-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)}+id}\text{Erfi}\left(\frac{2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - 2*c*x*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^((-3*I)*d + (9*e^2)
/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[L
og[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(I*d + e^2/(4*c*Log[f]))*f^a*Sqr
t[Pi]*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt
[Log[f]]) + (E^((3*I)*d + (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3*I)*e
+ 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx = \int\left(\frac{3}{8}e^{-id-iex}f^{a+cx^2}+\frac{3}{8}e^{id+iex}f^{a+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+cx^2}\right)dx$$

$$=\frac{1}{8}\int e^{-3id-3iex}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-id-iex}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{id+iex}f^{a+cx^2}$$

$$=\frac{1}{8}\int e^{-3id-3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{1}{8}\int e^{3id+3iex+a\log(f)+cx^2\log(f)}\,dx+\frac{3}{8}\int e^{-id-iex+a\log(f)+cx}$$

$$=\frac{1}{8}\left(3e^{-id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(3e^{id+\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(ie+2cx\log(f))^2}{4c\log(f)}}\,dx+\frac{1}{8}\left(e^{-3id+\frac{9}{4c\log}}\right.$$

$$=-\frac{3e^{-id+\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-3id+\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3ie-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3e^{id+\frac{e^2}{4c\log(f)}}f}{16}$$

Mathematica [A] time = 0.422011, size = 218, normalized size = 0.74

$$\frac{\sqrt{\pi}f^ae^{\frac{e^2}{4c\log(f)}}\left(e^{\frac{2e^2}{c\log(f)}}\left((\cos(3d)-i\sin(3d))\mathrm{Erfi}\left(\frac{2cx\log(f)-3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(3d)+i\sin(3d))\mathrm{Erfi}\left(\frac{2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)+3(\cos(d)-i\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^(e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*(3*Erfi[((-I)*e + 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])]*(Cos[d] - I*Sin[d]) + 3*Erfi[(I*e + 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cos[d] + I*Sin[d]) + E^((2*e^2)/(c*Log[f]))*(Erfi[((-3*
I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] - I*Sin[3*d]) + Er
fi[((3*I)*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[3*d] + I*Sin[3*d
]))))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.202, size = 242, normalized size = 0.8

$$\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{12\,id\ln(f)c-9\,e^2}{4\,c\ln(f)}}\,\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)}x+\frac{3\,i}{2}e\frac{1}{\sqrt{-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{-c\ln\left(f\right)}}+\frac{3\,f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{4\,id\ln(f)c-e^2}{4\,c\ln(f)}}\,\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)}x+\frac{i}{2}e\frac{1}{\sqrt{-c\ln \ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c-3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*
erf((-c*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))+3/16*Pi^(1/2)*f^a*exp(-1/4
*(4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x+1/2*I
*e/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c+e^2)/ln(f)/c)
/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*I*e/(-c*ln(f))^(1/2))-1/16*Pi
^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c+3*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c
*ln(f))^(1/2)*x+3/2*I*e/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.528473, size = 815, normalized size = 2.78

$$-\frac{\sqrt{\pi}\sqrt{-c\log\left(f\right)}\,\mathrm{erf}\left(\frac{\left(2\,cx\log\left(f\right)+3i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(\frac{4\,ac\log\left(f\right)^{2}+12i\,cd\log\left(f\right)+9\,e^{2}}{4\,c\log\left(f\right)}\right)}+3\,\sqrt{\pi}\sqrt{-c\log\left(f\right)}\,\mathrm{erf}\left(\frac{\left(2\,cx\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + 3*I*e)*sqrt(-c*log(
f))/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 12*I*c*d*log(f) + 9*e^2)/(c*log(f)
)) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) + I*e)*sqrt(-c*log(f)
)/(c*log(f)))*e^(1/4*(4*a*c*log(f)^2 + 4*I*c*d*log(f) + e^2)/(c*log(f))) +
3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - I*e)*sqrt(-c*log(f))/(c*
log(f)))*e^(1/4*(4*a*c*log(f)^2 - 4*I*c*d*log(f) + e^2)/(c*log(f))) + sqrt(
pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x*log(f) - 3*I*e)*sqrt(-c*log(f))/(c*log(f
)))*e^(1/4*(4*a*c*log(f)^2 - 12*I*c*d*log(f) + 9*e^2)/(c*log(f))))/(c*log(f
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos^3(d+ex)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(e*x + d)^3, x)
```

3.119 $\int f^{a+cx^2} \cos\left(d + fx^2\right) dx$

Optimal. Leaf size=103

$$\frac{\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(4*E^(I*d)*Sqrt[I*f - c*Log[f]])
 + (E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/(4*Sqrt[I*f + c*Log[
f]])
```

Rubi [A] time = 0.154279, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4473, 2287, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(4*E^(I*d)*Sqrt[I*f - c*Log[f]])
 + (E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])/(4*Sqrt[I*f + c*Log[
f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int e^{-id+a\log(f)-x^2(if-c\log(f))}\,dx+\frac{1}{2}\int e^{id+a\log(f)+x^2(if+c\log(f))}\,dx\\ &= \frac{e^{-id}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{if-c\log(f)}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{if+c\log(f)}\right)}{4\sqrt{if+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.476822, size = 170, normalized size = 1.65

$$-\frac{(-1)^{3/4}\sqrt{\pi}f^a\left(\sqrt{f+ic\log(f)}\left(f\cos(d)\mathrm{Erf}\left(\frac{(1+i)x\sqrt{f+ic\log(f)}}{\sqrt{2}}\right)-\mathrm{Erfi}\left((-1)^{3/4}x\sqrt{f+ic\log(f)}\right)(c\cos(d)\log(f)+\sin(d)(f\right.\right.}{4\left(c^2\log^2(f)+f^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -((-1)^(3/4)*f^a*Sqrt[Pi]*(Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f -
 I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d] + I*Sin[d]) + Sqrt[f + I*c*Log[f]]*(f
*Cos[d]*Erf[((1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]] - Erfi[(-1)^(3/4)*x*S
qrt[f + I*c*Log[f]]]*(c*Cos[d]*Log[f] + (f - I*c*Log[f])*Sin[d]))))/(4*(f^2
 + c^2*Log[f]^2))
```

Maple [A] time = 0.063, size = 82, normalized size = 0.8

$$\frac{f^{a}\sqrt{\pi}\mathrm{e}^{-id}}{4}\mathrm{Erf}\left(x\sqrt{if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{if-c\ln\left(f\right)}}+\frac{f^{a}\sqrt{\pi}\mathrm{e}^{id}}{4}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-if}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-if}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2))+1
/4*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.504205, size = 301, normalized size = 2.92

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,f\right)\sqrt{-c\log\left(f\right)-i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-i\,f}x\right)e^{\left(a\log\left(f\right)+i\,d\right)}+\sqrt{\pi}\left(c\log\left(f\right)+i\,f\right)\sqrt{-c\log\left(f\right)+i\,f}\,\mathrm{erf}\left(\right.}{4\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) -
I*f)*x)*e^(a*log(f) + I*d) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*log(f) + I*f
)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos\left(fx^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d), x)
```

3.120 $\int f^{a+cx^2} \cos^2\left(d+fx^2\right) dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\pi}e^{-2id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}e^{2id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) + (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.188145, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4473, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}e^{-2id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+2if}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}e^{2id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+2if}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^a
*Sqrt[Pi]*Erf[x*Sqrt[(2*I)*f - c*Log[f]]])/(8*E^((2*I)*d)*Sqrt[(2*I)*f - c*
Log[f]]) + (E^((2*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(2*I)*f + c*Log[f]]])/(8*S
qrt[(2*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos^2\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2id+a\log(f)-x^2(2if-c\log(f))\right)dx+\frac{1}{4}\int\exp\\ &= \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{2if-c\log(f)}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{2if+c\,\text{l}}\right.}{8\sqrt{2if+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.82727, size = 189, normalized size = 1.35

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\sqrt{2f-ic\log(f)}(c\log(f)-2if)(\cos(2d)+i\sin(2d))\text{Erfi}\left(\sqrt[4]{-1}x\sqrt{2f-ic\log(f)}\right)-\right.}{c^2\log^2(f)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(Erfi[(-1)^(3/4)*x*Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*S
qrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d])) + Erfi[(-1)^(1/4)*x*Sqrt[2*f
 - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]]*((-2*I)*f + c*Log[f])*(Cos[2*d] + I*
Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/8
```

Maple [A] time = 0.082, size = 107, normalized size = 0.8

$$\frac{f^{a}\sqrt{\pi}\mathrm{e}^{-2\,id}}{8}\mathrm{Erf}\left(x\sqrt{2\,if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{2\,if-c\ln\left(f\right)}}+\frac{f^{a}\sqrt{\pi}\mathrm{e}^{2\,id}}{8}\mathrm{Erf}\left(\sqrt{-c\ln\left(f\right)-2\,if}x\right)\frac{1}{\sqrt{-c\ln\left(f\right)-2\,if}}+\frac{f^{a}\sqrt{\pi}}{4}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-2*I*d)/(2*I*f-c*ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1
/2))+1/8*Pi^(1/2)*f^a*exp(2*I*d)/(-c*ln(f)-2*I*f)^(1/2)*erf((-c*ln(f)-2*I*f
)^(1/2)*x)+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.519641, size = 479, normalized size = 3.42

$$-\frac{2\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}+4\,f^{2}\right)\sqrt{-c\log\left(f\right)}f^{a}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\left(c^{2}\log\left(f\right)^{2}-2i\,cf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2i\,f}\,\mathrm{erf}\left(\sqrt{}\right.}{8\left(c^{3}\log\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf(sqrt(-c*log(f) - 2*I*f)*x)*e^(a*log(f) + 2*I*d) + sqrt(pi)*(c^2*log(f)^
2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(sqrt(-c*log(f) + 2*I*f)*x)*
```

```
e^(a*log(f) - 2*I*d))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2} \cos^2\left(d+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + f*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a} \cos\left(fx^2+d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^2, x)
```

3.121 $\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx$

Optimal. Leaf size=205

$$\frac{3\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{-3id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}e^{3id}f\cdots}{1\cdots}$$

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(16*E^(I*d)*Sqrt[I*f - c*Log[f
]]) + (f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(16*E^((3*I)*d)*Sqrt[(
3*I)*f - c*Log[f]]) + (3*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])
/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(3*I)*f
+ c*Log[f]]])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rubi [A] time = 0.287936, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4473, 2287, 2205, 2204}

$$\frac{3\sqrt{\pi}e^{-id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+if}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}e^{-3id}f^a\text{Erf}\left(x\sqrt{-c\log(f)+3if}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi}e^{id}f^a\text{Erfi}\left(x\sqrt{c\log(f)+if}\right)}{16\sqrt{c\log(f)+if}}+\frac{\sqrt{\pi}e^{3id}f\cdots}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[I*f - c*Log[f]]])/(16*E^(I*d)*Sqrt[I*f - c*Log[f
]]) + (f^a*Sqrt[Pi]*Erf[x*Sqrt[(3*I)*f - c*Log[f]]])/(16*E^((3*I)*d)*Sqrt[(
3*I)*f - c*Log[f]]) + (3*E^(I*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[I*f + c*Log[f]]])
/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[(3*I)*f
+ c*Log[f]]])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cos^3\left(d+fx^2\right)dx=\int\left(\frac{3}{8}e^{-id-ifx^2}f^{a+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+cx^2}+\frac{1}{8}e^{3id+3ifx^2}f^{a+cx^2}\right)dx$$
$$=\frac{1}{8}\int e^{-3id-3ifx^2}f^{a+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a+cx^2}\,dx+\frac{3}{8}\int e^{id+ifx^2}f$$
$$=\frac{1}{8}\int\exp\left(-3id+a\log(f)-x^2(3if-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3id+a\log(f)+x^2(3if+c\right.$$
$$=\frac{3e^{-id}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{if-c\log(f)}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{3if-c\log(f)}\right)}{16\sqrt{3if-c\log(f)}}+\frac{3e^{id}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{i\right.}}{16\sqrt{if+c\,l}}$$

Mathematica [A] time = 2.20998, size = 389, normalized size = 1.9

$$\frac{\sqrt[4]{-1}\sqrt{\pi}f^a\left((f-ic\log(f))\left(\sqrt{3f-ic\log(f)}\left(ic^2\log^2(f)+4cf\log(f)-3if^2\right)(\cos(3d)+i\sin(3d))\text{Erfi}\left(\sqrt[4]{-1}x\sqrt{3f-ic\log\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] ((-1)^(1/4)*f^a*Sqrt[Pi]*(3*Erfi[(-1)^(1/4)*x*Sqrt[f - I*c*Log[f]]]*Sqrt[f
- I*c*Log[f]]*((-9*I)*f^3 + 9*c*f^2*Log[f] - I*c^2*f*Log[f]^2 + c^3*Log[f]^
3)*(Cos[d] + I*Sin[d]) + (f - I*c*Log[f])*(-((3*f - I*c*Log[f])*(9*f*Erf[((
1 + I)*x*Sqrt[f + I*c*Log[f]])/Sqrt[2]]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*Erf
i[(-1)^(3/4)*x*Sqrt[f + I*c*Log[f]]]*Sqrt[f + I*c*Log[f]]*(Cos[d]*(3*f + I*
c*Log[f]) + c*Log[f]*Sin[d]) + Erfi[(-1)^(3/4)*x*Sqrt[3*f + I*c*Log[f]]]*(f
```

```
 + I*c*Log[f])*Sqrt[3*f + I*c*Log[f]]*(Cos[3*d] - I*Sin[3*d]))) + Erfi[(-1)
^(1/4)*x*Sqrt[3*f - I*c*Log[f]]]*Sqrt[3*f - I*c*Log[f]]*((-3*I)*f^2 + 4*c*f
*Log[f] + I*c^2*Log[f]^2)*(Cos[3*d] + I*Sin[3*d]))))/(16*(9*f^4 + 10*c^2*f^
2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] time = 0.195, size = 162, normalized size = 0.8

$$\frac{f^a\sqrt{\pi}\mathrm{e}^{-3\,id}}{16}\mathrm{Erf}\left(x\sqrt{3\,if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}+\frac{3\,f^a\sqrt{\pi}\mathrm{e}^{-id}}{16}\mathrm{Erf}\left(x\sqrt{if-c\ln\left(f\right)}\right)\frac{1}{\sqrt{if-c\ln\left(f\right)}}+\frac{3\,f^a\sqrt{\pi}\mathrm{e}^{id}}{16}\mathrm{Er}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3*I*d)/(3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(-I*d)/(I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(
1/2))+3/16*Pi^(1/2)*f^a*exp(I*d)/(-c*ln(f)-I*f)^(1/2)*erf((-c*ln(f)-I*f)^(1
/2)*x)+1/16*Pi^(1/2)*f^a*exp(3*I*d)/(-c*ln(f)-3*I*f)^(1/2)*erf((-c*ln(f)-3*
I*f)^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.547518, size = 869, normalized size = 4.24

$$-\frac{\sqrt{\pi}\left(c^3\log\left(f\right)^3-3i\,c^2f\log\left(f\right)^2+cf^2\log\left(f\right)-3i\,f^3\right)\sqrt{-c\log\left(f\right)-3i\,f}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3i\,f}x\right)e^{\left(a\log\left(f\right)+3i\,d\right)}+\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(sqrt(-c*log(f) - 3*I*f)*x)*e^(a*log(f) + 3*I*
d) + sqrt(pi)*(3*c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + 27*c*f^2*log(f) - 27*I
*f^3)*sqrt(-c*log(f) - I*f)*erf(sqrt(-c*log(f) - I*f)*x)*e^(a*log(f) + I*d)
 + sqrt(pi)*(3*c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + 27*c*f^2*log(f) + 27*I*f
^3)*sqrt(-c*log(f) + I*f)*erf(sqrt(-c*log(f) + I*f)*x)*e^(a*log(f) - I*d) +
 sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt
(-c*log(f) + 3*I*f)*erf(sqrt(-c*log(f) + 3*I*f)*x)*e^(a*log(f) - 3*I*d))/(c
^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos\left(fx^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + d)^3, x)
```

3.122 $\int f^{a+cx^2} \cos\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=183

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

```
[Out] (E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f -
 c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log[f]]) + (E^(I*d +
 e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))
/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

Rubi [A] time = 0.304128, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\operatorname{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}+id}\operatorname{Erfi}\left(\frac{2x(c\log(f)+if)+ie}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f -
 c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log[f]]) + (E^(I*d +
 e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + 2*x*(I*f + c*Log[f]))
/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-id-iex+a\log(f)-x^2(if-c\log(f))\right)dx+\frac{1}{2}\int\exp\left(id+iex+a\log(f)\right.\\ &= \frac{1}{2}\left(e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx+\frac{1}{2}\left(e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\right)\int\\ &= \frac{e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id+\frac{e^2}{4if+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie+2x(if+c\log(f))}{2\sqrt{if+c\log(f)}}\right)}{4\sqrt{if+c\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.941907, size = 217, normalized size = 1.19

$$\frac{\sqrt[4]{-1}\sqrt{\pi}f^a e^{\frac{e^2}{4c\log(f)+4if}}\left(\sqrt{f-ic\log(f)}(c\log(f)-if)(\cos(d)+i\sin(d))\operatorname{Erfi}\left(\frac{\sqrt[4]{-1}(-2icx\log(f)+e+2fx)}{2\sqrt{f-ic\log(f)}}\right)-(f-ic\log(f))\sqrt{f+i\right.}{4\left(c^2\log^2(f)+f^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*E^(e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*(-(E^(((I/2)*e^2*f)
/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Si
n[d])) + Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Sqrt[f - I*c*Log[f]]*((-I)*f + c*Log[f])*(Cos[d] + I*Sin[d])))/(4*
(f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.09, size = 167, normalized size = 0.9

$$\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{4id\ln(f)c+4df-e^2}{4c\ln(f)-4if}}\mathrm{Erf}\left(x\sqrt{if-c\ln(f)}+\frac{i}{2}e\frac{1}{\sqrt{if-c\ln(f)}}\right)\frac{1}{\sqrt{if-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{\frac{4id\ln(f)c-4df+e^2}{4if+4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d),x)
```

```
[Out] 1/4*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*
ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-1/4*Pi^
(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(
1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.548367, size = 765, normalized size = 4.18

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-if\right)\sqrt{-c\log\left(f\right)-if}\,\operatorname{erf}\left(\frac{\left(2c^2x\log\left(f\right)^2+2f^2x+ice\log\left(f\right)+ef\right)\sqrt{-c\log\left(f\right)-if}}{2\left(c^2\log\left(f\right)^2+f^2\right)}\right)e^{\left(\frac{4ac^2\log\left(f\right)^3+4ic^2d\log\left(f\right)^2-ie^2f+4idf^2+\left(ce^2+4a\right.}{4\left(c^2\log\left(f\right)^2+f^2\right)}\right.}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(
f)^2 + 2*f^2*x + I*c*e*log(f) + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 +
f^2))*e^(1/4*(4*a*c^2*log(f)^3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 +
 (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f
)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x - I*c*e*log(f)
+ e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^3
 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^
2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d), x)
```

3.123 $\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\operatorname{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\operatorname{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.3583, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4473, 2204, 2287, 2234, 2205}

$$\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{-c\log(f)+2if}-2id}\operatorname{Erf}\left(\frac{x(-c\log(f)+2if)+ie}{\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{\frac{e^2}{c\log(f)+2if}+2id}\operatorname{Erfi}\left(\frac{x(c\log(f)+2if)+ie}{\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\operatorname{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```


```
[Out] (f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(
(-2*I)*d - e^2/((2*I)*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + x*((2*I)*f - c
*Log[f]))/Sqrt[(2*I)*f - c*Log[f]]])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*
I)*d + e^2/((2*I)*f + c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + x*((2*I)*f + c*Lo
g[f]))/Sqrt[(2*I)*f + c*Log[f]]])/(8*Sqrt[(2*I)*f + c*Log[f]])
```


Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+cx^2}\right)dx$$

$$=\frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+cx^2}\,dx+\frac{1}{2}\int f^{a+cx^2}\,dx$$

$$=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int\exp\left(-2id-2iex+a\log(f)-x^2(2if-c\log(f))\right)d$$

$$=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\right)\int\exp\left(\frac{(-2ie+2x(-2if+c\log(f}{4(-2if+c\log(f))}\right.$$

$$=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2id-\frac{e^2}{2if-c\log(f)}}f^a\sqrt{\pi}\text{erf}\left(\frac{ie+x(2if-c\log(f))}{\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id+\frac{e^2}{2if+c\log(f}}}{8}$$

Mathematica [A] time = 2.29754, size = 252, normalized size = 1.19

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2\text{Erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}\left(\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\sin(2d)-i\cos(2d))e^{\frac{e^2}{c\log(f)+2if}}\text{Erfi}\left(\frac{\sqrt[4]{-1}(-icx\log(f)}{\sqrt{2f-ic\log}}\right.\right.}{c^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + ((
-1)^(1/4)*(-(E^(e^2/((-2*I)*f + c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*
c*x*Log[f]))/Sqrt[2*f + I*c*Log[f]]]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[
f]]*(Cos[2*d] - I*Sin[2*d])) + E^(e^2/((2*I)*f + c*Log[f]))*Erfi[((-1)^(1/4
)*(e + 2*f*x - I*c*x*Log[f]))/Sqrt[2*f - I*c*Log[f]]]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A] time = 0.138, size = 191, normalized size = 0.9

$$\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{2id\ln(f)c+4df-e^{2}}{-2if+c\ln(f)}}\mathrm{Erf}\left(x\sqrt{2if-c\ln(f)}+ie\frac{1}{\sqrt{2if-c\ln(f)}}\right)\frac{1}{\sqrt{2if-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{\frac{2id\ln(f)c-4df+e^{2}}{2if+c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x)
```

```
[Out] 1/8*Pi^(1/2)*f^a*exp(-(2*I*d*ln(f)*c+4*d*f-e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*
ln(f))^(1/2)*erf(x*(2*I*f-c*ln(f))^(1/2)+I*e/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^
(1/2)*f^a*exp((2*I*d*ln(f)*c-4*d*f+e^2)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(
1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+I*e/(-c*ln(f)-2*I*f)^(1/2))+1/4*f^a*Pi^(
1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.560042, size = 927, normalized size = 4.39

$$-\frac{2\sqrt{\pi}\left(c^2\log\left(f\right)^2+4f^2\right)\sqrt{-c\log\left(f\right)}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-2if}\operatorname{erf}\left(\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*f^a*erf(sqrt(-c*log
(f))*x) + sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*
erf((c^2*x*log(f)^2 + 4*f^2*x + I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) - 2*I*
f)/(c^2*log(f)^2 + 4*f^2))*e^((a*c^2*log(f)^3 + 2*I*c^2*d*log(f)^2 - 2*I*e^
2*f + 8*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + sqrt(
pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf((c^2*x*log(
f)^2 + 4*f^2*x - I*c*e*log(f) + 2*e*f)*sqrt(-c*log(f) + 2*I*f)/(c^2*log(f)^
2 + 4*f^2))*e^((a*c^2*log(f)^3 - 2*I*c^2*d*log(f)^2 + 2*I*e^2*f - 8*I*d*f^2
 + (c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)))/(c^3*log(f)^3 + 4*c*f
^2*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+cx^2}\cos^2\left(d+ex+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*cos(d + e*x + f*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^2, x)
```

3.124 $\int f^{a+cx^2} \cos^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=369

$$\frac{\sqrt{\pi} f^a \exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi} f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{3\sqrt{\pi} f^a e^{\frac{\cdots}{4c\log\cdots}}\cdots}{\cdots}$$

```
[Out] (3*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f
 - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*Log[f]]) + (E^((-
3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e + 2*x*
((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*L
og[f]]) + (3*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e +
2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
+ (E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*
e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)
*f + c*Log[f]])
```

Rubi [A] time = 0.582931, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a \exp\left(-\frac{9e^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{2x(-c\log(f)+3if)+3ie}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{3\sqrt{\pi} f^a e^{-\frac{e^2}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{3\sqrt{\pi} f^a e^{\frac{\cdots}{4c\log\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - e^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I*e + 2*x*(I*f
 - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*Log[f]]) + (E^((-
3*I)*d - (9*e^2)/(4*((3*I)*f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e + 2*x*
((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*L
og[f]]) + (3*E^(I*d + e^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e +
2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
+ (E^((3*I)*d + (9*e^2)/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[((3*I)*
e + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)
*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+cx^2}\cos^3\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3i\left(d+ex+fx^2\right)}f^{a+cx^2}+\frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+cx^2}+\frac{3}{8}\right.$$

$$= \frac{1}{8}\int e^{-3i\left(d+ex+fx^2\right)}f^{a+cx^2}dx+\frac{1}{8}\int\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+cx^2}$$

$$= \frac{1}{8}\int\exp\left(-3id-3iex+a\log(f)-x^2(3if-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(3id+3iex+a\right.$$

$$= \frac{1}{8}\left(3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\right)\int\exp\left(\frac{(-ie+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f\right.$$

$$= \frac{3e^{-id-\frac{e^2}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{e^{-3id-\frac{9e^2}{4(3if-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3ie+2x(3if-c\log(}{2\sqrt{3if-c\log(f)}}\right.}{16\sqrt{3if-c\log(f)}}$$

Mathematica [B] time = 7.01844, size = 2997, normalized size = 8.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-27*(-1)^(3/4)*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*
I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e
^2)/(f - I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2
*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log
[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) - (3*(-1)^(3/4)*c^2*f*Cos[d]*Erfi[((
-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*c^
3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*L
og[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]])/E^(((I/4)*e^2)/(f - I*c*Log[f])) -
(3*(-1)^(3/4)*f^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]
))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(
3*f - I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f
*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*L
og[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) - (3*(-1)^(3/4)*c^2*f*Cos[3*
d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log
[f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f
])) + ((-1)^(1/4)*c^3*Cos[3*d]*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]])/E^((((9
*I)/4)*e^2)/(3*f - I*c*Log[f])) - 27*(-1)^(1/4)*E^(((I/4)*e^2)/(f + I*c*Log
[f]))*f^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f
 + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]] + 27*(-1)^(3/4)*c*E^(((I/4)*e^2)/(f +
 I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(
2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]] - 3*(-1)^(1/4)*c^2*E^(
((I/4)*e^2)/(f + I*c*Log[f]))*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]] + 3*(-
1)^(3/4)*c^3*E^(((I/4)*e^2)/(f + I*c*Log[f]))*Cos[d]*Erfi[((-1)^(3/4)*(e +
2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*
Log[f]] - 3*(-1)^(1/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f^3*Cos[3*d]*
Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]
])]*Sqrt[3*f + I*c*Log[f]] + (-1)^(3/4)*c*E^((((9*I)/4)*e^2)/(3*f + I*c*Log
[f]))*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sq
rt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]] - 3*(-1)^(1/4)*c^2*E^(
(((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*
x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*
Log[f]] + (-1)^(3/4)*c^3*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*Cos[3*d]*Er
fi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
```

```
]*Log[f]^3*Sqrt[3*f + I*c*Log[f]] + (27*(-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(e
+ 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]
*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Erfi[((-1)
^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqr
t[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])) + (3*(-1)^(1/4)*
c^2*f*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[
f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/(f - I*c*Log[f])
) + (3*(-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(e + 2*f*x - (2*I)*c*x*Log[f]))/(2*S
qrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]]*Sin[d])/E^(((I/4)*e^2)/
(f - I*c*Log[f])) + 27*(-1)^(3/4)*E^(((I/4)*e^2)/(f + I*c*Log[f]))*f^3*Erfi
[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt
[f + I*c*Log[f]]*Sin[d] + 27*(-1)^(1/4)*c*E^(((I/4)*e^2)/(f + I*c*Log[f]))*
f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]
])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^2*E^(((I/4)*e^2)/(f
 + I*c*Log[f]))*f*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[
f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^3*E
^(((I/4)*e^2)/(f + I*c*Log[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + (2*I)*c*x*Log
[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d] + (3*(
-1)^(1/4)*f^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*
f - I*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f
- I*c*Log[f])) + ((-1)^(3/4)*c*f^2*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*
x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*
d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f])) + (3*(-1)^(1/4)*c^2*f*Erfi[((-1)
^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/4)*e^2)/(3*f - I*c*Log[f]))
+ ((-1)^(3/4)*c^3*Erfi[((-1)^(1/4)*(3*e + 6*f*x - (2*I)*c*x*Log[f]))/(2*Sqr
t[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d])/E^((((9*I)/
4)*e^2)/(3*f - I*c*Log[f])) + 3*(-1)^(3/4)*E^((((9*I)/4)*e^2)/(3*f + I*c*Lo
g[f]))*f^3*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f +
 I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d] + (-1)^(1/4)*c*E^((((9*I)/4)
*e^2)/(3*f + I*c*Log[f]))*f^2*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log
[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d] +
3*(-1)^(3/4)*c^2*E^((((9*I)/4)*e^2)/(3*f + I*c*Log[f]))*f*Erfi[((-1)^(3/4)*
(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt
[3*f + I*c*Log[f]]*Sin[3*d] + (-1)^(1/4)*c^3*E^((((9*I)/4)*e^2)/(3*f + I*c*
Log[f]))*Erfi[((-1)^(3/4)*(3*e + 6*f*x + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I
*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d]))/(16*(f - I*c*Log[f]
)*(3*f - I*c*Log[f])*(f + I*c*Log[f])*(3*f + I*c*Log[f]))
```

Maple [A] time = 0.31, size = 334, normalized size = 0.9

$$\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{12\,id\ln\left(f\right)c+36\,df-9\,e^{2}}{4\,c\ln\left(f\right)-12\,if}}\,\mathrm{Erf}\left(x\sqrt{3\,if-c\ln\left(f\right)}+\frac{3\,i}{2}e\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}+\frac{3\,f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{4\,id\ln\left(f\right)c+4\,df-e^{2}}{4\,c\ln\left(f\right)-4\,if}}\,\mathrm{Erf}\Bigg(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x)
```

```
[Out] 1/16*Pi^(1/2)*f^a*exp(-3/4*(4*I*d*ln(f)*c+12*d*f-3*e^2)/(-3*I*f+c*ln(f)))/(
3*I*f-c*ln(f))^(1/2)*erf(x*(3*I*f-c*ln(f))^(1/2)+3/2*I*e/(3*I*f-c*ln(f))^(1
/2))+3/16*Pi^(1/2)*f^a*exp(-1/4*(4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(
I*f-c*ln(f))^(1/2)*erf(x*(I*f-c*ln(f))^(1/2)+1/2*I*e/(I*f-c*ln(f))^(1/2))-3
/16*Pi^(1/2)*f^a*exp(1/4*(4*I*d*ln(f)*c-4*d*f+e^2)/(I*f+c*ln(f)))/(-c*ln(f)
-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*I*e/(-c*ln(f)-I*f)^(1/2))-1/16*
Pi^(1/2)*f^a*exp(3/4*(4*I*d*ln(f)*c-12*d*f+3*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f
)-3*I*f)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+3/2*I*e/(-c*ln(f)-3*I*f)^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.662307, size = 1839, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 18*f^2*x + 3*I*c*e*lo
```

```
g(f) + 9*e*f)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c
^2*log(f)^3 + 12*I*c^2*d*log(f)^2 - 27*I*e^2*f + 108*I*d*f^2 + 9*(c*e^2 + 4
*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(2*c^2
*x*log(f)^2 + 18*f^2*x - 3*I*c*e*log(f) + 9*e*f)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(4*a*c^2*log(f)^3 - 12*I*c^2*d*log(f)^2 + 27*I
*e^2*f - 108*I*d*f^2 + 9*(c*e^2 + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^2))
+ sqrt(pi)*(3*c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + 27*c*f^2*log(f) - 27*I*f^
3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*f^2*x + I*c*e*log(f)
 + e*f)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*c^2*log(f)^
3 + 4*I*c^2*d*log(f)^2 - I*e^2*f + 4*I*d*f^2 + (c*e^2 + 4*a*f^2)*log(f))/(c
^2*log(f)^2 + f^2)) + sqrt(pi)*(3*c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 + 27*c*
f^2*log(f) + 27*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*c^2*x*log(f)^2 + 2*
f^2*x - I*c*e*log(f) + e*f)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(
1/4*(4*a*c^2*log(f)^3 - 4*I*c^2*d*log(f)^2 + I*e^2*f - 4*I*d*f^2 + (c*e^2 +
 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^
2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+a}\cos\left(fx^2+ex+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + a)*cos(f*x^2 + e*x + d)^3, x)
```

3.125 $\int f^{a+bx+cx^2} \cos(d+ex)\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{\pi} f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id} \operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id} \operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -(E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b*L
og[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) +
 (E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[
f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.258078, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4473, 2287, 2234, 2204}

$$\frac{\sqrt{\pi} f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+id} \operatorname{Erfi}\left(\frac{b\log(f)+2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id} \operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] -(E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b*L
og[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) +
 (E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[
f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cos(d+ex)\,dx &= \int\left(\frac{1}{2}e^{-id-iex}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-id-iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+iex}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int \exp\left(-id+a\log(f)+cx^2\log(f)-x(ie-b\log(f))\right)dx+\frac{1}{2}\int \exp\left(id+a\log(f)+cx^2\right.\\ &= \frac{1}{2}\left(e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{2}\left(e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right)\int \mathrm{ex}\\ &= -\frac{e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ie+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right.}{4\sqrt{c}\sqrt{\log(f)}}\end{aligned}$$

Mathematica [A] time = 0.327174, size = 151, normalized size = 0.88

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{\frac{e(e-2ib\log(f))}{4c\log(f)}}\left(e^{\frac{ibe}{c}}(\cos(d)-i\sin(d))\mathrm{Erfi}\left(\frac{\log(f)(b+2cx)-ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(d)+i\sin(d))\mathrm{Erfi}\left(\frac{\log(f)(b+2cx)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x],x]
```

```
[Out] (E^((e*(e - (2*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
I*b*e)/c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos
[d] - I*Sin[d]) + Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]
*(Cos[d] + I*Sin[d])))/(4*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.07, size = 168, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-2i\ln(f)be+4id\ln(f)c-e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-ie}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+2i\ln(f)b}{4c\ln(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c-e^2)/ln
(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-I*e)/(-c*ln(f)
)^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c
-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(I*e+b*ln(f))/(
-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.496283, size = 483, normalized size = 2.81

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)+ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2-e^2-(4icd-2ibe)\log(f)}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)-ie)}{2c\log(f)}\right.}{4c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(-c*l
og(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 - (4*I*c*d - 2*I*b
*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log
(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^
2 - (-4*I*c*d + 2*I*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\cos{(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos{(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d), x)
```

3.126 $\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.281563, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4473, 2234, 2204, 2287}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(2e+ib\log(f))^2}{4c\log(f)}-2id}\text{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2id-\frac{(b\log(f)+2ie)^2}{4c\log(f)}}\text{Erfi}\left(\frac{b\log(f)+2cx\log(f)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (2*e + I*b*Log[f])^2/(4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[((2*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[
f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^((2*I)*d - ((2*I)*e + b*Log[f])^2/(4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos^2(d+ex)\,dx = \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex}f^{a+bx+cx^2}\right)dx$$

$$= \frac{1}{4}\int e^{-2id-2iex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx$$

$$= \frac{1}{4}\int \exp\left(-2id+a\log(f)+cx^2\log(f)-x(2ie-b\log(f))\right)dx+\frac{1}{4}\int \exp\left(2id+a\log(f)\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-2ie+b\log}{4}\right.$$

$$= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\exp\left(-2id+\frac{(2e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\text{erfi}\left(\frac{2ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+$$

Mathematica [A] time = 0.638655, size = 204, normalized size = 0.88

$$\frac{\sqrt{\pi}e^{-\frac{ibe}{c}}f^{a-\frac{b^2}{4c}}\left((\cos(2d)+i\sin(2d))e^{\frac{e^2}{c\log(f)}}\text{Erfi}\left(\frac{\log(f)(b+2cx)+2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+(\cos(2d)-i\sin(2d))e^{\frac{e(e+2ib\log(f))}{c\log(f)}}\text{Erfi}\left(\frac{\log(f)(b+2cx)-2ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((I*b*e)/c)*Erfi[((b + 2*c*x)*Sqrt[Log[f]]
)/(2*Sqrt[c])] + E^((e*(e + (2*I)*b*Log[f]))/(c*Log[f]))*Erfi[((-2*I)*e + (
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[2*d] - I*Sin[2*d]) + E^(e
^2/(c*Log[f]))*Erfi[((2*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])
]*(Cos[2*d] + I*Sin[2*d])))/(8*Sqrt[c]*E^((I*b*e)/c)*Sqrt[Log[f]])
```

Maple [A] time = 0.106, size = 217, normalized size = 0.9

$$-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4i\ln(f)be+8id\ln(f)c-4e^2}{4c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}x+\frac{b\ln(f)-2ie}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2+4i\ln}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c-4*e^2)/
ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*I*e)/(-c*l
n(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(
f)*c-4*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*I*e+b*
ln(f))/(-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*e
rf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.507513, size = 618, normalized size = 2.68

$$-\frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)+2ie)\sqrt{-c\log(f)}}{2c\log(f)}\right)e^{\left(-\frac{(b^2-4ac)\log(f)^2-4e^2-(8icd-4ibe)\log(f)}{4c\log(f)}\right)}+\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{((2cx+b)\log(f)-2}{2c\log(}\right.}{8c\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + 2*I*e)*sqrt(-c
*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*e^2 - (8*I*c*d - 4
*I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)
*log(f) - 2*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*e^2 - (-8*I*c*d + 4*I*b*e)*log(f))/(c*log(f))) + 2*sqrt(pi)*sqrt(-c*l
og(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c*l
og(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos(ex+d)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^2, x)
```

3.127 $\int f^{a+bx+cx^2} \cos^3(d+ex)\,dx$

Optimal. Leaf size=346

$$-\frac{3\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+}}{16}$$

```
[Out] (-3*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b
*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]
) - (E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3
*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (3*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*S
qrt[c]*Sqrt[Log[f]])
```

Rubi [A] time = 0.415833, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4473, 2287, 2234, 2204}

$$-\frac{3\sqrt{\pi}f^a e^{\frac{(e+ib\log(f))^2}{4c\log(f)}-id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{\frac{(3e+ib\log(f))^2}{4c\log(f)}-3id}\operatorname{Erfi}\left(\frac{-b\log(f)-2cx\log(f)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-ib\log(f))^2}{4c\log(f)}+}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (-3*E^((-I)*d + (e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e - b
*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]
) - (E^((-3*I)*d + (3*e + I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((3
*I)*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (3*E^(I*d + (e - I*b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi
[(I*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqr
t[Log[f]]) + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[((3*I)*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*S
qrt[c]*Sqrt[Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos^3(d+ex)\,dx = \int\left(\frac{3}{8}e^{-id-iex}f^{a+bx+cx^2}+\frac{3}{8}e^{id+iex}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3iex}f^{a+bx+cx^2}+\frac{1}{8}e^{3id+3iex}f^{a+bx+cx^2}\right)dx$$

$$= \frac{1}{8}\int e^{-3id-3iex}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-id-iex}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{id}$$

$$= \frac{1}{8}\int \exp\left(-3id+a\log(f)+cx^2\log(f)-x(3ie-b\log(f))\right)dx+\frac{1}{8}\int \exp\left(3id+a\log(f)\right.$$

$$= \frac{1}{8}\left(3e^{id+\frac{(e-ib\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(ie+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx+\frac{1}{8}\left(3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\right)\int$$

$$= -\frac{3e^{-id+\frac{(e+ib\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{ie-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\exp\left(-3id+\frac{(3e+ib\log(f))^2}{4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3ie}{}\right.}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] time = 0.966964, size = 386, normalized size = 1.12

$$\sqrt{\pi}f^{a-\frac{b^2}{4c}}e^{\frac{e(e-6ib\log(f))}{4c\log(f)}}\left(i\sin(3d)e^{\frac{2e^2}{c\log(f)}}\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)+\cos(3d)e^{\frac{2e^2}{c\log(f)}}\operatorname{Erfi}\left(\frac{\log(f)(b+2cx)+3ie}{2\sqrt{c}\sqrt{\log(f)}}\right)-i\sin(3d)e^{\frac{e(2e+3ib\log(f))}{c\log(f)}}\mathrm{E}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x]^3,x]
```

```
[Out] (E^((e*(e - (6*I)*b*Log[f]))/(4*c*Log[f]))*f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((
e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))*Cos[3*d]*Erfi[((-3*I)*e + (b + 2*c*x)
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + E^((2*e^2)/(c*Log[f]))*Cos[3*d]*Erfi[(
(3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^(((2*I)*b*e)/
c)*Erfi[((-I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cos[d] - I
*Sin[d]) + 3*E^((I*b*e)/c)*Erfi[(I*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[
Log[f]])]*(Cos[d] + I*Sin[d]) - I*E^((e*(2*e + (3*I)*b*Log[f]))/(c*Log[f]))
*Erfi[((-3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*Sin[3*d] +
I*E^((2*e^2)/(c*Log[f]))*Erfi[((3*I)*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqr
t[Log[f]])]*Sin[3*d]))/(16*Sqrt[c]*Sqrt[Log[f]])
```

Maple [A] time = 0.237, size = 334, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2 b^2-6\,i\ln(f)be+12\,id\ln(f)c-9\,e^2}{4\,c\ln(f)}}\mathrm{Erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-3\,ie}{2}\frac{1}{\sqrt{-c\ln(f)}}\right)\frac{1}{\sqrt{-c\ln(f)}}-\frac{3\,f^a\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^2 b^2-2}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c-9*e^2
)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*I*e)/(-c
*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*
ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)
-I*e)/(-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b
*e-4*I*d*ln(f)*c-e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2
*(I*e+b*ln(f))/(-c*ln(f))^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+6*
I*ln(f)*b*e-12*I*d*ln(f)*c-9*e^2)/ln(f)/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))
^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [A] time = 0.521183, size = 964, normalized size = 2.79

$$-\frac{3\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)+i\,e\right)\sqrt{-c\log\left(f\right)}}{2\,c\log\left(f\right)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log\left(f\right)^2-e^2-\left(4i\,cd-2i\,be\right)\log\left(f\right)}{4\,c\log\left(f\right)}\right)}+3\sqrt{\pi}\sqrt{-c\log\left(f\right)}\operatorname{erf}\left(\frac{\left(\left(2\,cx+b\right)\log\left(f\right)-i\,\cdots}{2\,c\log\left(\cdots\right.}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) + I*e)*sqrt(-
c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - e^2 - (4*I*c*d - 2*
I*b*e)*log(f))/(c*log(f))) + 3*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b
)*log(f) - I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2
 - e^2 - (-4*I*c*d + 2*I*b*e)*log(f))/(c*log(f))) + sqrt(pi)*sqrt(-c*log(f)
)*erf(1/2*((2*c*x + b)*log(f) + 3*I*e)*sqrt(-c*log(f))/(c*log(f)))*e^(-1/4*
((b^2 - 4*a*c)*log(f)^2 - 9*e^2 - (12*I*c*d - 6*I*b*e)*log(f))/(c*log(f)))
+ sqrt(pi)*sqrt(-c*log(f))*erf(1/2*((2*c*x + b)*log(f) - 3*I*e)*sqrt(-c*log
(f))/(c*log(f)))*e^(-1/4*((b^2 - 4*a*c)*log(f)^2 - 9*e^2 - (-12*I*c*d + 6*I
*b*e)*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos(ex+d)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x + d)^3, x)
```

3.128 $\int f^{a+bx+cx^2} \cos\left(d+fx^2\right) dx$

Optimal. Leaf size=189

$$\frac{\sqrt{\pi} f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}$$

```
[Out] -(E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Lo
g[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log
[f]]) + (E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[
(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f +
c*Log[f]])
```

Rubi [A] time = 0.265494, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a e^{id-\frac{b^2\log^2(f)}{4c\log(f)+4if}}\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)}{2\sqrt{c\log(f)+if}}\right)}{4\sqrt{c\log(f)+if}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}-id}\text{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -(E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Lo
g[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*f - c*Log
[f]]) + (E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[
(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(4*Sqrt[I*f +
c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{2}\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+ifx^2}f^{a+bx+cx^2}\,dx\\
&= \frac{1}{2}\int \exp\left(-id+a\log(f)+bx\log(f)-x^2(if-c\log(f))\right)dx+\frac{1}{2}\int \exp\left(id+a\log(f)\right.\\
&= \frac{1}{2}\left(e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx+\frac{1}{2}\left(e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f\right.\\
&= -\frac{e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{e^{id-\frac{b^2\log^2(f)}{4if+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(if+c}{2\sqrt{if+c\log}}\right.}{4\sqrt{if+c\log(f)}}
\end{aligned}$$

Mathematica [A] time = 0.982861, size = 231, normalized size = 1.22

$$-\frac{(-1)^{3/4}\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+4if}}\left(\sqrt{f-ic\log(f)}(f+ic\log(f))(\cos(d)+i\sin(d))e^{\frac{ib^2f\log^2(f)}{2\left(c^2\log^2(f)+f^2\right)}}\mathrm{Erfi}\left(\frac{\sqrt[4]{-1}(2fx-i\log(f)(b+2cx))}{2\sqrt{f-ic\log(f)}}\right)+(f\right.}{4\left(c^2\log^2(f)+f^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2],x]
```

```
[Out] -((-1)^(3/4)*E^((b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*(Erfi[(
(-1)^(3/4)*(2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*(f - I
*c*Log[f])*Sqrt[f + I*c*Log[f]]*((-I)*Cos[d] - Sin[d]) + E^(((I/2)*b^2*f*Lo
g[f]^2)/(f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(2*f*x - I*(b + 2*c*x)*Log[f
]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*(Cos[d]
 + I*Sin[d])))/(4*(f^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.089, size = 178, normalized size = 0.9

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+4id\ln(f)c+4df}{4c\ln(f)-4if}}\mathrm{Erf}\left(-x\sqrt{if-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{if-c\ln(f)}}\right)\frac{1}{\sqrt{if-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2-4id\ln(f)c+4}{4if+4c\ln(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d*f)/(-I*f+c*ln(f))
)/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(I*f-c*ln(f))^
(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f)*c+4*d*f)/(I*f+c*l
n(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(
f)-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.547952, size = 778, normalized size = 4.12

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-if\right)\sqrt{-c\log\left(f\right)-if}\,\mathrm{erf}\left(\frac{\left(2f^2x-ibf\log\left(f\right)+\left(2c^2x+bc\right)\log\left(f\right)^2\right)\sqrt{-c\log\left(f\right)-if}}{2\left(c^2\log\left(f\right)^2+f^2\right)}\right)e^{\left(\frac{4af^2\log\left(f\right)-\left(b^2c-4ac^2\right)\log\left(f\right)^3+4idf^2+\left(4ic^2\cdots\right.}{4\left(c^2\log\left(f\right)^2+f^2\right)}\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*
b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2
+ f^2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (
4*I*c^2*d + I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) +
 I*f)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*log(f) + (2*c^2*x + b*
c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*lo
g(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I*c^2*d - I*b^2*f)*log(
f)^2)/(c^2*log(f)^2 + f^2)))/(c^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\cos\left(d+fx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d), x)
```

3.129 $\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)\,dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\sqrt{\cdots}\right.}{4\sqrt{c}\sqrt{\log\cdots}}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.405915, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4473, 2234, 2204, 2287, 2205}

$$-\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}-2id}\operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+2if)}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a e^{2id-\frac{b^2\log^2(f)}{4c\log(f)+8if}}\operatorname{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+2if)}{2\sqrt{c\log(f)+2if}}\right)}{8\sqrt{c\log(f)+2if}}+\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\sqrt{\cdots}\right.}{4\sqrt{c}\sqrt{\log\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) - (E^((-2*I)*d + (b^2*Log[f]^2)/((8*I)*f - 4*c*Log[
f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*((2*I)*f - c*Log[f]))/(2*Sqrt[(2*I)*f
 - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d - (b^2*Log[f]^2)
/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((2*I)*f + c*Log
[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqrt[(2*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos^2\left(d+fx^2\right)dx &= \int \left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2ifx^2}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2id-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)+bx\log(f)-x^2(2if-c\log(f))\right)dx+\frac{1}{4}\int \exp\left(2id+a\log(\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-2if+c\log}{4(-2if+c\log(f))}\right.\\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2id+\frac{b^2\log^2(f)}{8if-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(2if-c\log(f))}{2\sqrt{2if-c\log(f)}}\right)}{8\sqrt{2if-c\log(f)}}+\frac{e^{2id-\frac{b}{8i}}}{}
\end{aligned}$$

Mathematica [A] time = 3.0957, size = 301, normalized size = 1.23

$$\frac{1}{8}\sqrt{\pi}f^a\left(\frac{2f^{-\frac{b^2}{4c}}\text{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt[4]{-1}e^{\frac{b^2\log^2(f)}{-4c\log(f)+8if}}\left(\sqrt{2f-ic\log(f)}(2f+ic\log(f))(\sin(2d)-i\cos(2d))e^{\frac{ib^2f\log^2(f)}{c^2\log^2(f)+4f^2}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f^
(b^2/(4*c))*Sqrt[Log[f]]) + ((-1)^(1/4)*E^((b^2*Log[f]^2)/((8*I)*f - 4*c*Lo
g[f]))*(-(Erfi[((-1)^(3/4)*(4*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f + I*
c*Log[f]])]*(2*f - I*c*Log[f])*Sqrt[2*f + I*c*Log[f]]*(Cos[2*d] - I*Sin[2*d
])) + E^((I*b^2*f*Log[f]^2)/(4*f^2 + c^2*Log[f]^2))*Erfi[((-1)^(1/4)*(4*f*x
 - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Sqrt[2*f - I*c*Log[f]
]*(2*f + I*c*Log[f])*((-I)*Cos[2*d] + Sin[2*d])))/(4*f^2 + c^2*Log[f]^2)))/
8
```

Maple [A] time = 0.137, size = 227, normalized size = 0.9

$$-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2+8id\ln(f)c+16df}{4c\ln(f)-8if}}\mathrm{Erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{b\ln(f)}{2}\frac{1}{\sqrt{2if-c\ln(f)}}\right)\frac{1}{\sqrt{2if-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^2b^2-8}{8if+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+8*I*d*ln(f)*c+16*d*f)/(-2*I*f+c*ln(
f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*I*f-
c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-8*I*d*ln(f)*c+16*d*f
)/(2*I*f+c*ln(f)))/(-c*ln(f)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2
*ln(f)*b/(-c*ln(f)-2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))
^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.560231, size = 1026, normalized size = 4.19

$$-\frac{\sqrt{\pi}\left(c^2\log\left(f\right)^2 - 2icf\log\left(f\right)\right)\sqrt{-c\log\left(f\right) - 2if}\,\mathrm{erf}\left(\frac{\left(8f^2x - 2ibf\log(f) + \left(2c^2x + bc\right)\log(f)^2\right)\sqrt{-c\log(f) - 2if}}{2\left(c^2\log(f)^2 + 4f^2\right)}\right)e^{\left(\frac{16af^2\log(f) - \left(b^2c - 4ac^2\right)\cdots}{4\left(\cdots\right.}\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x - 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) -
2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^2)*
log(f)^3 + 32*I*d*f^2 + (8*I*c^2*d + 2*I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 4
*f^2)) + sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*e
rf(1/2*(8*f^2*x + 2*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f)
 + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(1/4*(16*a*f^2*log(f) - (b^2*c - 4*a*c^
2)*log(f)^3 - 32*I*d*f^2 + (-8*I*c^2*d - 2*I*b^2*f)*log(f)^2)/(c^2*log(f)^2
 + 4*f^2)) + 2*sqrt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c
*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2
*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos\left(fx^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^2, x)
```

3.130 $\int f^{a+bx+cx^2} \cos^3\left(d+fx^2\right) dx$

Optimal. Leaf size=378

$$-\frac{3\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{-4c\log(f)+4if}-id} \operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id} \operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{\sqrt{\pi} f^a \exp\left(3id - \right.}{}$$

```
[Out] (-3*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*
Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*
Log[f]]) - (E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f]))*f^a*Sqrt[
Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])]
)/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
 c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d - (b^2*Log[f]^2)/(4*(
(3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((3*I)*f + c*Log[f]
))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rubi [A] time = 0.521957, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4473, 2287, 2234, 2205, 2204}

$$-\frac{3\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{-4c\log(f)+4if}-id} \operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+if)}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{-4c\log(f)+12if}-3id} \operatorname{Erf}\left(\frac{b\log(f)-2x(-c\log(f)+3if)}{2\sqrt{-c\log(f)+3if}}\right)}{16\sqrt{-c\log(f)+3if}}+\frac{\sqrt{\pi} f^a \exp\left(3id - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (-3*E^((-I)*d + (b^2*Log[f]^2)/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*
Log[f] - 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt[I*f - c*
Log[f]]) - (E^((-3*I)*d + (b^2*Log[f]^2)/((12*I)*f - 4*c*Log[f]))*f^a*Sqrt[
Pi]*Erf[(b*Log[f] - 2*x*((3*I)*f - c*Log[f]))/(2*Sqrt[(3*I)*f - c*Log[f]])]
)/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d - (b^2*Log[f]^2)/((4*I)*f + 4*c
*Log[f]))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f +
 c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]]) + (E^((3*I)*d - (b^2*Log[f]^2)/(4*(
(3*I)*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*((3*I)*f + c*Log[f]
))/(2*Sqrt[(3*I)*f + c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos^3\left(d+fx^2\right)dx = \int \left(\frac{3}{8}e^{-id-ifx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{id+ifx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{-3id-3ifx^2}f^{a+bx+cx^2}+\frac{1}{8}e^{3id+3ifx^2}f^{a+bx\cdots}\right.$$

$$=\frac{1}{8}\int e^{-3id-3ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int e^{3id+3ifx^2}f^{a+bx+cx^2}\,dx+\frac{3}{8}\int e^{-id-ifx^2}f^{a+bx+cx^2}\,dx+\cdots$$

$$=\frac{1}{8}\int \exp\left(-3id+a\log(f)+bx\log(f)-x^2(3if-c\log(f))\right)dx+\frac{1}{8}\int \exp\left(3id+a\log\cdots\right.$$

$$=\frac{1}{8}\left(3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right)dx+\frac{1}{8}\left(e^{-3id+\frac{b^2\log^2\cdots}{12if-4c\log\cdots}}\right.$$

$$=-\frac{3e^{-id+\frac{b^2\log^2(f)}{4if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}-\frac{e^{-3id+\frac{b^2\log^2(f)}{12if-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x\cdots}{2\sqrt{3if\cdots}}\right)}{16\sqrt{3if-c\log(f)}}$$

Mathematica [B] time = 6.97349, size = 3285, normalized size = 8.69

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3
*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f
 - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*(-1)^(1/4)*c*E^(((I/4)*b^2*Log[f
]^2)/(f - I*c*Log[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2
*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*
(-1)^(3/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f*Cos[d]*Erfi[((-1
)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*
Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f
 - I*c*Log[f]))*Cos[d]*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4
)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-1)^(1/4)
*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Sqrt[
3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f])
)*f^2*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2
*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*
E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4)*(6*
f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^2*
Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*
Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))
/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/
4)*f^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*
Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I
*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log
[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]
*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*
Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (
2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/
E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) - (3*(-1)^(1/4)*f^3*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ ((-1)^(3/4)*c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^((
(I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) - (3*(-1)^(1/4)*c^2*f*Cos[3*d]*Erfi
[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log
```

```
[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f])) + ((-1)^(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] +
(2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[
f]])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + 27*(-1)^(1/4)*E^(((I/4)*
b^2*Log[f]^2)/(f - I*c*Log[f]))*f^3*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] -
(2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] +
27*(-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*f^2*Erfi[((-1)^(1
/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[
f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(
f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f])
)/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(
3/4)*c^3*E^(((I/4)*b^2*Log[f]^2)/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*f*x
- I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f
 - I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(2*f*x + I*b*Lo
g[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*Si
n[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (27*(-1)^(1/4)*c*f^2*Erfi
[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f
]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*L
og[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(2*f*x + I*b*Log[f] + (2*I)*
c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d]
)/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f])) + (3*(-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*b^2*Log[f]^2)/(f + I*c*Log[f]
)) + 3*(-1)^(1/4)*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f^3*Erfi[((-1
)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c*E^(((I/4)*b^2*Log[f]^2)/(3
*f - I*c*Log[f]))*f^2*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + 3
*(-1)^(1/4)*c^2*E^(((I/4)*b^2*Log[f]^2)/(3*f - I*c*Log[f]))*f*Erfi[((-1)^(1
/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Lo
g[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c^3*E^(((I/4)*b^2*Log[f
]^2)/(3*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(6*f*x - I*b*Log[f] - (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f - I*c*Log[f]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d
] + (3*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)
*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(6*
f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sq
rt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f]))
+ (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[
f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/
E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^(3
/4)*(6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Lo
g[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*b^2*Log[f]^2)/(3*f + I*c*
Log[f]))))/(16*(f - I*c*Log[f])*(3*f - I*c*Log[f])*(f + I*c*Log[f])*(3*f +
I*c*Log[f]))
```

Maple [A] time = 0.316, size = 354, normalized size = 0.9

$$-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+12\,id\ln(f)c+36\,df}{4\,c\ln(f)-12\,if}}\mathrm{Erf}\left(-x\sqrt{3\,if-c\ln\left(f\right)}+\frac{b\ln\left(f\right)}{2}\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{3\,if-c\ln\left(f\right)}}-\frac{3\,f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}+}{4\,c\ln}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+12*I*d*ln(f)*c+36*d*f)/(-3*I*f+c*l
n(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*I*
f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*d*ln(f)*c+4*d
*f)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*ln(f
)*b/(I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*d*ln(f
)*c+4*d*f)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+
1/2*ln(f)*b/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-1
2*I*d*ln(f)*c+36*d*f)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f)^(1/2)*erf(-(-c*ln(f
)-3*I*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.660477, size = 1852, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x - 3*I*b*f*log(f) + (2*c^2*x + b
*c)*log(f)^2)*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*
f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 + 108*I*d*f^2 + (12*I*c^2*d + 3*I*b
^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(c^3*log(f)^3 + 3*I*c^2*
f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(1/2*(18*f^
2*x + 3*I*b*f*log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + 3*I*f)/(c
^2*log(f)^2 + 9*f^2))*e^(1/4*(36*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3
- 108*I*d*f^2 + (-12*I*c^2*d - 3*I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + 9*f^2))
 + sqrt(pi)*(3*c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + 27*c*f^2*log(f) - 27*I*f
^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x - I*b*f*log(f) + (2*c^2*x + b*c)
*log(f)^2)*sqrt(-c*log(f) - I*f)/(c^2*log(f)^2 + f^2))*e^(1/4*(4*a*f^2*log(
f) - (b^2*c - 4*a*c^2)*log(f)^3 + 4*I*d*f^2 + (4*I*c^2*d + I*b^2*f)*log(f)^
2)/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(3*c^3*log(f)^3 + 3*I*c^2*f*log(f)^2 +
27*c*f^2*log(f) + 27*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2*f^2*x + I*b*f*
log(f) + (2*c^2*x + b*c)*log(f)^2)*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^
2))*e^(1/4*(4*a*f^2*log(f) - (b^2*c - 4*a*c^2)*log(f)^3 - 4*I*d*f^2 + (-4*I
*c^2*d - I*b^2*f)*log(f)^2)/(c^2*log(f)^2 + f^2)))/(c^4*log(f)^4 + 10*c^2*f
^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \cos\left(fx^2+d\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + d)^3, x)
```

3.131 $\int f^{a+bx+cx^2} \cos\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=208

$$\frac{\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)-}{2\sqrt{c\log(f)+if}}\right.}{4\sqrt{c\log(f)+if}}$$

```
[Out] (E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I
*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*
f - c*Log[f]]) + (E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]
])])/(4*Sqrt[I*f + c*Log[f]])
```

Rubi [A] time = 0.394717, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi} f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{4\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(e-ib\log(f))^2}{4c\log(f)+4if}+id\right)\text{Erfi}\left(\frac{b\log(f)+2x(c\log(f)+if)-}{2\sqrt{c\log(f)+if}}\right.}{4\sqrt{c\log(f)+if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] (E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(I
*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(4*Sqrt[I*
f - c*Log[f]]) + (E^(I*d + (e - I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(I*e + b*Log[f] + 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]
])])/(4*Sqrt[I*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int f^{a+bx+cx^2}\cos\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}e^{-id-iex-ifx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{id+iex+ifx^2}f^{a+bx+cx^2}\right)dx\\ &= \frac{1}{2}\int e^{-id-iex-ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{id+iex+ifx^2}f^{a+bx+cx^2}\,dx\\ &= \frac{1}{2}\int\exp\left(-id+a\log(f)-x(ie-b\log(f))-x^2(if-c\log(f))\right)dx+\frac{1}{2}\int\exp\left(id+\right.\\ &= \frac{1}{2}\left(\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f)))^2}{4(-if+c\log(f))}\right.\\ &= \frac{\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{4\sqrt{if-c\log(f)}}+\frac{\exp\left(id+\frac{(e-ib\log(f))^2}{4if+4c\log(f)}\right)\cdots}{4\sqrt{\cdots}}\end{aligned}$$

Mathematica [A] time = 2.12557, size = 348, normalized size = 1.67

$$\frac{\sqrt[4]{-1}\sqrt{\pi}f^{\frac{f(af-be)+ac^2\log^2(f)}{c^2\log^2(f)+f^2}}\exp\left(-\frac{1}{4}i\left(\frac{b^2\log^2(f)}{f+ic\log(f)}+\frac{e^2}{f-ic\log(f)}\right)\right)\left(\sqrt{f-ic\log(f)}(f+ic\log(f))(\sin(d)-i\cos(d))e^{\frac{ib^2f\log^2(f)}{2\left(c^2\log^2(f)+f^2\right)}}f\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2],x]
```

```
[Out] ((-1)^(1/4)*f^((f*(-(b*e) + a*f) + a*c^2*Log[f]^2)/(f^2 + c^2*Log[f]^2))*Sq
rt[Pi]*(-(E^(((I/2)*e^2*f)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f - (2*I)*c*Lo
g[f]))*Erfi[((-1)^(3/4)*(e + 2*f*x + I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f + I*c
*Log[f]])]*(f - I*c*Log[f])*Sqrt[f + I*c*Log[f]]*(Cos[d] - I*Sin[d])) + E^(
((I/2)*b^2*f*Log[f]^2)/(f^2 + c^2*Log[f]^2))*f^((b*e)/(2*f + (2*I)*c*Log[f]
))*Erfi[((-1)^(1/4)*(e + 2*f*x - I*(b + 2*c*x)*Log[f]))/(2*Sqrt[f - I*c*Log
[f]])]*Sqrt[f - I*c*Log[f]]*(f + I*c*Log[f])*((-I)*Cos[d] + Sin[d])))/(4*E^
((I/4)*(e^2/(f - I*c*Log[f]) + (b^2*Log[f]^2)/(f + I*c*Log[f])))*(f^2 + c^2
*Log[f]^2))
```

Maple [A] time = 0.1, size = 214, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-2i\ln(f)be+4id\ln(f)c+4df-e^{2}}{4c\ln(f)-4if}}\operatorname{Erf}\left(-x\sqrt{if-c\ln(f)}+\frac{b\ln(f)-ie}{2}\frac{1}{\sqrt{if-c\ln(f)}}\right)\frac{1}{\sqrt{if-c\ln(f)}}-\frac{f^{a}\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e
^2)/(-I*f+c*ln(f)))/(I*f-c*ln(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*l
n(f)-I*e)/(I*f-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*l
n(f)*b*e-4*I*d*ln(f)*c+4*d*f-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-
(-c*ln(f)-I*f)^(1/2)*x+1/2*(I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.560543, size = 938, normalized size = 4.51

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-if\right)\sqrt{-c\log\left(f\right)-if}\,\operatorname{erf}\left(\frac{\left(2\,f^{2}x+\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+ef+\left(ice-ibf\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-if}}{2\left(c^{2}\log\left(f\right)^{2}+f^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}+ie^{2}f-4i\,df^{2}-\left(\cdots\right.}{\cdots}\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*f)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x + (2
*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) - I*f
)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f - 4*I
*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*
a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c*log(f) + I*f)*sqrt(-c*lo
g(f) + I*f)*erf(1/2*(2*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I
*b*f)*log(f))*sqrt(-c*log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c -
 4*a*c^2)*log(f)^3 - I*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*
f)*log(f)^2 - (c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)))/(c
^2*log(f)^2 + f^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^{2}}\cos\left(d+ex+fx^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^{2}+bx+a}\cos\left(fx^{2}+ex+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d), x)
```

3.132 $\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)\,dx$

Optimal. Leaf size=268

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f}{4c\log(f)+8}\right.}{}$$

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rubi [A] time = 0.460815, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4473, 2234, 2204, 2287, 2205}

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{Erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a\exp\left(-\frac{(2e+ib\log(f))^2}{-4c\log(f)+8if}-2id\right)\operatorname{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+2if)+2ie}{2\sqrt{-c\log(f)+2if}}\right)}{8\sqrt{-c\log(f)+2if}}+\frac{\sqrt{\pi}f^a\exp\left(\frac{(2e-ib\log(f}{4c\log(f)+8}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(
4*Sqrt[c]*Sqrt[Log[f]]) + (E^((-2*I)*d - (2*e + I*b*Log[f])^2/((8*I)*f - 4*
c*Log[f]))*f^a*Sqrt[Pi]*Erf[((2*I)*e - b*Log[f] + 2*x*((2*I)*f - c*Log[f]))
/(2*Sqrt[(2*I)*f - c*Log[f]])])/(8*Sqrt[(2*I)*f - c*Log[f]]) + (E^((2*I)*d
+ (2*e - I*b*Log[f])^2/((8*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[((2*I)*e +
 b*Log[f] + 2*x*((2*I)*f + c*Log[f]))/(2*Sqrt[(2*I)*f + c*Log[f]])])/(8*Sqr
t[(2*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos^2\left(d+ex+fx^2\right)dx &= \int\left(\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{4}\int e^{-2id-2iex-2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2id+2iex+2ifx^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int f^{a+bx+cx^2} \\
&= \frac{1}{4}\int \exp\left(-2id+a\log(f)-x(2ie-b\log(f))-x^2(2if-c\log(f))\right)dx+\frac{1}{4}\int \exp \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\right)\int \exp\left(\frac{(-
\right. \\
&= \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\exp\left(-2id-\frac{(2e+ib\log(f))^2}{8if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2ie-b\log(f)+2x}{2\sqrt{2if-c}}\right.}{8\sqrt{2if-c\log(f)}}
\end{aligned}$$

Mathematica [B] time = 6.73518, size = 1118, normalized size = 4.17

$$f^a\sqrt{\pi}\left(8\sqrt{c}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}f^{2-\frac{b^2}{4c}}+2c^{5/2}\text{Erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{5}{2}}(f)f^{-\frac{b^2}{4c}}+2\sqrt[4]{-1}ce^{\frac{i\left(-4e^2+4ib\log(f)e+b^2\log^2(f)\right)}{4(2f-ic\log(f))}}\text{Erfi}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(8*Sqrt[c]*f^(2 - b^2/(4*c))*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/
(2*Sqrt[c])]*Sqrt[Log[f]] + (2*c^(5/2)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])]*Log[f]^(5/2))/f^(b^2/(4*c)) - 2*(-1)^(3/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Cos[2*d]*Erfi[((-1)^
(1/4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[
f]])]*Log[f]*Sqrt[2*f - I*c*Log[f]] + (-1)^(1/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Cos[2*d]*Erfi[((-1)^(1/
4)*(2*e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]]
)]*Log[f]^2*Sqrt[2*f - I*c*Log[f]] - (2*(-1)^(1/4)*c*f*Cos[2*d]*Erfi[((-1)^
(3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[
f]])]*Log[f]*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(3/4)*c^2*Cos[2*d]*Erfi[((-1)^(
3/4)*(2*e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[2*f + I*c*Log[f]])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f])) + 2*(-1)^(1/4)*c*E^(((I/4)*(-4*e^2 + (
4*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(2*
e + 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log
[f]*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (-1)^(3/4)*c^2*E^(((I/4)*(-4*e^2 + (4
*I)*b*e*Log[f] + b^2*Log[f]^2))/(2*f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(2*e +
 4*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[2*f - I*c*Log[f]])]*Log[f]
^2*Sqrt[2*f - I*c*Log[f]]*Sin[2*d] + (2*(-1)^(3/4)*c*f*Erfi[((-1)^(3/4)*(2*
e + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log
[f]*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f] +
 b^2*Log[f]^2))/(2*f + I*c*Log[f])) + ((-1)^(1/4)*c^2*Erfi[((-1)^(3/4)*(2*e
 + 4*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[2*f + I*c*Log[f]])]*Log[
f]^2*Sqrt[2*f + I*c*Log[f]]*Sin[2*d])/E^(((I/4)*(-4*e^2 - (4*I)*b*e*Log[f]
+ b^2*Log[f]^2))/(2*f + I*c*Log[f]))))/(8*c*Log[f]*(2*f - I*c*Log[f])*(2*f
+ I*c*Log[f]))
```

Maple [A] time = 0.176, size = 263, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{8}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-4i\ln(f)be+8id\ln(f)c+16df-4e^{2}}{4c\ln(f)-8if}}\mathrm{Erf}\left(-x\sqrt{2if-c\ln(f)}+\frac{b\ln(f)-2ie}{2}\frac{1}{\sqrt{2if-c\ln(f)}}\right)\frac{1}{\sqrt{2if-c\ln(f)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x)
```

```
[Out] -1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*ln(f)*b*e+8*I*d*ln(f)*c+16*d*f-
4*e^2)/(-2*I*f+c*ln(f)))/(2*I*f-c*ln(f))^(1/2)*erf(-x*(2*I*f-c*ln(f))^(1/2)
+1/2*(b*ln(f)-2*I*e)/(2*I*f-c*ln(f))^(1/2))-1/8*Pi^(1/2)*f^a*exp(-1/4*(ln(f
)^2*b^2+4*I*ln(f)*b*e-8*I*d*ln(f)*c+16*d*f-4*e^2)/(2*I*f+c*ln(f)))/(-c*ln(f
)-2*I*f)^(1/2)*erf(-(-c*ln(f)-2*I*f)^(1/2)*x+1/2*(2*I*e+b*ln(f))/(-c*ln(f)-
2*I*f)^(1/2))-1/4*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(
f))^(1/2)*x+1/2/(-c*ln(f))^(1/2)*b*ln(f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.575358, size = 1206, normalized size = 4.5

$$-\frac{\sqrt{\pi}\left(c^{2}\log(f)^{2}-2icf\log(f)\right)\sqrt{-c\log(f)-2if}\,\mathrm{erf}\left(\frac{\left(8f^{2}x+\left(2c^{2}x+bc\right)\log(f)^{2}+4ef+\left(2ice-2ibf\right)\log(f)\right)\sqrt{-c\log(f)-2if}}{2\left(c^{2}\log(f)^{2}+4f^{2}\right)}\right)e^{\left(-\frac{(b^{2}c-4}{}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(c^2*log(f)^2 - 2*I*c*f*log(f))*sqrt(-c*log(f) - 2*I*f)*erf(
1/2*(8*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f + (2*I*c*e - 2*I*b*f)*log(f
))*sqrt(-c*log(f) - 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^
2)*log(f)^3 + 8*I*e^2*f - 32*I*d*f^2 - (8*I*c^2*d - 4*I*b*c*e + 2*I*b^2*f)*
log(f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) +
sqrt(pi)*(c^2*log(f)^2 + 2*I*c*f*log(f))*sqrt(-c*log(f) + 2*I*f)*erf(1/2*(8
*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 4*e*f + (-2*I*c*e + 2*I*b*f)*log(f))*sq
rt(-c*log(f) + 2*I*f)/(c^2*log(f)^2 + 4*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*lo
g(f)^3 - 8*I*e^2*f + 32*I*d*f^2 - (-8*I*c^2*d + 4*I*b*c*e - 2*I*b^2*f)*log(
f)^2 - 4*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 4*f^2)) + 2*sq
rt(pi)*(c^2*log(f)^2 + 4*f^2)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*l
og(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log(f)^3 + 4*c*f^2*log(f))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \cos \left(fx^2+ex+d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^2, x)
```

3.133 $\int f^{a+bx+cx^2} \cos^3\left(d+ex+fx^2\right) dx$

Optimal. Leaf size=422

$$\frac{3\sqrt{\pi}f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a \exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\,\text{l}}{2\sqrt{-c\log(}}\right.}{16\sqrt{-c\log(f)+3if}}$$

```
[Out] (3*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[
(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt
[I*f - c*Log[f]]) + (E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I)*f - c*Log
[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*Log[f]))/(2*
Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d + (e
- I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] +
 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
 + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[
Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f +
c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rubi [A] time = 0.6647, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{3\sqrt{\pi}f^a \exp\left(-\frac{(e+ib\log(f))^2}{-4c\log(f)+4if}-id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\log(f)+if)+ie}{2\sqrt{-c\log(f)+if}}\right)}{16\sqrt{-c\log(f)+if}}+\frac{\sqrt{\pi}f^a \exp\left(-\frac{(3e+ib\log(f))^2}{4(-c\log(f)+3if)}-3id\right)\text{Erf}\left(\frac{-b\log(f)+2x(-c\,\text{l}}{2\sqrt{-c\log(}}\right.}{16\sqrt{-c\log(f)+3if}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^((-I)*d - (e + I*b*Log[f])^2/((4*I)*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[
(I*e - b*Log[f] + 2*x*(I*f - c*Log[f]))/(2*Sqrt[I*f - c*Log[f]])])/(16*Sqrt
[I*f - c*Log[f]]) + (E^((-3*I)*d - (3*e + I*b*Log[f])^2/(4*((3*I)*f - c*Log
[f])))*f^a*Sqrt[Pi]*Erf[((3*I)*e - b*Log[f] + 2*x*((3*I)*f - c*Log[f]))/(2*
Sqrt[(3*I)*f - c*Log[f]])])/(16*Sqrt[(3*I)*f - c*Log[f]]) + (3*E^(I*d + (e
- I*b*Log[f])^2/((4*I)*f + 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(I*e + b*Log[f] +
 2*x*(I*f + c*Log[f]))/(2*Sqrt[I*f + c*Log[f]])])/(16*Sqrt[I*f + c*Log[f]])
 + (E^((3*I)*d - ((3*I)*e + b*Log[f])^2/(4*((3*I)*f + c*Log[f])))*f^a*Sqrt[
Pi]*Erfi[((3*I)*e + b*Log[f] + 2*x*((3*I)*f + c*Log[f]))/(2*Sqrt[(3*I)*f +
c*Log[f]])])/(16*Sqrt[(3*I)*f + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int f^{a+bx+cx^2}\cos^3\left(d+ex+fx^2\right)dx = \int\left(\frac{1}{8}e^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2id+2iex+2ifx^2-3i\left(d+ex+fx^2\right)\right)f^{a+bx}\right.$$

$$=\frac{1}{8}\int e^{-3i\left(d+ex+fx^2\right)}f^{a+bx+cx^2}\,dx+\frac{1}{8}\int\exp\left(6id+6iex+6ifx^2-3i\left(d+ex+fx^2\right)\right)$$

$$=\frac{1}{8}\int\exp\left(-3id+a\log(f)-x(3ie-b\log(f))-x^2(3if-c\log(f))\right)dx+\frac{1}{8}\int\exp\left(\right.$$

$$=\frac{1}{8}\left(3\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-ie+b\log(f)+2x(-if+c\log(f))}{4(-if+c\log(f))}\right.$$

$$=\frac{3\exp\left(-id-\frac{(e+ib\log(f))^2}{4if-4c\log(f)}\right)f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{ie-b\log(f)+2x(if-c\log(f))}{2\sqrt{if-c\log(f)}}\right)}{16\sqrt{if-c\log(f)}}+\frac{\exp\left(-3id-\frac{(3e+ib\log}{4(3if-c\log}\right.}{}$$

Mathematica [B] time = 7.1826, size = 3829, normalized size = 9.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-27*(-1)^(3/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[
f]^2))/(f - I*c*Log[f]))*f^3*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f
] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]] + 27*
(-1)^(1/4)*c*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*L
og[f]))*f^2*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log
[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*
c^2*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f
*Cos[d]*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Log[f]^2*Sqrt[f - I*c*Log[f]] + 3*(-1)^(1/4)*c^3*E^(((
I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Cos[d]*Erf
i[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*
Log[f]])]*Log[f]^3*Sqrt[f - I*c*Log[f]] - 3*(-1)^(3/4)*E^(((I/4)*(-9*e^2 +
(6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Cos[3*d]*Erfi[((-
1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*L
og[f]])]*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c*E^(((I/4)*(-9*e^2 + (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]*Sqrt[3*f - I*c*Log[f]] - 3*(-1)^(3/4)*c^2*E^(((I/4)*(-9*e^2 + (6*I)
*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Cos[3*d]*Erfi[((-1)^(1/4
)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])
]*Log[f]^2*Sqrt[3*f - I*c*Log[f]] + (-1)^(1/4)*c^3*E^(((I/4)*(-9*e^2 + (6*I
)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Cos[3*d]*Erfi[((-1)^(1/4)
*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f]])]
*Log[f]^3*Sqrt[3*f - I*c*Log[f]] - (27*(-1)^(1/4)*f^3*Cos[d]*Erfi[((-1)^(3/
4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*S
qrt[f + I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f
 + I*c*Log[f])) + (27*(-1)^(3/4)*c*f^2*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x +
 I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f +
I*c*Log[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*L
og[f])) - (3*(-1)^(1/4)*c^2*f*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[
f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log
[f]])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f]))
 + (3*(-1)^(3/4)*c^3*Cos[d]*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I
)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]])/E^(
((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) - (3*(-1
)^(1/4)*f^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x
```

```
*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-9
*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(3/4)*
c*f^2*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f
]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]*Sqrt[3*f + I*c*Log[f]])/E^(((I/4)*(-
9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) - (3*(-1)^(1/
4)*c^2*f*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Lo
g[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]])/E^(((I/
4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^
(3/4)*c^3*Cos[3*d]*Erfi[((-1)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*L
og[f]))/(2*Sqrt[3*f + I*c*Log[f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]])/E^(((I
/4)*(-9*e^2 - (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + 27*(-
1)^(1/4)*E^(((I/4)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f
]))*f^3*Erfi[((-1)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sq
rt[f - I*c*Log[f]])]*Sqrt[f - I*c*Log[f]]*Sin[d] + 27*(-1)^(3/4)*c*E^(((I/4
)*(-e^2 + (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f^2*Erfi[((-1
)^(1/4)*(e + 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]
])]*Log[f]*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-e^2 +
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*f*Erfi[((-1)^(1/4)*(e
+ 2*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^
2*Sqrt[f - I*c*Log[f]]*Sin[d] + 3*(-1)^(3/4)*c^3*E^(((I/4)*(-e^2 + (2*I)*b*
e*Log[f] + b^2*Log[f]^2))/(f - I*c*Log[f]))*Erfi[((-1)^(1/4)*(e + 2*f*x - I
*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[f - I*c*Log[f]])]*Log[f]^3*Sqrt[f -
I*c*Log[f]]*Sin[d] + (27*(-1)^(3/4)*f^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*L
og[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Sqrt[f + I*c*Log[f]]*S
in[d])/E^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])
) + (27*(-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c
*x*Log[f]))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]*Sqrt[f + I*c*Log[f]]*Sin[d])/E
^(((I/4)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + (3*(
-1)^(3/4)*c^2*f*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]
))/(2*Sqrt[f + I*c*Log[f]])]*Log[f]^2*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4
)*(-e^2 - (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + (3*(-1)^(1/
4)*c^3*Erfi[((-1)^(3/4)*(e + 2*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqr
t[f + I*c*Log[f]])]*Log[f]^3*Sqrt[f + I*c*Log[f]]*Sin[d])/E^(((I/4)*(-e^2 -
 (2*I)*b*e*Log[f] + b^2*Log[f]^2))/(f + I*c*Log[f])) + 3*(-1)^(1/4)*E^(((I/
4)*(-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^3*Erfi
[((-1)^(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I
*c*Log[f]])]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c*E^(((I/4)*(-9*e
^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f^2*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
]])]*Log[f]*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + 3*(-1)^(1/4)*c^2*E^(((I/4)*(-
9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*f*Erfi[((-1)^
(1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[
f]])]*Log[f]^2*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (-1)^(3/4)*c^3*E^(((I/4)*(
-9*e^2 + (6*I)*b*e*Log[f] + b^2*Log[f]^2))/(3*f - I*c*Log[f]))*Erfi[((-1)^(
1/4)*(3*e + 6*f*x - I*b*Log[f] - (2*I)*c*x*Log[f]))/(2*Sqrt[3*f - I*c*Log[f
```

```
]])]*Log[f]^3*Sqrt[3*f - I*c*Log[f]]*Sin[3*d] + (3*(-1)^(3/4)*f^3*Erfi[((-1
)^(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Lo
g[f]])]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log[
f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c*f^2*Erfi[((-1)^(3/4
)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f]])
]*Log[f]*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*e*Log
[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + (3*(-1)^(3/4)*c^2*f*Erfi[((-1)^(
3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[f
]])]*Log[f]^2*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b*
e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f])) + ((-1)^(1/4)*c^3*Erfi[((-1)^
(3/4)*(3*e + 6*f*x + I*b*Log[f] + (2*I)*c*x*Log[f]))/(2*Sqrt[3*f + I*c*Log[
f]])]*Log[f]^3*Sqrt[3*f + I*c*Log[f]]*Sin[3*d])/E^(((I/4)*(-9*e^2 - (6*I)*b
*e*Log[f] + b^2*Log[f]^2))/(3*f + I*c*Log[f]))))/(16*(f - I*c*Log[f])*(3*f
- I*c*Log[f])*(f + I*c*Log[f])*(3*f + I*c*Log[f]))
```

Maple [A] time = 0.422, size = 426, normalized size = 1.

$$-\frac{f^{a}\sqrt{\pi}}{16}\mathrm{e}^{-\frac{(\ln(f))^{2}b^{2}-6i\ln(f)be+12id\ln(f)c+36df-9e^{2}}{4c\ln(f)-12if}}\operatorname{Erf}\left(-x\sqrt{3if-c\ln\left(f\right)}+\frac{b\ln\left(f\right)-3ie}{2}\frac{1}{\sqrt{3if-c\ln\left(f\right)}}\right)\frac{1}{\sqrt{3if-c\ln\left(f\right)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x)
```

```
[Out] -1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-6*I*ln(f)*b*e+12*I*d*ln(f)*c+36*d*
f-9*e^2)/(-3*I*f+c*ln(f)))/(3*I*f-c*ln(f))^(1/2)*erf(-x*(3*I*f-c*ln(f))^(1/
2)+1/2*(b*ln(f)-3*I*e)/(3*I*f-c*ln(f))^(1/2))-3/16*Pi^(1/2)*f^a*exp(-1/4*(l
n(f)^2*b^2-2*I*ln(f)*b*e+4*I*d*ln(f)*c+4*d*f-e^2)/(-I*f+c*ln(f)))/(I*f-c*ln
(f))^(1/2)*erf(-x*(I*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-I*e)/(I*f-c*ln(f))^(1/2)
)-3/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+2*I*ln(f)*b*e-4*I*d*ln(f)*c+4*d*f
-e^2)/(I*f+c*ln(f)))/(-c*ln(f)-I*f)^(1/2)*erf(-(-c*ln(f)-I*f)^(1/2)*x+1/2*(
I*e+b*ln(f))/(-c*ln(f)-I*f)^(1/2))-1/16*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+
6*I*ln(f)*b*e-12*I*d*ln(f)*c+36*d*f-9*e^2)/(3*I*f+c*ln(f)))/(-c*ln(f)-3*I*f
)^(1/2)*erf(-(-c*ln(f)-3*I*f)^(1/2)*x+1/2*(3*I*e+b*ln(f))/(-c*ln(f)-3*I*f)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.695348, size = 2195, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(pi)*(c^3*log(f)^3 - 3*I*c^2*f*log(f)^2 + c*f^2*log(f) - 3*I*f^3
)*sqrt(-c*log(f) - 3*I*f)*erf(1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*
e*f + (3*I*c*e - 3*I*b*f)*log(f))*sqrt(-c*log(f) - 3*I*f)/(c^2*log(f)^2 + 9
*f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + 27*I*e^2*f - 108*I*d*f^2 - (12
*I*c^2*d - 6*I*b*c*e + 3*I*b^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*
log(f))/(c^2*log(f)^2 + 9*f^2)) + sqrt(pi)*(3*c^3*log(f)^3 - 3*I*c^2*f*log(
f)^2 + 27*c*f^2*log(f) - 27*I*f^3)*sqrt(-c*log(f) - I*f)*erf(1/2*(2*f^2*x +
 (2*c^2*x + b*c)*log(f)^2 + e*f + (I*c*e - I*b*f)*log(f))*sqrt(-c*log(f) -
I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*e^2*f -
4*I*d*f^2 - (4*I*c^2*d - 2*I*b*c*e + I*b^2*f)*log(f)^2 - (c*e^2 - 2*b*e*f +
 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(3*c^3*log(f)^3 + 3*I*c^
2*f*log(f)^2 + 27*c*f^2*log(f) + 27*I*f^3)*sqrt(-c*log(f) + I*f)*erf(1/2*(2
*f^2*x + (2*c^2*x + b*c)*log(f)^2 + e*f + (-I*c*e + I*b*f)*log(f))*sqrt(-c*
log(f) + I*f)/(c^2*log(f)^2 + f^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 - I
*e^2*f + 4*I*d*f^2 - (-4*I*c^2*d + 2*I*b*c*e - I*b^2*f)*log(f)^2 - (c*e^2 -
 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + f^2)) + sqrt(pi)*(c^3*log(f)^3
+ 3*I*c^2*f*log(f)^2 + c*f^2*log(f) + 3*I*f^3)*sqrt(-c*log(f) + 3*I*f)*erf(
1/2*(18*f^2*x + (2*c^2*x + b*c)*log(f)^2 + 9*e*f + (-3*I*c*e + 3*I*b*f)*log
(f))*sqrt(-c*log(f) + 3*I*f)/(c^2*log(f)^2 + 9*f^2))*e^(-1/4*((b^2*c - 4*a*
c^2)*log(f)^3 - 27*I*e^2*f + 108*I*d*f^2 - (-12*I*c^2*d + 6*I*b*c*e - 3*I*b
^2*f)*log(f)^2 - 9*(c*e^2 - 2*b*e*f + 4*a*f^2)*log(f))/(c^2*log(f)^2 + 9*f^
2)))/(c^4*log(f)^4 + 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a} \cos\left(fx^2+ex+d\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(f*x^2 + e*x + d)^3, x)
```

3.134 $\int f^{a+bx+cx^2} \cos\left(a+bx+ex^2\right) dx$

Optimal. Leaf size=209

$$\frac{\sqrt{\pi}\exp\left(-(-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\operatorname{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}+\frac{\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\operatorname{Erf}}{4\sqrt{c\log(f)+ie}}$$

```
[Out] (Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c*Log[f
]])])/(4*E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])))*S
qrt[I*e - c*Log[f]]) + (E^((I + Log[f])*(a - (b^2*(I + Log[f]))/((4*I)*e +
4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]))/(2*Sqrt
[I*e + c*Log[f]])])/(4*Sqrt[I*e + c*Log[f]])
```

Rubi [A] time = 0.481805, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4473, 2287, 2234, 2205, 2204}

$$\frac{\sqrt{\pi}\exp\left(-(-\log(f)+i)\left(a-\frac{b^2(-\log(f)+i)}{-4c\log(f)+4ie}\right)\right)\operatorname{Erf}\left(\frac{b(-\log(f)+i)+2x(-c\log(f)+ie)}{2\sqrt{-c\log(f)+ie}}\right)}{4\sqrt{-c\log(f)+ie}}+\frac{\sqrt{\pi}\exp\left((\log(f)+i)\left(a-\frac{b^2(\log(f)+i)}{4c\log(f)+4ie}\right)\right)\operatorname{Erf}}{4\sqrt{c\log(f)+ie}}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[(b*(I - Log[f]) + 2*x*(I*e - c*Log[f]))/(2*Sqrt[I*e - c*Log[f
]])])/(4*E^((I - Log[f])*(a - (b^2*(I - Log[f]))/((4*I)*e - 4*c*Log[f])))*S
qrt[I*e - c*Log[f]]) + (E^((I + Log[f])*(a - (b^2*(I + Log[f]))/((4*I)*e +
4*c*Log[f])))*Sqrt[Pi]*Erfi[(b*(I + Log[f]) + 2*x*(I*e + c*Log[f]))/(2*Sqrt
[I*e + c*Log[f]])])/(4*Sqrt[I*e + c*Log[f]])
```

Rule 4473

```
Int[Cos[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cos[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
```

```
x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)dx &= \int\left(\frac{1}{2}e^{-ia-ibx-iex^2}f^{a+bx+cx^2}+\frac{1}{2}e^{ia+ibx+iex^2}f^{a+bx+cx^2}\right)dx \\
&= \frac{1}{2}\int e^{-ia-ibx-iex^2}f^{a+bx+cx^2}\,dx+\frac{1}{2}\int e^{ia+ibx+iex^2}f^{a+bx+cx^2}\,dx \\
&= \frac{1}{2}\int \exp\left(-a(i-\log(f))-bx(i-\log(f))-x^2(ie-c\log(f))\right)dx+\frac{1}{2}\int \exp\left(a(i+\right. \\
&= \frac{1}{2}\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\int \exp\left(\frac{(-b(i-\log(f))+2x(-ie+c\,\mathrm{lo}}{4(-ie+c\log(f))}\right. \\
&= \frac{\exp\left(-(i-\log(f))\left(a-\frac{b^2(i-\log(f))}{4ie-4c\log(f)}\right)\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b(i-\log(f))+2x(ie-c\log(f))}{2\sqrt{ie-c\log(f)}}\right)}{4\sqrt{ie-c\log(f)}}+\frac{\exp\left((i+\mathrm{lo}\right.}{}
\end{aligned}$$

Mathematica [A] time = 1.79565, size = 325, normalized size = 1.56

$$-\frac{i\sqrt{\pi}e^{-\frac{b^2c\log^3(f)}{2\left(c^2\log^2(f)+e^2\right)}}f^{a-\frac{b^2}{2(e-ic\log(f))}}\left((\cos(a)+i\sin(a))(e+ic\log(f))\sqrt{c\log(f)+ie}\exp\left(\frac{1}{4}b^2\left(\frac{\log^2(f)}{c\log(f)-ie}+\frac{1}{c\log(f)+ie}\right)\right)\mathbf{Erfi}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[f^(a + b*x + c*x^2)*Cos[a + b*x + e*x^2],x]
```

```
[Out] ((-I/4)*f^(a - b^2/(2*(e - I*c*Log[f])))*Sqrt[Pi]*(-(E^((b^2*(((-I)*e + c*L
og[f])^(-1) + Log[f]^2/(I*e + c*Log[f])))/4)*f^((I*b^2*c*Log[f])/(e^2 + c^2
*Log[f]^2))*Erfi[((-I)*(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[(-I)*e + c
*Log[f]])]*(e - I*c*Log[f])*Sqrt[(-I)*e + c*Log[f]]*(Cos[a] - I*Sin[a])) +
E^((b^2*(Log[f]^2/((-I)*e + c*Log[f]) + (I*e + c*Log[f])^(-1)))/4)*Erfi[(I*
(b + 2*e*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[I*e + c*Log[f]])]*(e + I*c*Log[f]
)*Sqrt[I*e + c*Log[f]]*(Cos[a] + I*Sin[a])))/(E^((b^2*c*Log[f]^3)/(2*(e^2 +
 c^2*Log[f]^2)))*(e^2 + c^2*Log[f]^2))
```

Maple [A] time = 0.099, size = 216, normalized size = 1.

$$-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f))^2b^2+4i\ln(f)ac-2i\ln(f)b^2+4ae-b^2}{4c\ln(f)-4ie}}\mathrm{Erf}\left(-\sqrt{ie-c\ln(f)}x+\frac{b\ln(f)-ib}{2}\frac{1}{\sqrt{ie-c\ln(f)}}\right)\frac{1}{\sqrt{ie-c\ln(f)}}-\frac{f^a\sqrt{\pi}}{4}\mathrm{e}^{-\frac{(\ln(f}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x)
```

```
[Out] -1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2+4*I*ln(f)*a*c-2*I*ln(f)*b^2+4*a*e-b
^2)/(-I*e+c*ln(f)))/(I*e-c*ln(f))^(1/2)*erf(-(I*e-c*ln(f))^(1/2)*x+1/2*(b*l
n(f)-I*b)/(I*e-c*ln(f))^(1/2))-1/4*Pi^(1/2)*f^a*exp(-1/4*(ln(f)^2*b^2-4*I*l
n(f)*a*c+2*I*ln(f)*b^2+4*a*e-b^2)/(I*e+c*ln(f)))/(-c*ln(f)-I*e)^(1/2)*erf(-
(-c*ln(f)-I*e)^(1/2)*x+1/2*(b*ln(f)+I*b)/(-c*ln(f)-I*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: IndexError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: IndexError
```

Fricas [B] time = 0.55359, size = 938, normalized size = 4.49

$$-\frac{\sqrt{\pi}\left(c\log\left(f\right)-i\,e\right)\sqrt{-c\log\left(f\right)-i\,e}\,\mathrm{erf}\left(\frac{\left(2\,e^{2}x+\left(2\,c^{2}x+bc\right)\log\left(f\right)^{2}+be+\left(i\,bc-i\,be\right)\log\left(f\right)\right)\sqrt{-c\log\left(f\right)-i\,e}}{2\left(c^{2}\log\left(f\right)^{2}+e^{2}\right)}\right)e^{\left(-\frac{\left(b^{2}c-4\,ac^{2}\right)\log\left(f\right)^{3}+i\,b^{2}e-4i\,ae^{2}-(-\cdots}{4\cdots}\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*(c*log(f) - I*e)*sqrt(-c*log(f) - I*e)*erf(1/2*(2*e^2*x + (2
*c^2*x + b*c)*log(f)^2 + b*e + (I*b*c - I*b*e)*log(f))*sqrt(-c*log(f) - I*e
)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c - 4*a*c^2)*log(f)^3 + I*b^2*e - 4*I
*a*e^2 - (-2*I*b^2*c + 4*I*a*c^2 + I*b^2*e)*log(f)^2 - (b^2*c - 2*b^2*e + 4
*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)) + sqrt(pi)*(c*log(f) + I*e)*sqrt(-c*l
og(f) + I*e)*erf(1/2*(2*e^2*x + (2*c^2*x + b*c)*log(f)^2 + b*e + (-I*b*c +
I*b*e)*log(f))*sqrt(-c*log(f) + I*e)/(c^2*log(f)^2 + e^2))*e^(-1/4*((b^2*c
- 4*a*c^2)*log(f)^3 - I*b^2*e + 4*I*a*e^2 - (2*I*b^2*c - 4*I*a*c^2 - I*b^2*
e)*log(f)^2 - (b^2*c - 2*b^2*e + 4*a*e^2)*log(f))/(c^2*log(f)^2 + e^2)))/(c
^2*log(f)^2 + e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int f^{a+bx+cx^2}\cos\left(a+bx+ex^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(c*x**2+b*x+a)*cos(e*x**2+b*x+a),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*cos(a + b*x + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int f^{cx^2+bx+a}\cos\left(ex^2+bx+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(c*x^2+b*x+a)*cos(e*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(f^(c*x^2 + b*x + a)*cos(e*x^2 + b*x + a), x)
```

3.135 $\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx$

Optimal. Leaf size=245

$$\frac{bcf^2\log(F)\sin^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\sin(d+ex)\cos(d+}{b^2c^2\log^2(F)+4}$$

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x])
/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2
+ b^2*c^2*Log[F]^2)) + (2*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2
 + b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/
(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x]^2
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.358488, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6741, 12, 6742, 2194, 4432, 4434}

$$\frac{bcf^2\log(F)\sin^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{2ef^2\sin(d+ex)\cos(d+}{b^2c^2\log^2(F)+4}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```


```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x])
/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F]*(4*e^2
+ b^2*c^2*Log[F]^2)) + (2*b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2
 + b^2*c^2*Log[F]^2) - (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x])/
(4*e^2 + b^2*c^2*Log[F]^2) + (b*c*f^2*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x]^2
)/(4*e^2 + b^2*c^2*Log[F]^2)
```


Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int F^{c(a+bx)}(f+f\sin(d+ex))^2\,dx = \int f^2F^{ac+bcx}(1+\sin(d+ex))^2\,dx$$

$$= f^2\int F^{ac+bcx}(1+\sin(d+ex))^2\,dx$$

$$= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\sin(d+ex)+F^{ac+bcx}\sin^2(d+ex)\right)dx$$

$$= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\sin^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\sin(d+ex)\,dx$$

$$= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2bcf^2F^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}-\frac{2ef^2F^{ac+bcx}}{4e^2}$$

$$= \frac{f^2F^{ac+bcx}}{bc\log(F)}-\frac{2ef^2F^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{2bcf^2F^{ac+bcx}}{e^2+b}$$

Mathematica [A] time = 1.60363, size = 180, normalized size = 0.73

$$\frac{f^2(\sin(d+ex)+1)^2F^{c(a+bx)}\left(\frac{4bc\log(F)\sin(d+ex)}{b^2c^2\log^2(F)+e^2}-\frac{2e\sin(2(d+ex))}{b^2c^2\log^2(F)+4e^2}-\frac{4e\cos(d+ex)}{b^2c^2\log^2(F)+e^2}-\frac{bc\log(F)\cos(2(d+ex))}{b^2c^2\log^2(F)+4e^2}+\frac{3}{bc\log(F)}\right)}{2\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(1 + Sin[d + e*x])^2*(3/(b*c*Log[F]) - (4*e*Cos[d + e*
x])/(e^2 + b^2*c^2*Log[F]^2) - (b*c*Cos[2*(d + e*x)]*Log[F])/(4*e^2 + b^2*c
^2*Log[F]^2) + (4*b*c*Log[F]*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2) - (2*e*
Sin[2*(d + e*x)])/(4*e^2 + b^2*c^2*Log[F]^2)))/(2*(Cos[(d + e*x)/2] + Sin[(
d + e*x)/2])^4)
```

Maple [A] time = 0.099, size = 368, normalized size = 1.5

$$\frac{3\,F^{ac}f^2F^{bcx}}{2\,bc\ln(F)}-\frac{F^{ac}f^2\ln(F)\,bce^{bcx\ln(F)}}{\left(2+2\,(\tan(ex+d))^2\right)\left(4\,e^2+b^2c^2\,(\ln(F))^2\right)}-2\,\frac{F^{ac}f^2ee^{bcx\ln(F)}\tan(ex+d)}{\left(1+(\tan(ex+d))^2\right)\left(4\,e^2+b^2c^2\,(\ln(F))^2\right)}+\frac{F^{ac}f^2}{\left(2+2\,(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x)
```

```
[Out] 3/2*F^(a*c)*f^2/b/c/ln(F)*F^(b*c*x)-1/2*F^(a*c)*f^2/(1+tan(e*x+d)^2)/(4*e^2
+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))-2*F^(a*c)*f^2/(1+tan(e*x+d)^2)
/(4*e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(e*x+d)+1/2*F^(a*c)*f^2/(1+t
an(e*x+d)^2)/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))*tan(e*x+d)^
2-2*F^(a*c)*f^2/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*
ln(F))+2*F^(a*c)*f^2/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln(F)^2)*e*exp(b
*c*x*ln(F))*tan(1/2*d+1/2*e*x)^2+4*F^(a*c)*f^2/(1+tan(1/2*d+1/2*e*x)^2)*ln(
F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)
```

Maxima [B] time = 1.1844, size = 784, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))
*F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*
e*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*s
in(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*
b^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(
2*e*x + 4*d) - 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log
(F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*
x))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c
*cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) - ((F^(a*c)*b*c*log(F)*sin
(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*d) - (F^(a*c)*b*c*log(F)*sin(
d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F^(a*c)*b*c*cos(d)*log(F) - F^
(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*cos(d)*log(F) + F^(
a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2
*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x + a*c)*f^2/(b*c*
log(F))
```

Fricas [A] time = 0.511159, size = 581, normalized size = 2.37

$$-\frac{\left(2\,b^{3}c^{3}ef^{2}\cos\left(ex+d\right)\log\left(F\right)^{3}+8\,bce^{3}f^{2}\cos\left(ex+d\right)\log\left(F\right)-6\,e^{4}f^{2}+\left(b^{4}c^{4}f^{2}\cos\left(ex+d\right)^{2}-2\,b^{4}c^{4}f^{2}\right)\log\left(F\right)^{4}+\left(b^{2}\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 8*b*c*e^3*f^2*cos(e*x + d)*log(F)
 - 6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 - 2*b^4*c^4*f^2)*log(F)^4 + (b^2
*c^2*e^2*f^2*cos(e*x + d)^2 - 8*b^2*c^2*e^2*f^2)*log(F)^2 - 2*(b^4*c^4*f^2*
log(F)^4 - b^3*c^3*e*f^2*cos(e*x + d)*log(F)^3 + 4*b^2*c^2*e^2*f^2*log(F)^2
 - b*c*e^3*f^2*cos(e*x + d)*log(F))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*
log(F)^5 + 5*b^3*c^3*e^2*log(F)^3 + 4*b*c*e^4*log(F))
```

Sympy [A] time = 162.303, size = 1760, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x - f
**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(F, 1)), (zo
o*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*sin(d + e*x
)**2 + zoo*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*si
n(d + e*x)*cos(d + e*x) + zoo*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e
/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F, exp(-2*I*e/(b*c)))), (zoo*e**4*f**2
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**4*f*
*2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(
-I*e/(b*c)))), (zoo*e**4*f**2*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)
*sin(d + e*x) + zoo*e**4*f**2*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)
*cos(d + e*x), Eq(F, exp(I*e/(b*c)))), (zoo*e**4*f**2*exp(2*I*e/(b*c))**(a*
c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2 + zoo*e**4*f**2*exp(2*I*e/(b*c
))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)*cos(d + e*x) + zoo*e**4*f*
*2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F,
 exp(2*I*e/(b*c)))), (F**(a*c)*(f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e
*x)**2/2 + f**2*x - f**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e
*x)/e), Eq(b, 0)), (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f
**2*x - f**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*f**2*cos(d + e*x)/e, Eq(c,
 0)), (F**(a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*sin(d + e*x)**2/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c
)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*sin(d + e*x)/(b**5*c**5*log(F)**5 + 5
*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c
**4*f**2*log(F)**4/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*
c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sin(d + e
*x)*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*
e**4*log(F)) - 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*cos(d + e*x
)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) +
3*F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sin(d + e*x)**2/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 8*F**(a*c
)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sin(d + e*x)/(b**5*c**5*log(F)**
5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)
*b**2*c**2*e**2*f**2*log(F)**2*cos(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**
3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 5*F**(a*c)*F**(b*c*x)*b**2*c**
2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4
*b*c*e**4*log(F)) - 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)
*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**
4*log(F)) - 8*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*cos(d + e*x)/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c
)*F**(b*c*x)*e**4*f**2*sin(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e
**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f**2*cos(d
+ e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*lo
g(F)) + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*
```

```
e**2*log(F)**3 + 4*b*c*e**4*log(F)), True))
```

Giac [C] time = 1.45405, size = 2395, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c + 2*x*e + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*
sgn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b
*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*x*e + 2*d)/
(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(
abs(F)) + a*c*log(abs(F))) - 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*p
i*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*x*e - 2*d)*log(abs(F))/(4*b^2*
c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi
*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F)
- 1/2*pi*a*c - 2*x*e - 2*d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 3*(2*b*c*f^2*cos(-
1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(ab
s(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(
F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F
) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(
b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*log(abs(F))*sin(1/2*pi*
b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*
b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b*c*sgn(F)
- pi*b*c + 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn
(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 2*(2*b*c*f^2*log(a
bs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*
a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2)
 - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*
x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) -
1/2*I*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b
*c*log(abs(F)) + 16*I*e) - 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c
*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(-4*I*pi*b*c*sgn
(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*
log(abs(F))) + 1/2*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2
```

```
*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c
+ 2*b*c*log(abs(F)) + 2*I*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi
*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-I*pi*b*c*sgn(F
) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(a
bs(F))) + 1/2*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*p
i*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*
b*c*log(abs(F)) - 2*I*e) - 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c
*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-I*pi*b*c*sgn(F) +
I*pi*b*c + 2*b*c*log(abs(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F
))) - 1/2*I*(2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a
*c*sgn(F) - 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c
 + 8*b*c*log(abs(F)) - 16*I*e) - 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(-4*I*pi*b
*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(abs(F))
+ a*c*log(abs(F))) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*
c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c +
 4*b*c*log(abs(F))) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c + 4*b*
c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f^2*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a
*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f^2*e^(-1/2*I*pi
*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi
*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log
(abs(F)))
```

3.136 $\int F^{c(a+bx)}(f + f\sin(d + ex))\,dx$

Optimal. Leaf size=99

$$\frac{bcf\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{ef\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (e*f*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2
+ b^2*c^2*Log[F]^2) + (b*c*f*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rubi [A] time = 0.158877, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6741, 12, 6742, 2194, 4432}

$$\frac{bcf\log(F)\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}-\frac{ef\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) - (e*f*F^(a*c + b*c*x)*Cos[d + e*x])/(e^2
+ b^2*c^2*Log[F]^2) + (b*c*f*F^(a*c + b*c*x)*Log[F]*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}(f+f\sin(d+ex))\,dx &= \int fF^{ac+bcx}(1+\sin(d+ex))\,dx\\ &= f\int F^{ac+bcx}(1+\sin(d+ex))\,dx\\ &= f\int \left(F^{ac+bcx}+F^{ac+bcx}\sin(d+ex)\right)\,dx\\ &= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\sin(d+ex)\,dx\\ &= \frac{fF^{ac+bcx}}{bc\log(F)}-\frac{efF^{ac+bcx}\cos(d+ex)}{e^2+b^2c^2\log^2(F)}+\frac{bcfF^{ac+bcx}\log(F)\sin(d+ex)}{e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.577816, size = 83, normalized size = 0.84

$$\frac{fF^{c(a+bx)}\left(b^2c^2\log^2(F)\sin(d+ex)+b^2c^2\log^2(F)-bce\log(F)\cos(d+ex)+e^2\right)}{bc\log(F)\left(b^2c^2\log^2(F)+e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Sin[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(e^2 - b*c*e*Cos[d + e*x]*Log[F] + b^2*c^2*Log[F]^2 + b^
2*c^2*Log[F]^2*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A] time = 0.014, size = 183, normalized size = 1.9

$$\frac{fF^{c(bx+a)}}{bc\ln(F)}+\frac{ef\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)^{-1}-\frac{ef\mathrm{e}^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x)
```

```
[Out] f/b/c/ln(F)*F^(c*(b*x+a))+f/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln(F)^2)*
e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2-f/(1+tan(1/2*d+1/2*e*x)^2)/(e^2
+b^2*c^2*ln(F)^2)*e*exp(c*(b*x+a)*ln(F))+2*f/(1+tan(1/2*d+1/2*e*x)^2)*ln(F)
*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)
```

Maxima [B] time = 1.05947, size = 294, normalized size = 2.97

$$-\frac{\left((F^{ac}bc\log(F)\sin(d) + F^{ac}e\cos(d))F^{bcx}\cos(ex+2d) - (F^{ac}bc\log(F)\sin(d) - F^{ac}e\cos(d))F^{bcx}\cos(ex) - (F^{ac}bc\cos(d\right.}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\log(F)^2\sin(d)^2 + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*((F^(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x + 2*
d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*cos(e*x) - (F
^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^
(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*sin(e*x))*f/(b^2*c^2*
cos(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2)
+ F^(b*c*x + a*c)*f/(b*c*log(F))
```

Fricas [A] time = 0.487338, size = 197, normalized size = 1.99

$$\frac{\left(b^2c^2f\log(F)^2\sin(ex+d) + b^2c^2f\log(F)^2 - bcef\cos(ex+d)\log(F) + e^2f\right)F^{bcx+ac}}{b^3c^3\log(F)^3 + bce^2\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] (b^2*c^2*f*log(F)^2*sin(e*x + d) + b^2*c^2*f*log(F)^2 - b*c*e*f*cos(e*x + d
)*log(F) + e^2*f)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [A] time = 12.888, size = 408, normalized size = 4.12

$$\begin{cases} fx - \frac{f\cos(d+ex)}{e} & \text{for } F = 1 \\ \tilde{\infty}e^2 f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\sin(d+ex) + \tilde{\infty}e^2 f\left(e^{-\frac{ie}{bc}}\right)^{ac}\left(e^{-\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F = e^{-\frac{ie}{bc}} \\ \tilde{\infty}e^2 f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\sin(d+ex) + \tilde{\infty}e^2 f\left(e^{\frac{ie}{bc}}\right)^{ac}\left(e^{\frac{ie}{bc}}\right)^{bcx}\cos(d+ex) & \text{for } F = e^{\frac{ie}{bc}} \\ F^{ac}\left(fx - \frac{f\cos(d+ex)}{e}\right) & \text{for } b = 0 \\ fx - \frac{f\cos(d+ex)}{e} & \text{for } c = 0 \\ \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2\sin(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3+bce^2\log(F)} - \frac{F^{ac}F^{bcx}bcef\log(F)\cos(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log(F)^3+bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*sin(e*x+d)),x)
```

```
[Out] Piecewise((f*x - f*cos(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-I*e/(b*c))**
(a*c)*exp(-I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**2*f*exp(-I*e/(b*c))**(
a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(-I*e/(b*c)))), (zoo*e
**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**2
*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(I*
e/(b*c)))), (F**(a*c)*(f*x - f*cos(d + e*x)/e), Eq(b, 0)), (f*x - f*cos(d +
 e*x)/e, Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*sin(d + e*x)
/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f*
log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) - F**(a*c)*F**(b*c*x)*b*c
*e*f*log(F)*cos(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)
*F**(b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

Giac [C] time = 1.26293, size = 1270, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/
2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2
) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - p
i*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + (2*b*c*f*log(abs(F))*s
```

```
in(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*
e + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) - (pi*b
*c*sgn(F) - pi*b*c + 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi
*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(
F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - (2*b*c*f*l
og(abs(F))*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2
*pi*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e
)^2) - (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*
c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 +
(pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
+ 1/2*(2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c + I*x*e + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log
(abs(F)) + 4*I*e) + 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*
b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F
) - 1/2*I*pi*a*c - I*x*e - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*log
(abs(F)) - 4*I*e) - 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*
I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*
b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) -
 1/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*
f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*
pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs
(F)) + a*c*log(abs(F)))
```

3.137 $\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx$

Optimal. Leaf size=80

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Rubi [A] time = 0.0655502, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4456, 4450}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+f\sin(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)dx}{2f}$$
$$= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Mathematica [A] time = 1.69873, size = 128, normalized size = 1.6

$$\frac{2F^{c(a+bx)}\left(-i\text{Hypergeometric2F1}\left(1,-\frac{ibc\log(F)}{e},1-\frac{ibc\log(F)}{e},-\sin(d+ex)+i\cos(d+ex)\right)+\frac{\sin\left(\frac{ex}{2}\right)}{\left(\sin\left(\frac{d}{2}\right)+\cos\left(\frac{d}{2}\right)\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+c\right.}}{ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x]),x]
```

```
[Out] (2*F^(c*(a + b*x))*((-I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b
*c*Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]] - (Cos[d] + I*(1 + Sin[d]))^(-
1) + Sin[(e*x)/2]/((Cos[d/2] + Sin[d/2])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/
2]))))/(e*f)
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{f+f\sin(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{F^{bcx+ac}}{f\sin\left(ex+d\right)+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f*sin(e*x + d) + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\sin\left(d+ex\right)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(sin(d + e*x) + 1), x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{f\sin\left(ex+d\right)+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f), x)
```

3.138 $\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},ie^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\right.}{6e^2f^2}$$

```
[Out] -(F^(c*(a + b*x))*Cot[d/2 + Pi/4 + (e*x)/2]*Csc[d/2 + Pi/4 + (e*x)/2]^2)/(6
*e*f^2) - (b*c*F^(c*(a + b*x))*Csc[d/2 + Pi/4 + (e*x)/2]^2*Log[F])/(6*e^2*f
^2) - (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Lo
g[F])/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))]*(e + I*b*c*Log[F]))/(3*e^
2*f^2)
```

Rubi [A] time = 0.0997605, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4456, 4448, 4450}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(e+ibc\log(F))\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};ie^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\csc^2\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)F^{c(a+bx)}}{6e^2f^2}-\frac{\cot\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```



```
[Out] -(F^(c*(a + b*x))*Cot[d/2 + Pi/4 + (e*x)/2]*Csc[d/2 + Pi/4 + (e*x)/2]^2)/(6
*e*f^2) - (b*c*F^(c*(a + b*x))*Csc[d/2 + Pi/4 + (e*x)/2]^2*Log[F])/(6*e^2*f
^2) - (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Lo
g[F])/e, 2 - (I*b*c*Log[F])/e, I*E^(I*(d + e*x))]*(e + I*b*c*Log[F]))/(3*e^
2*f^2)
```


Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\sin(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}-\frac{\pi}{4}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2} - \frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2} + \frac{\left(1+\frac{b}{}\right.}{}$$

$$= -\frac{F^{c(a+bx)}\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)}{6ef^2} - \frac{bcF^{c(a+bx)}\csc^2\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\log(F)}{6e^2f^2} - \frac{2e^{i(d+e}}{}$$

Mathematica [A] time = 3.23028, size = 240, normalized size = 1.3

$$\frac{F^{c(a+bx)}\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)\left((-1+i)\left(b^2c^2\log^2(F)+e^2\right)\left(\sin\left(\frac{1}{2}(d+ex)\right)+\cos\left(\frac{1}{2}(d+ex)\right)\right)^3\left(1-(1-i)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Sin[d + e*x])^2,x]
```

```
[Out] (F^(c*(a + b*x))*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*(2*e^2*Sin[(d + e*x)
/2] - e*(e + b*c*Log[F])*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + 2*(e^2 + b
^2*c^2*Log[F]^2)*Sin[(d + e*x)/2]*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2 -
 (1 - I)*(1 - (1 - I)*Hypergeometric2F1[1, ((-I)*b*c*Log[F])/e, 1 - (I*b*c*
```

```
Log[F])/e, I*Cos[d + e*x] - Sin[d + e*x]])*(e^2 + b^2*c^2*Log[F]^2)*(Cos[(d
 + e*x)/2] + Sin[(d + e*x)/2])^3))/(3*e^3*f^2*(1 + Sin[d + e*x])^2)
```

Maple [F] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{\left(f + f\sin\left(ex+d\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{F^{bcx+ac}}{f^2\cos\left(ex+d\right)^2 - 2\,f^2\sin\left(ex+d\right) - 2\,f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(-F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 - 2*f^2*sin(e*x + d) - 2*f^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\sin^2(d+ex)+2\sin(d+ex)+1}\,dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*sin(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(sin(d + e*x)**2 + 2*sin(d + e*x) + 1), x)/f**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{\left(f\sin(ex+d)+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*sin(e*x + d) + f)^2, x)
```

3.139 $\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx$

Optimal. Leaf size=245

$$\frac{2ef^2\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf^2\log(F)\cos^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}$$

```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + (2*b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x
]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F
]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x]^2*Log
[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sin[d + e*x])/(e
^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x]
)/(4*e^2 + b^2*c^2*Log[F]^2)
```

Rubi [A] time = 0.326041, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6741, 12, 6742, 2194, 4433, 4435}

$$\frac{2ef^2\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf^2\log(F)\cos^2(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}+\frac{2bcf^2\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{2ef^2\sin(d+ex)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```


```
[Out] (f^2*F^(a*c + b*c*x))/(b*c*Log[F]) + (2*b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x
]*Log[F])/(e^2 + b^2*c^2*Log[F]^2) + (2*e^2*f^2*F^(a*c + b*c*x))/(b*c*Log[F
]*(4*e^2 + b^2*c^2*Log[F]^2)) + (b*c*f^2*F^(a*c + b*c*x)*Cos[d + e*x]^2*Log
[F])/(4*e^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Sin[d + e*x])/(e
^2 + b^2*c^2*Log[F]^2) + (2*e*f^2*F^(a*c + b*c*x)*Cos[d + e*x]*Sin[d + e*x]
)/(4*e^2 + b^2*c^2*Log[F]^2)
```


Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int F^{c(a+bx)}(f+f\cos(d+ex))^2\,dx &= \int f^2F^{ac+bcx}(1+\cos(d+ex))^2\,dx \\
&= f^2\int F^{ac+bcx}(1+\cos(d+ex))^2\,dx \\
&= f^2\int \left(F^{ac+bcx}+2F^{ac+bcx}\cos(d+ex)+F^{ac+bcx}\cos^2(d+ex)\right)dx \\
&= f^2\int F^{ac+bcx}\,dx+f^2\int F^{ac+bcx}\cos^2(d+ex)\,dx+\left(2f^2\right)\int F^{ac+bcx}\cos(d+ex)\,dx \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{bcf^2F^{ac+bcx}\cos^2(d+ex)\log(F)}{4e^2+b^2c^2\log^2(F)}+\frac{2e\ldots}{\ldots} \\
&= \frac{f^2F^{ac+bcx}}{bc\log(F)}+\frac{2bcf^2F^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{2e^2f^2F^{ac+bcx}}{bc\log(F)\left(4e^2+b^2c^2\log^2(F)\right)}+\frac{bcf^2\ldots}{\ldots}
\end{aligned}$$

Mathematica [A] time = 0.629182, size = 228, normalized size = 0.93

$$\frac{f^2F^{c(a+bx)}\left(b^2c^2\log^2(F)\cos(2(d+ex))\left(b^2c^2\log^2(F)+e^2\right)+4b^2c^2\log^2(F)\cos(d+ex)\left(b^2c^2\log^2(F)+4e^2\right)+4b^3c^3e\log^3}{2\left(5b^3c^3e^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (f^2*F^(c*(a + b*x))*(12*e^4 + 15*b^2*c^2*e^2*Log[F]^2 + 3*b^4*c^4*Log[F]^4
 + b^2*c^2*Cos[2*(d + e*x)]*Log[F]^2*(e^2 + b^2*c^2*Log[F]^2) + 4*b^2*c^2*C
os[d + e*x]*Log[F]^2*(4*e^2 + b^2*c^2*Log[F]^2) + 16*b*c*e^3*Log[F]*Sin[d +
 e*x] + 4*b^3*c^3*e*Log[F]^3*Sin[d + e*x] + 2*b*c*e^3*Log[F]*Sin[2*(d + e*x
)] + 2*b^3*c^3*e*Log[F]^3*Sin[2*(d + e*x)]))/(2*(4*b*c*e^4*Log[F] + 5*b^3*c
^3*e^2*Log[F]^3 + b^5*c^5*Log[F]^5))
```

Maple [A] time = 0.076, size = 371, normalized size = 1.5

$$\frac{3F^{ac}f^2F^{bcx}}{2bc\ln(F)}+4\frac{F^{ac}f^2e\mathrm{e}^{bcx\ln(F)}\tan(d/2+1/2\,ex)}{\left(1+(\tan(d/2+1/2\,ex))^2\right)\left(e^2+b^2c^2(\ln(F))^2\right)}+2\frac{F^{ac}f^2\ln(F)\,bc\mathrm{e}^{bcx\ln(F)}}{\left(1+(\tan(d/2+1/2\,ex))^2\right)\left(e^2+b^2c^2(\ln(F))^2\right)}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x)
```

```
[Out] 3/2*F^(a*c)*f^2/b/c/ln(F)*F^(b*c*x)+4*F^(a*c)*f^2/(1+tan(1/2*d+1/2*e*x)^2)/
(e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(1/2*d+1/2*e*x)+2*F^(a*c)*f^2/(
1+tan(1/2*d+1/2*e*x)^2)*ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(b*c*x*ln(F))-2*
F^(a*c)*f^2/(1+tan(1/2*d+1/2*e*x)^2)*ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(b*
c*x*ln(F))*tan(1/2*d+1/2*e*x)^2+1/2*F^(a*c)*f^2/(1+tan(e*x+d)^2)/(4*e^2+b^2
*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))+2*F^(a*c)*f^2/(1+tan(e*x+d)^2)/(4*
e^2+b^2*c^2*ln(F)^2)*e*exp(b*c*x*ln(F))*tan(e*x+d)-1/2*F^(a*c)*f^2/(1+tan(e
*x+d)^2)/(4*e^2+b^2*c^2*ln(F)^2)*ln(F)*b*c*exp(b*c*x*ln(F))*tan(e*x+d)^2
```

Maxima [B] time = 1.28771, size = 780, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 + 2*F^(a*c)*b*c*e*log(F)*sin(2*d))*
F^(b*c*x)*cos(2*e*x) + (F^(a*c)*b^2*c^2*cos(2*d)*log(F)^2 - 2*F^(a*c)*b*c*e
*log(F)*sin(2*d))*F^(b*c*x)*cos(2*e*x + 4*d) - (F^(a*c)*b^2*c^2*log(F)^2*si
n(2*d) - 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2*e*x) + (F^(a*c)*b
^2*c^2*log(F)^2*sin(2*d) + 2*F^(a*c)*b*c*e*cos(2*d)*log(F))*F^(b*c*x)*sin(2
*e*x + 4*d) + 2*(F^(a*c)*b^2*c^2*cos(2*d)^2*log(F)^2 + F^(a*c)*b^2*c^2*log(
F)^2*sin(2*d)^2 + 4*(F^(a*c)*cos(2*d)^2 + F^(a*c)*sin(2*d)^2)*e^2)*F^(b*c*x
))*f^2/(b^3*c^3*cos(2*d)^2*log(F)^3 + b^3*c^3*log(F)^3*sin(2*d)^2 + 4*(b*c*
cos(2*d)^2*log(F) + b*c*log(F)*sin(2*d)^2)*e^2) + ((F^(a*c)*b*c*cos(d)*log(
F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d) + (F^(a*c)*b*c*cos(d)*log(F
) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^(a*c)*b*c*log(F)*sin(d) + F^(
a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(a*c)*b*c*log(F)*sin(d) - F^(a
*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f^2/(b^2*c^2*cos(d)^2*log(F)^2 + b^2*c^2*
log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) + F^(b*c*x + a*c)*f^2/(b*c*l
og(F))
```

Fricas [A] time = 0.499445, size = 536, normalized size = 2.19

$$\frac{\left(6 e^4 f^2 + \left(b^4 c^4 f^2 \cos(ex+d)^2 + 2 b^4 c^4 f^2 \cos(ex+d) + b^4 c^4 f^2\right) \log(F)^4 + \left(b^2 c^2 e^2 f^2 \cos(ex+d)^2 + 8 b^2 c^2 e^2 f^2 \cos(ex+\right.}{b^5 c^5 \log(F)^5 + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] (6*e^4*f^2 + (b^4*c^4*f^2*cos(e*x + d)^2 + 2*b^4*c^4*f^2*cos(e*x + d) + b^4
*c^4*f^2)*log(F)^4 + (b^2*c^2*e^2*f^2*cos(e*x + d)^2 + 8*b^2*c^2*e^2*f^2*co
s(e*x + d) + 7*b^2*c^2*e^2*f^2)*log(F)^2 + 2*((b^3*c^3*e*f^2*cos(e*x + d) +
 b^3*c^3*e*f^2)*log(F)^3 + (b*c*e^3*f^2*cos(e*x + d) + 4*b*c*e^3*f^2)*log(F
))*sin(e*x + d))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5 + 5*b^3*c^3*e^2*log(F)^3
 + 4*b*c*e^4*log(F))
```

Sympy [A] time = 169.745, size = 1760, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d))**2,x)
```

```
[Out] Piecewise((f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f**2*x + f
**2*sin(d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(F, 1)), (zo
o*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*sin(d + e*x
)**2 + zoo*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e/(b*c))**(b*c*x)*si
n(d + e*x)*cos(d + e*x) + zoo*e**4*f**2*exp(-2*I*e/(b*c))**(a*c)*exp(-2*I*e
/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F, exp(-2*I*e/(b*c)))), (zoo*e**4*f**2
*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**4*f*
*2*exp(-I*e/(b*c))**(a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(
-I*e/(b*c)))), (zoo*e**4*f**2*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)
*sin(d + e*x) + zoo*e**4*f**2*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)
*cos(d + e*x), Eq(F, exp(I*e/(b*c)))), (zoo*e**4*f**2*exp(2*I*e/(b*c))**(a*
c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)**2 + zoo*e**4*f**2*exp(2*I*e/(b*c
))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*sin(d + e*x)*cos(d + e*x) + zoo*e**4*f*
*2*exp(2*I*e/(b*c))**(a*c)*exp(2*I*e/(b*c))**(b*c*x)*cos(d + e*x)**2, Eq(F,
 exp(2*I*e/(b*c)))), (F**(a*c)*(f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e
*x)**2/2 + f**2*x + f**2*sin(d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e
*x)/e), Eq(b, 0)), (f**2*x*sin(d + e*x)**2/2 + f**2*x*cos(d + e*x)**2/2 + f
**2*x + f**2*sin(d + e*x)*cos(d + e*x)/(2*e) + 2*f**2*sin(d + e*x)/e, Eq(c,
 0)), (F**(a*c)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*cos(d + e*x)**2/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c
)*F**(b*c*x)*b**4*c**4*f**2*log(F)**4*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5
*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + F**(a*c)*F**(b*c*x)*b**4*c
**4*f**2*log(F)**4/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*
c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sin(d + e
*x)*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*
e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*b**3*c**3*e*f**2*log(F)**3*sin(d + e*x
)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) +
2*F**(a*c)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*sin(d + e*x)**2/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 3*F**(a*c
)*F**(b*c*x)*b**2*c**2*e**2*f**2*log(F)**2*cos(d + e*x)**2/(b**5*c**5*log(F
)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 8*F**(a*c)*F**(b*c
*x)*b**2*c**2*e**2*f**2*log(F)**2*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**
3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 5*F**(a*c)*F**(b*c*x)*b**2*c**
2*e**2*f**2*log(F)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4
*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)
*cos(d + e*x)/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**
4*log(F)) + 8*F**(a*c)*F**(b*c*x)*b*c*e**3*f**2*log(F)*sin(d + e*x)/(b**5*c
**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c
)*F**(b*c*x)*e**4*f**2*sin(d + e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e
**2*log(F)**3 + 4*b*c*e**4*log(F)) + 2*F**(a*c)*F**(b*c*x)*e**4*f**2*cos(d
+ e*x)**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*e**2*log(F)**3 + 4*b*c*e**4*lo
g(F)) + 4*F**(a*c)*F**(b*c*x)*e**4*f**2/(b**5*c**5*log(F)**5 + 5*b**3*c**3*
```

```
e**2*log(F)**3 + 4*b*c*e**4*log(F)), True))
```

Giac [C] time = 1.38787, size = 2392, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) -
 1/2*pi*a*c + 2*x*e + 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*s
gn(F) - pi*b*c + 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 4*e)*f^2*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + 2*x*e + 2*d)/(
4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 4*e)^2))*e^(b*c*x*log(a
bs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b
*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)*log(abs(F))/(4*b^2*c^2*log
(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c +
2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + x*e + d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^
2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 2*(2*b*c*f^2*cos(1/2*pi*b*c*x
*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)*log(abs(
F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*
sgn(F) - pi*b*c - 2*e)*f^2*sin(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*
a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F
) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2*b*c*
f^2*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c
 - 2*x*e - 2*d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c - 4*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 4*e)*f^2*sin(1/2*pi*b*c*x*sgn(F)
- 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - 2*x*e - 2*d)/(4*b^2*c^2*l
og(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 4*e)^2))*e^(b*c*x*log(abs(F)) + a*
c*log(abs(F))) + 3*(2*b*c*f^2*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2
*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f^2*sin(-1/2*pi*b*c*x*sgn(F
) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2
 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1
/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn
(F) - 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b
*c*log(abs(F)) + 16*I*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c
*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(-4*I*pi*b*c*sgn
(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*I*e))*e^(b*c*x*log(abs(F)) + a*c*
log(abs(F))) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
```

```
1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(I*pi*b*c*sgn(F) - I*pi*b
*c + 2*b*c*log(abs(F)) + 2*I*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I
*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-I*pi*b*c*sg
n(F) + I*pi*b*c + 2*b*c*log(abs(F)) - 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*lo
g(abs(F))) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/
2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(I*pi*b*c*sgn(F) - I*pi*b*c
 + 2*b*c*log(abs(F)) - 2*I*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*p
i*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-I*pi*b*c*sgn(
F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(
abs(F))) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*
I*pi*a*c*sgn(F) - 1/2*I*pi*a*c - 2*I*x*e - 2*I*d)/(4*I*pi*b*c*sgn(F) - 4*I*
pi*b*c + 8*b*c*log(abs(F)) - 16*I*e) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) +
1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + 2*I*x*e + 2*I*d)/(-4*
I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*I*e))*e^(b*c*x*log(ab
s(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I
*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(F) - 2*I*pi
*b*c + 4*b*c*log(abs(F))) + 2*I*f^2*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*
c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi*b*c
+ 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*(-2*I
*f^2*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) + 2*I*f^2*e^(-1/
2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/
(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a
*c*log(abs(F)))
```

3.140 $\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx$

Optimal. Leaf size=98

$$\frac{ef\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (b*c*f*F^(a*c + b*c*x)*Cos[d + e*x]*Log[
F])/(e^2 + b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rubi [A] time = 0.148457, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6741, 12, 6742, 2194, 4433}

$$\frac{ef\sin(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{bcf\log(F)\cos(d+ex)F^{ac+bcx}}{b^2c^2\log^2(F)+e^2}+\frac{fF^{ac+bcx}}{bc\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

```
[Out] (f*F^(a*c + b*c*x))/(b*c*Log[F]) + (b*c*f*F^(a*c + b*c*x)*Cos[d + e*x]*Log[
F])/(e^2 + b^2*c^2*Log[F]^2) + (e*f*F^(a*c + b*c*x)*Sin[d + e*x])/(e^2 + b^
2*c^2*Log[F]^2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int F^{c(a+bx)}(f+f\cos(d+ex))\,dx &= \int fF^{ac+bcx}(1+\cos(d+ex))\,dx\\ &= f\int F^{ac+bcx}(1+\cos(d+ex))\,dx\\ &= f\int \left(F^{ac+bcx}+F^{ac+bcx}\cos(d+ex)\right)\,dx\\ &= f\int F^{ac+bcx}\,dx+f\int F^{ac+bcx}\cos(d+ex)\,dx\\ &= \frac{fF^{ac+bcx}}{bc\log(F)}+\frac{bcfF^{ac+bcx}\cos(d+ex)\log(F)}{e^2+b^2c^2\log^2(F)}+\frac{efF^{ac+bcx}\sin(d+ex)}{e^2+b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.235257, size = 82, normalized size = 0.84

$$\frac{fF^{c(a+bx)}\left(b^2c^2\log^2(F)\cos(d+ex)+b^2c^2\log^2(F)+bce\log(F)\sin(d+ex)+e^2\right)}{bc\log(F)\left(b^2c^2\log^2(F)+e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))*(f + f*Cos[d + e*x]),x]
```

```
[Out] (f*F^(c*(a + b*x))*(e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cos[d + e*x]*Log[F]^2
+ b*c*e*Log[F]*Sin[d + e*x]))/(b*c*Log[F]*(e^2 + b^2*c^2*Log[F]^2))
```

Maple [A] time = 0.012, size = 186, normalized size = 1.9

$$\frac{fF^{c(bx+a)}}{bc\ln(F)}+\frac{f\ln(F)\,bce^{c(bx+a)\ln(F)}}{e^2+b^2c^2(\ln(F))^2}\left(1+\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\right)^{-1}+2\,\frac{efe^{c(bx+a)\ln(F)}\tan(d/2+1/2\,ex)}{\left(1+(\tan(d/2+1/2\,ex))^2\right)\left(e^2+b^2c^2(\ln(F))^2\right)}-\frac{f\ln(F}{e^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x)
```

```
[Out] f/b/c/ln(F)*F^(c*(b*x+a))+f/(1+tan(1/2*d+1/2*e*x)^2)*ln(F)*b*c/(e^2+b^2*c^2
*ln(F)^2)*exp(c*(b*x+a)*ln(F))+2*f/(1+tan(1/2*d+1/2*e*x)^2)/(e^2+b^2*c^2*ln
(F)^2)*e*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)-f/(1+tan(1/2*d+1/2*e*x)^2)
*ln(F)*b*c/(e^2+b^2*c^2*ln(F)^2)*exp(c*(b*x+a)*ln(F))*tan(1/2*d+1/2*e*x)^2
```

Maxima [B] time = 1.13489, size = 292, normalized size = 2.98

$$\frac{\left((F^{ac}bc\cos(d)\log(F) - F^{ac}e\sin(d))F^{bcx}\cos(ex+2d) + (F^{ac}bc\cos(d)\log(F) + F^{ac}e\sin(d))F^{bcx}\cos(ex) + (F^{ac}bc\log(F)\right.}{2\left(b^2c^2\cos(d)^2\log(F)^2 + b^2c^2\log(F)^2\sin(d)^2 + \left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*((F^(a*c)*b*c*cos(d)*log(F) - F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x + 2*d
) + (F^(a*c)*b*c*cos(d)*log(F) + F^(a*c)*e*sin(d))*F^(b*c*x)*cos(e*x) + (F^
(a*c)*b*c*log(F)*sin(d) + F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x + 2*d) - (F^(
a*c)*b*c*log(F)*sin(d) - F^(a*c)*e*cos(d))*F^(b*c*x)*sin(e*x))*f/(b^2*c^2*c
os(d)^2*log(F)^2 + b^2*c^2*log(F)^2*sin(d)^2 + (cos(d)^2 + sin(d)^2)*e^2) +
 F^(b*c*x + a*c)*f/(b*c*log(F))
```

Fricas [A] time = 0.478642, size = 188, normalized size = 1.92

$$\frac{\left(bcef\log(F)\sin(ex+d) + e^2f + \left(b^2c^2f\cos(ex+d) + b^2c^2f\right)\log(F)^2\right)F^{bcx+ac}}{b^3c^3\log(F)^3 + bce^2\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="fricas")
```

```
[Out] (b*c*e*f*log(F)*sin(e*x + d) + e^2*f + (b^2*c^2*f*cos(e*x + d) + b^2*c^2*f)
*log(F)^2)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3 + b*c*e^2*log(F))
```

Sympy [A] time = 14.1783, size = 408, normalized size = 4.16

$$\begin{cases} fx + \frac{f\sin(d+ex)}{e} & \text{for } F = 1 \\ \tilde{\infty} e^2 f \left(e^{-\frac{ie}{bc}}\right)^{ac} \left(e^{-\frac{ie}{bc}}\right)^{bcx} \sin(d+ex) + \tilde{\infty} e^2 f \left(e^{-\frac{ie}{bc}}\right)^{ac} \left(e^{-\frac{ie}{bc}}\right)^{bcx} \cos(d+ex) & \text{for } F = e^{-\frac{ie}{bc}} \\ \tilde{\infty} e^2 f \left(e^{\frac{ie}{bc}}\right)^{ac} \left(e^{\frac{ie}{bc}}\right)^{bcx} \sin(d+ex) + \tilde{\infty} e^2 f \left(e^{\frac{ie}{bc}}\right)^{ac} \left(e^{\frac{ie}{bc}}\right)^{bcx} \cos(d+ex) & \text{for } F = e^{\frac{ie}{bc}} \\ F^{ac}\left(fx + \frac{f\sin(d+ex)}{e}\right) & \text{for } b = 0 \\ fx + \frac{f\sin(d+ex)}{e} & \text{for } c = 0 \\ \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2\cos(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}b^2c^2f\log(F)^2}{b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}bcef\log(F)\sin(d+ex)}{b^3c^3\log(F)^3+bce^2\log(F)} + \frac{F^{ac}F^{bcx}e^2f}{b^3c^3\log(F)^3+bce^2\log(F)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))*(f+f*cos(e*x+d)),x)
```

```
[Out] Piecewise((f*x + f*sin(d + e*x)/e, Eq(F, 1)), (zoo*e**2*f*exp(-I*e/(b*c))**
(a*c)*exp(-I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**2*f*exp(-I*e/(b*c))**(
a*c)*exp(-I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(-I*e/(b*c)))), (zoo*e
**2*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*sin(d + e*x) + zoo*e**2
*f*exp(I*e/(b*c))**(a*c)*exp(I*e/(b*c))**(b*c*x)*cos(d + e*x), Eq(F, exp(I*
e/(b*c)))), (F**(a*c)*(f*x + f*sin(d + e*x)/e), Eq(b, 0)), (f*x + f*sin(d +
 e*x)/e, Eq(c, 0)), (F**(a*c)*F**(b*c*x)*b**2*c**2*f*log(F)**2*cos(d + e*x)
/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b**2*c**2*f*
log(F)**2/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)*F**(b*c*x)*b*c
*e*f*log(F)*sin(d + e*x)/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)) + F**(a*c)
*F**(b*c*x)*e**2*f/(b**3*c**3*log(F)**3 + b*c*e**2*log(F)), True))
```

Giac [C] time = 1.25457, size = 1266, normalized size = 12.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))*(f+f*cos(e*x+d)),x, algorithm="giac")
```

```
[Out] (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*p
i*a*c + x*e + d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi
*b*c + 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c + 2*e)*f*sin(1/2*pi*b*c*x*sgn(F) -
 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c + x*e + d)/(4*b^2*c^2*log(ab
s(F))^2 + (pi*b*c*sgn(F) - pi*b*c + 2*e)^2))*e^(b*c*x*log(abs(F)) + a*c*log
```

```
(abs(F))) + (2*b*c*f*cos(1/2*pi*b*c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sg
n(F) - 1/2*pi*a*c - x*e - d)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c
*sgn(F) - pi*b*c - 2*e)^2) + (pi*b*c*sgn(F) - pi*b*c - 2*e)*f*sin(1/2*pi*b*
c*x*sgn(F) - 1/2*pi*b*c*x + 1/2*pi*a*c*sgn(F) - 1/2*pi*a*c - x*e - d)/(4*b^
2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c - 2*e)^2))*e^(b*c*x*log(abs(F
)) + a*c*log(abs(F))) + 2*(2*b*c*f*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (p
i*b*c*sgn(F) - pi*b*c)^2) - (pi*b*c*sgn(F) - pi*b*c)*f*sin(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F)
)^2 + (pi*b*c*sgn(F) - pi*b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F)))
- 1/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sg
n(F) - 1/2*I*pi*a*c + I*x*e + I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*c*
log(abs(F)) + 4*I*e) + 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c - I*x*e - I*d)/(-2*I*pi*b*c*sgn(F) + 2*I*
pi*b*c + 4*b*c*log(abs(F)) - 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))
) - 1/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c - I*x*e - I*d)/(2*I*pi*b*c*sgn(F) - 2*I*pi*b*c + 4*b*
c*log(abs(F)) - 4*I*e) + 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x -
 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c + I*x*e + I*d)/(-2*I*pi*b*c*sgn(F) + 2*
I*pi*b*c + 4*b*c*log(abs(F)) + 4*I*e))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F
))) - 1/2*I*(-2*I*f*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*
c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F))) +
 2*I*f*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1
/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))))*e^(b*c*x*lo
g(abs(F)) + a*c*log(abs(F)))
```

3.141 $\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx$

Optimal. Leaf size=79

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(bc\log(F)+ie)}$$

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/
e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(I*e + b*c*Log[F]))
```

Rubi [A] time = 0.0602341, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4457, 4451}

$$\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(bc\log(F)+ie)}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

```
[Out] (2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/
e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(I*e + b*c*Log[F]))
```

Rule 4457

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{f+f\cos(d+ex)}\,dx = \frac{\int F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{2f}$$
$$= \frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{f(ie+bc\log(F))}$$

Mathematica [A] time = 0.0501889, size = 80, normalized size = 1.01

$$-\frac{2ie^{i(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{f(e-ibc\log(F))}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x]),x]
```

```
[Out] ((-2*I)*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log
[F])/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))])/(f*(e - I*b*c*Log[F]))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{f+f\cos(ex+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{F^{bcx+ac}}{f\cos\left(ex+d\right)+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f*cos(e*x + d) + f), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cos\left(d+ex\right)+1}\,dx}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d)),x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cos(d + e*x) + 1), x)/f
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{f\cos\left(ex+d\right)+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d)),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f), x)
```

3.142 $\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx$

Optimal. Leaf size=169

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-e^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F}{6e^2f^2}$$

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*(I*e - b*c*Log[F]))/(3*e^2*f^2)
 - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a
 + b*x))*Sec[d/2 + (e*x)/2]^2*Tan[d/2 + (e*x)/2])/(6*e*f^2)
```

Rubi [A] time = 0.0966504, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4457, 4448, 4451}

$$-\frac{2e^{i(d+ex)}F^{c(a+bx)}(-bc\log(F)+ie)\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)}{3e^2f^2}-\frac{bc\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)F^{c(a+bx)}}{6e^2f^2}+\frac{\tan\left(\frac{d}{2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (-2*E^(I*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])
/e, 2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*(I*e - b*c*Log[F]))/(3*e^2*f^2)
 - (b*c*F^(c*(a + b*x))*Log[F]*Sec[d/2 + (e*x)/2]^2)/(6*e^2*f^2) + (F^(c*(a
 + b*x))*Sec[d/2 + (e*x)/2]^2*Tan[d/2 + (e*x)/2])/(6*e*f^2)
```

Rule 4457

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
```

```
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{F^{c(a+bx)}}{(f+f\cos(d+ex))^2}\,dx = \frac{\int F^{c(a+bx)}\sec^4\left(\frac{d}{2}+\frac{ex}{2}\right)dx}{4f^2}$$

$$= -\frac{bcF^{c(a+bx)}\log(F)\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{6e^2f^2} + \frac{F^{c(a+bx)}\sec^2\left(\frac{d}{2}+\frac{ex}{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{6ef^2} + \frac{\left(1+\frac{b^2c^2\log^2(F)}{e^2}\right)\int$$

$$= -\frac{2e^{i(d+ex)}F^{c(a+bx)}\,{}_2F_1\left(2,1-\frac{ibc\log(F)}{e};2-\frac{ibc\log(F)}{e};-e^{i(d+ex)}\right)(ie-bc\log(F))}{3e^2f^2} - \frac{bcF^{c(a+bx)}\log}{6}$$

Mathematica [A] time = 0.392507, size = 145, normalized size = 0.86

$$\frac{2\cos\left(\frac{1}{2}(d+ex)\right)F^{c(a+bx)}\left(4e^{i(d+ex)}\cos^3\left(\frac{1}{2}(d+ex)\right)(bc\log(F)-ie)\text{Hypergeometric2F1}\left(2,1-\frac{ibc\log(F)}{e},2-\frac{ibc\log(F)}{e},-\right.\right.}{3e^2f^2(\cos(d+ex)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[F^(c*(a + b*x))/(f + f*Cos[d + e*x])^2,x]
```

```
[Out] (2*F^(c*(a + b*x))*Cos[(d + e*x)/2]*(-(b*c*Cos[(d + e*x)/2]*Log[F]) + 4*E^(
I*(d + e*x))*Cos[(d + e*x)/2]^3*Hypergeometric2F1[2, 1 - (I*b*c*Log[F])/e,
2 - (I*b*c*Log[F])/e, -E^(I*(d + e*x))]*((-I)*e + b*c*Log[F]) + e*Sin[(d +
e*x)/2]))/(3*e^2*f^2*(1 + Cos[d + e*x])^2)
```

Maple [F] time = 0.114, size = 0, normalized size = 0.

$$\int \frac{F^{c(bx+a)}}{\left(f + f\cos\left(ex + d\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

```
[Out] int(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{F^{bcx+ac}}{f^{2}\cos\left(ex + d\right)^{2} + 2\,f^{2}\cos\left(ex + d\right) + f^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)/(f^2*cos(e*x + d)^2 + 2*f^2*cos(e*x + d) + f^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{F^{ac}F^{bcx}}{\cos^{2}\left(d+ex\right)+2\cos\left(d+ex\right)+1}\, dx}{f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F**(c*(b*x+a))/(f+f*cos(e*x+d))**2,x)
```

```
[Out] Integral(F**(a*c)*F**(b*c*x)/(cos(d + e*x)**2 + 2*cos(d + e*x) + 1), x)/f**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{F^{(bx+a)c}}{\left(f\cos\left(ex+d\right)+f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F^(c*(b*x+a))/(f+f*cos(e*x+d))^2,x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)/(f*cos(e*x + d) + f)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```